Supplementary Materials for Online Publication:
Labor Coercion and the Accumulation of Human Capital

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This supplemental appendix develops a general equilibrium model of labor market coercion of unskilled workers, public provision of education, and workers' endogenous human capital accumulation decisions in a small open economy based on the material presented in Bobonis and Morrow (2011).

Section A describes a model without coercion in which potential students weigh the return from schooling against the opportunity cost of not working as an unskilled worker during matriculation. In addition, government expenditure directly affects the quality of education. If a commodity boom affects both relative wages and government expenditure on schooling through land tax revenue, equilibrium educational attainment is determined by the interaction of these factors that affect the demand for and supply of education, respectively. We explicitly examine the case in which rising coffee prices lead to a lower skill premium and a lower level of educational attainment consistent with the demand side dominating.

Section B extends the analysis to a coercive regime in which the government can devote resources toward reducing the wage paid to unskilled laborers, thus increasing the skill premium. In response to a given increase in land tax revenue, educational capital increases less than in the non-coercive regime as the elite controlled government devotes some resources to coercive efforts. Educational attainment falls by less (or increases) in response to rising coffee prices during the coercive period than in the non-coercive period due to a higher skill premium and a greater incentive to invest in human capital despite lesser schooling expenditure by the government.

Section C shows that, despite the possibility of agents accumulating more human capital, both unskilled and skilled workers are worse off with coercion than without. This is due to coercion avoidance behavior in which more workers become skilled, but the provision of educational capital by the government falls relative to the non-coercive case leading to a diminished effectiveness of education. We also show that it is possible for the elite to be better off under coercion as they receive wages that are coerced away from unskilled workers. Proofs and details for calibration exercises are relegated to a technical appendix.

A. Model of Public Education Provision and Skills Accumulation without Coercion

The three factors of production are unskilled labor (U), skilled labor (S), and land (L) with factor prices w_u, w_s, and r, respectively. We include the two types of labor to discretize the human capital accumulation decision. Land is included because of the importance of land rents in government tax revenue from which education and coercion were financed.[3] While total population size is exogenous and fixed, its composition between unskilled and skilled labor is determined endogenously. The stock of land is exogenous and fixed.

[3] The budget data collected by the authors indicates that property taxes were the main source of local government revenue.

We follow Findlay and Kierzkowski (1983) in examining the endogeneity of the stocks of skilled and unskilled labor in an overlapping generations model. Individuals are born each period t and live for two periods. We define a household as two generations co-existing at a given point in time: one youth (0) and one adult (1). In the following period, the adult dies, the youth becomes an adult, and a new youth is born. $N_{0,t}$ is the total number of youth and $N_{1,t} \equiv N_{0,t-1}$ is the total number of adults in each period t such that the total population at any period is $N_t \equiv N_{0,t} + N_{1,t}$. The total number of unskilled workers is equal to the number of unskilled youth and unskilled adults: $U_t = U_{0,t} + U_{1,t}$. The number of skilled workers is equal to the number of adults who invested in human capital as youth $S_t = E_{1,t}$. Finally, $E_{0,t}$ is the number of students such that the total population is $N_t = U_t + S_t + E_{0,t}$.

Determination of Factor Prices

Motivated by the fact that Puerto Rico is a small open economy, we assume for simplicity that there are three goods, one of which is coffee, whose prices are set exogenously on world markets. With an equal number of factors as goods, no factor intensity reversals, and all goods being produced, exogenous goods prices determine factor prices that are insensitive to domestic factor endowments. We refer to these pinned down factor prices (w_s,w_u,r) as 'shadow wages' for the entirety of this document. The technical appendix derives explicit conditions under which an increase in coffee prices leads to increase in both land rents and the relative wage of unskilled labor and we assume that this is the case. We do not do this to generate a specific theoretical result but rather to 'match the moments' in which a rise in coffee prices led to increases in the wages of unskilled labor and land rents during the coffee boom in Puerto Rico during this time, based on historical accounts (Bergad 1983).[4]

Provision of Public Education

An education sector combines students $E_{0,t}$ with educational capital K_t (e.g., schools) to produce skilled workers. The total flow of skills Q_t is described by the following constant returns to scale production function

$$Q_t = F(K_t, E_{0,t}), \tag{1}$$

which is increasing in each of its arguments with diminishing marginal products and complementarity between factors. The effectiveness of a worker, as measured by skill per worker, is denoted by $q_t = Q_t/E_{0,t} = f(k_t)$ where $k_t = K_t/E_{0,t}$. The wage that each physical skilled worker earns as an adult at time t is

[4] Some of the model's complexity arises because there are three goods and three factors. A more tractable model with two goods and two factors would clearly deliver the demand side responses in the presence and absence of labor coercion, but would obscure local elites and their capabilities to engage in the enforcement of labor coercion. In addition, if taxes came from skilled workers, this model would have the counterfactual implication that tax revenue fell with coffee prices increasing as skilled wages must fall if unskilled wages rise in a two-factor two –good model (Jones and Scheinkman, 1977).

comprised of the wage per effective skilled worker ($w_{s,t}$) and the number of effective skilled workers each physical skilled worker comprises [$f(k_{t-1})$]. Educational capital depreciates fully each period.

The elite controlled government maximizes the sum of landowners' utility and skilled workers' altruistic utility, which are both linear in income given an exogenous linear tax rate on land, τ. This is motivated by the fact that landowners and skilled workers were given the right to vote and they comprised the entirety of the legislative body. Consequently the government maximizes the following function with respect to educational capital (K_t) at time t subject to the constraint that spending can not exceed tax revenue $\tau r_t L$, where β is the exogenous discount factor and $E_t[\bullet]$ is the expectations operator:

$$(1-\tau)r_t L + w_{s,t} f(k_{t-1})E_{1,t} + \beta E_t[w_{s,t+1} f(k_t)E_{0,t}]. \tag{2}$$

Trivially, this leads to a solution in which all tax revenue is spent on educational capital:

$$K_t = \tau r_t L. \tag{3}$$

The Human Capital Accumulation Decision

The altruistic adult in the household chooses the youth's education level to maximize the youth's lifetime income. The choice is discrete – the youth either attends school or does not. If the youth does not go to school, they earn the unskilled wage for two periods. If the child attends school, they attend school in the first period and work in the second period earning the going wage for skilled labor. An education can only be obtained during youth, as workers never enter school in the final period of life because they die before returns are realized. The key tradeoff for households' human capital accumulation decision is that education brings a higher but deferred wage. At the time of birth, the present discounted value of the stream of payments to a skilled worker is $\beta E_t[w_{s,t+1} f(k_t)]$. Because education is a public good, its only cost is the stream of forgone unskilled wages $w_{u,t} + \beta E_t[w_{u,t+1}]$. We assume that goods prices, and therefore factor prices, follow a random walk such that the current wage is each individual's best guess of the wage that will prevail when they are adults.[5] In equilibrium, the discounted stream of payments to a skilled worker equals the discounted stream of forgone unskilled wages. This leads to the following *educational indifference condition*:

$$f(k_t) = \frac{w_{u,t}}{w_{s,t}}\left[\frac{1+\beta}{\beta}\right]. \tag{4}$$

[5] All agents are risk-neutral such that uncertainty plays no role. In addition, we have studied the time path of coffee prices during the period and find that they are well represented by a random walk – the point estimate on a model with multiple lags is 0.996 (standard error = 0.167).

For a given set of factor prices, the tax revenue and equilibrium skill per worker conditions [equations (3) and (4)] determine equilibrium values of K_t and k_t which determine the equilibrium level of $E_{0,t}$, and, subsequently $E_{1,t+1}$.

The Response of Education to a Change in the Price of Coffee without Coercion

We now consider the impact of an increase in coffee prices on educational attainment and educational capital spending as determined by an increase in both land rents and the relative wage of unskilled labor. Based on equation (3), educational capital provision rises ($K \uparrow$) in response to increasing land tax revenue. From equation (4), equilibrium educational capital per student rises ($k \uparrow$) as the unskilled-skilled wage gap narrows. The equilibrium change in E is ambiguous. On one hand, a larger quantity of educational capital increases the attractiveness of education for a given set of wages. However, as the relative value of remaining unskilled increases ($w_u/w_s\uparrow$), households demand relatively less education. We can see this by taking proportional changes of equations (3) and (4), suppressing time subscripts, and combining them to deliver:

$$\hat{E} = \hat{r} - \frac{1}{\varepsilon_{f(k),k}}\left[\hat{w}_u - \hat{w}_s\right] \tag{5}$$

where hats represent proportional changes and $\varepsilon_{f(k),k}>0$ is the elasticity of $f(k)$ with respect to k.

This expression is useful as it decomposes changes in educational attainment into 'supply-side' and 'demand-side' effects. On the supply side, higher land rents increase tax revenue and the provision of educational capital, increasing the effectiveness of an education. On the demand side, the incentive to acquire an education declines as the skill premium falls. If the latter effect dominates, this provides a simple model of how increased coffee prices can lead to falling educational attainment. As this is consistent with contemporary evidence (e.g., Soares, Kruger, and Berthelon 2006; Kruger 2007), and with our empirical evidence of falling educational attainment in response to rising coffee prices in the non-coercive regime, we maintain the assumption of the demand side dominating in the non-coercive case.

B. Public Education Provision and Skills Accumulation with Endogenous Coercion

We now introduce the possibility that tax revenue can also be allocated towards the enforcement of coercive labor market regulations through which unskilled workers are paid below-market wages. The government now spends its land tax revenue on some combination of educational (K) and coercive (V) capital subject to the budget constraint $\tau r_t L = V_t + K_t$.

Motivated by the discussion in Section II.B of the main text, we assume that unskilled laborers can work for a landowner under a coercive contract, or attempt to breach this contract and work in some household production at the market-clearing wage. Given an attempt at breaching the contract, the laborer is caught with probability $\pi(V)$ in which case they are imprisoned for a period with a payoff that we normalize to zero. With probability $1 - \pi(V)$, such an attempt is successful, in which case the worker earns the non-coercive wage w_u. Greater coercive capital expenditures increase the probability that laborers in breach of the law are caught, but this is subject to decreasing marginal effectiveness $[\pi'(V) > 0, \pi''(V) < 0]$. Coercive capital fully depreciates at the end of each time period. Because unskilled workers are necessary in the production process of all goods produced, they are paid a wage that is incentive compatible for them to supply labor $w_c \equiv [1 - \pi(V)]w_u$.[6] Consequently, the educational indifference condition becomes:

$$f(k_t) = \frac{[1-\pi(V_t)]w_{u,t}}{w_{s,t}}\left[\frac{1+\beta}{\beta}\right]. \tag{6}$$

Analogous to the case without coercion, the government maximizes the utility of landowners and the altruistic utility of skilled workers:[7]

$$(1-\tau)r_tL + \pi(V_t)w_{u,t}U_{1,t} + w_{s,t}f(k_{t-1})E_{1,t} + \beta E_t[w_{s,t+1}f(k_t)E_{0,t}]. \tag{7}$$

The first term reflects landowners' after-tax income. The second term reflects the value of coerced adult unskilled income that is reallocated to landowners. The last two terms are components of the utility of the altruistic adult skilled workers: the wage income of adult skilled workers and the discounted anticipated income of youth who attend school in the current period.[8] This objective function is maximized with respect to current coercive (V_t) and educational (K_t) expenditures subject to the budget constraint $\tau r_tL = V_t + K_t$.[9]

Maximizing the constrained objective function yields a first order condition relating expenditures on educational and coercive capital where we suppress time subscripts:

$$\pi'(V)w_uU = \beta w_sf'(k), \tag{8}$$

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[6] More generally, any coercive wage $w_c(w_u,V)$ that satisfies $\partial w_c(w_u,V)/\partial w_u > 0$, $\partial w_c(w_u,V)/V < 0$, and $\partial w_c^2(w_u,V)/\partial V\partial w_u < 0$ yields identical results. This cross-partial restriction captures the idea that the marginal return to coercion for the elites is increasing in the shadow wage of unskilled labor, consistent with Domar (1970).

[7] We depart from median voter-determined policy models because they have stark and counterfactual implications. For example, if the median voter is a landowner all tax revenue will go towards coercive activity. Allowing for the government objective function to consist of a weighted average of social welfare and income to municipal council members (à la Grossman and Helpman (1994)) delivers nearly identical results with minimal restrictions. The model also abstracts from another mechanism for under-investments in education – in order to restrict the electoral franchise (Bourguignon and Verdier 2000; Galor, Moav, and Vollrath 2009). Although this mechanism may be at play, the short-run price variations should not induce large incentives for 21 year later franchise adjustments based on a voting age of 21.

[8] Even if allowed, there will be no coercion of skilled workers because coercion reallocates income to landowners from skilled laborers. However, as income is reallocated from one group of voters to another there will be no gain in the objective function but there will be a positive opportunity cost in that they are not providing educational capital nor coercing unskilled workers.

[9] We assume that skilled laborers live in the same households as students for simplicity given that we examine a stationary equilibrium. Assuming that skilled parents get (higher) utility value from having skilled children would satisfy this. Because our model does not predict *who* will obtain an education, we do this to avoid cases where some unskilled youth live with skilled parents such that some skilled parents have no incentive to provide educational capital.

such that the marginal return to coercion equals the discounted marginal return to educational capital. A stationary equilibrium where $E_{1,t} = E_{1,t+1}$ can be solved recursively given that exogenous world prices determine a unique vector of equilibrium factor prices. The technical appendix discusses the existence, uniqueness, and stability of the stationary equilibrium.

The Response of Education to a Change in the Price of Coffee with Coercion

Combining the new educational indifference condition [equation (6)] and the new government budget constraint, we can derive the response of educational attainment to a change in coffee prices under the coercive regime:

$$\widehat{E} = \underbrace{\left(\frac{\tau r L}{K}\right)\widehat{r} - \frac{1}{\varepsilon_{f(k),k}}\left(\widehat{w}_u - \widehat{w}_s\right)}_{A} + \underbrace{\frac{V}{K}\left[\underbrace{\left(\frac{K}{V}\right)\left(\frac{-\varepsilon_{(1-\pi(V)),V}}{\varepsilon_{f(k),k}}\right)}_{c.a.} - \underbrace{1}_{e.q.}\right]\widehat{V}}_{B}, \tag{9}$$

where $\varepsilon_{(1-\pi(V)),V}<0$ is the elasticity of the coercive wage with respect to coercion holding the shadow wage constant. Although V is still an endogenous variable, equation (9) is illustrative. The first term (A) is nearly identical to the no-coercion case. Educational attainment is increasing in the land rental rate as tax revenue allows for more educational capital but is declining in the relative wage of unskilled labor as the opportunity cost of matriculation rises.

The second term (B) represents two conflicting forces that determine how educational attainment responds to increased coercion. The first part (*c.a.*) is a *coercion avoidance* effect through which, as coercion increases, the payoff to being an unskilled laborer diminishes relative to becoming a skilled worker. This increases the incentive to obtain an education, *ceteris paribus*. However, the *educational quality* effect (*e.q.*) states that every extra peso spent on coercion necessarily reduces expenditures on educational capital and thereby diminishes the effectiveness of and the incentive to obtain an education. The overall effect of increased coercion on educational attainment is based on the sum of these two effects.

Although the net effect is ambiguous in theory, we calibrate this expression using available data for the period to develop a prior about the net effect of increased coercion on educational attainment. The technical appendix discusses this calibration in detail and shows that, for reasonable parameter values based on historical data for this episode, the coercion avoidance effect dominates such that increased coercion leads to greater educational attainment. We assume that this holds for the entirety of the section while emphasizing that the educational quality effect might dominate in other cases.

To eliminate the endogenous variable V, we combine equations (8) and (9) to obtain the following expression:

$$\hat{E} = \underbrace{\frac{1}{\kappa}\left[\left(\frac{\tau r L}{K}\right)\hat{r} - \frac{1}{\varepsilon_{f(k),k}}\left(\hat{w}_u - \hat{w}_s\right)\right]}_{A} + \underbrace{\alpha_0 \hat{r}}_{B} + \underbrace{\alpha_1\left(\hat{w}_u - \hat{w}_s\right)}_{C} \tag{10}$$

where κ, α_0, and α_1 are all positive constants and $\kappa > 1$. The technical appendix discusses the structural composition of these coefficients in detail.

Relative to the no-coercion case, coercion leads to the presence of the additional additive terms (*B*) and (*C*). Term (*B*) represents an 'income effect' in which increased land tax revenue leads to greater coercion and increased human capital accumulation to avoid this coercion. Term (*C*) represents how an increased relative wage of unskilled labor provides greater incentive for government coercion and the increased efforts to avoid it. The final term is consistent with Domar (1970) who argues that increased wages for unskilled workers lead to greater returns to slavery and serfdom through cost savings. Both (*B*) and (*C*) lead to educational attainment falling by less or rising in response to an increase in coffee prices relative to the non-coercion case.

For a given set shadow wages, government provision of educational capital (*K*) is lower under coercion than without. This is because all additional tax revenue is spent on educational capital in the former but some is spent on coercion in the latter. Whether the absolute level of educational capital *K* increases or falls under coercion in response to higher coffee prices is generally ambiguous. The technical appendix discusses this in detail.

C. *Welfare as Measured by Lifetime Income under Non-Coercion and Coercion*

Using the structure of the model described above, we now show that unskilled workers are *not* better off under coercion even if educational attainment (*E*) increases. To do this, we compare the present discounted value of unskilled and skilled workers' wages in the coercive and non-coercive regimes, and then the lifetime income of the elite, the latter of which comes from land income, redistributed wages from unskilled workers, and skilled wages.

Ignoring uncertainty and suppressing time subscripts, the present discounted value of lifetime income for an unskilled worker in the non-coercive regime is $(1+\beta)w_u$. In the coercive regime, their lifetime income is $(1+\beta)[1-\pi(V)]w_u$. Consequently, for a given set of shadow wages, the relative lifetime income for an unskilled worker in the coercive relative to the non-coercive regime is $1-\pi(V)<1$. Therefore unskilled workers are worse off under coercion.

To see the effect of coercion on the lifetime income of skilled workers, start by defining educational capital per worker in the non-coercive and coercive periods as k^{NC} and k^{C}, respectively. The present discounted value of lifetime income for an skilled worker at birth in the non-coercive regime is $\beta w_s f(k^{NC})$. In the coercive regime, this present discounted value is $\beta w_s f(k^{C})$. Using the educational indifference conditions [equations (4) and (6)] for a given set of shadow wages, the relative lifetime income of a skilled worker under the coercive relative to the non-coercive regime is

$$\frac{f\left(k^{C}\right)}{f\left(k^{NC}\right)} = 1 - \pi\left(V\right) < 1 .$$

Consequently, skilled workers are also worse off under coercion. The reason for this is that free entry into education requires equal utility from being unskilled or skilled. More intuitively, because *f(k)* is increasing in *k*, there is less educational capital per worker with coercion than in the non-coercive regime ($k^{C}<k^{NC}$). Workers increasingly enroll in schools that receive less funding (a lower *K* under coercion than without), driving down the effectiveness of schooling for skilled workers relative to its non-coercive level.[10] Therefore, both skilled and unskilled workers are worse off under coercion than non-coercion, even though the latter can engage in avoidance behavior and accumulate human capital to avoid coercion.

If unskilled and skilled workers each possessed lower lifetime wages under the coercion than under the non-coercive regime, why was this policy undertaken? We can answer this by looking at the value of the elite controlled government's objective function with and without coercion. For a given set of shadow factor prices, we subtract the elite's objective function under coercion [equation (7)] from its non-coercive analog [equation (2)], impose a stationary equilibrium in which $E_{0,t}= E_{1,t}$ for each regime, and suppress time subscripts and uncertainty to obtain:

$$\pi\left(V\right)w_{u}U^{C}+\left(1+\beta\right)w_{s}\left[f\left(k^{C}\right)E^{C}-f\left(k^{NC}\right)E^{NC}\right]$$

where superscripts *C* and *NC* label values of endogenous variables with and without coercion, respectively. Using the education production function [equation (1)], we obtain:

$$\pi\left(V_{t}\right)w_{u,t}U_{1,t}^{C}+\left(1+\beta\right)w_{s,t}\left[Q_{t}^{C}-Q_{t}^{NC}\right]$$

where *Q* is the total flow of skills. The first term above represents the total gain to landowners from the introduction of a wedge between unskilled workers' shadow wage and the coerced wage. This is strictly positive. The second term represents the change in the elite's welfare coming from changes in total skilled income.[11] Notice that the sign of this second term is ambiguous and depends on whether *total* skills increase or fall under coercion relative to no-coercion. While effectiveness of schooling declines as $k^{C}<k^{NC}$, the magnitude of the change in the equilibrium number of skilled workers depends on functional form assumptions. If $Q^{C}>Q^{NC}$, it is unambiguous that the elite prefer the coercive regime. If $Q^{C}<Q^{NC}$, this is consistent with a observed preference for this policy by the elite as long as the first term is more positive than the second term is negative.[12]

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[10] It is interesting that this is consistent with the result that there were fewer schools per student during the coercive regime.
[11] This is not the same as the change in the wage per skilled worker because equilibrium educational attainment (*E*) changes.
[12] Numerical simulations are available upon request that show that both cases are possible.

Main Elements of the Model

Element 1. In a non-coercive regime, a higher price of coffee increases the provision of education ($K\uparrow$). Equilibrium educational attainment falls ($E\downarrow$), as demand for education falls more than supply rises.

Element 2. In a coercive regime, the provision of education may increase or decrease ($K\uparrow\downarrow$) in response to an increase in coffee prices. This response will be strictly less than in the case without coercion as some government expenditure is allocated to coercion. The change in equilibrium educational attainment will be *strictly greater than* in the case without coercion.

Element 3. Because the increased accumulation of human capital under the coercive regime results from avoidance behavior, unskilled workers are worse off despite increased accumulation of human capital. In equilibrium, agents are indifferent about whether they are skilled or unskilled. A lower value of being an unskilled worker equates with a lower present discounted value of being skilled worker due to a lower equilibrium amount of schooling capital per student.

Element 4. While landowners do not see the value of their land holdings increase under coercion, their welfare is greater due to the rents extracted from unskilled workers' wages.

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Theoretical Appendix

Determination of Factor Prices

As is well known (e.g., Ethier, 1983) the Stolper-Samuelson theorem loses a great deal of intuitive content and simplicity when there are more than two goods and two factors. Consequently, we restrict the cost share parameter space such that an increase in the price of coffee will lead to an increase in the wage of unskilled labor, an increase in the rental rate of land, and a fall in the wage of skilled labor. As stated in the text, we do so not to generate a specific theoretical result but rather to "match the moments" in which a rise in coffee prices led to increases in the wages of unskilled labor and land rents during the coffee boom in Puerto Rico during this time, based on historical accounts (Bergad 1983).

We assume that there are three goods: coffee (*c*), food (*f*), and tobacco (*b*). Technically, with zero profits, an equal number of factors and goods, and no factor intensity reversals, the following three zero profit equations uniquely determine equilibrium factor prices as a function of international goods prices where we take tobacco as the numeraire good and suppress time subscripts for convenience and assume production of all goods:

$$\begin{aligned} p_c &= a_{sc}w_s + a_{uc}w_u + a_{lc}r, \\ p_f &= a_{sf}w_s + a_{uf}w_u + a_{lf}r, \\ 1 &= a_{sb}w_s + a_{ub}w_u + a_{lb}r, \end{aligned}$$

where a_{ij} is the unit input requirement of factor $i \in \{s,u,l\}$ in production of good $j \in \{c,f,b\}$. Taking proportional changes of the system of zero profit conditions and expressing it in matrix notation yields

$$\begin{pmatrix} \hat{p}_c \\ \hat{p}_f \\ 0 \end{pmatrix} = \begin{pmatrix} \theta_{sc} & \theta_{uc} & \theta_{lc} \\ \theta_{sf} & \theta_{uf} & \theta_{lf} \\ \theta_{sb} & \theta_{ub} & \theta_{lb} \end{pmatrix} \begin{pmatrix} \hat{w}_s \\ \hat{w}_u \\ \hat{r} \end{pmatrix},$$

where θ_{ij} represents the relevant cost share for factor *i* in production of good *j*. Inverting this system to obtain a matrix of Stolper-Samuelson derivatives yields

$$\begin{pmatrix} \hat{r} \\ \hat{w}_u \\ \hat{w}_s \end{pmatrix} = \frac{1}{|M|} \begin{pmatrix} \theta_{uf}\theta_{sb} - \theta_{ub}\theta_{sf} & \theta_{sc}\theta_{ub} - \theta_{uc}\theta_{sb} & \theta_{uc}\theta_{sf} - \theta_{uf}\theta_{sc} \\ \theta_{lb}\theta_{sf} - \theta_{lf}\theta_{sb} & \theta_{lc}\theta_{sb} - \theta_{lb}\theta_{sc} & \theta_{lf}\theta_{sc} - \theta_{lc}\theta_{sf} \\ \theta_{lf}\theta_{ub} - \theta_{lb}\theta_{uf} & \theta_{lb}\theta_{uc} - \theta_{lc}\theta_{ub} & \theta_{lc}\theta_{uf} - \theta_{lf}\theta_{uc} \end{pmatrix} \begin{pmatrix} \hat{p}_c \\ \hat{p}_f \\ 0 \end{pmatrix},$$

where

$$|M| = \theta_{lc}(\theta_{uf}\theta_{sb} - \theta_{ub}\theta_{sf}) + \theta_{uc}(\theta_{lb}\theta_{sf} - \theta_{lf}\theta_{sb}) + \theta_{sc}(\theta_{lf}\theta_{ub} - \theta_{lb}\theta_{uf}). \qquad \text{(A1)}$$

We now invoke the following factor intensity restrictions: $(\theta_{sf} / \theta_{uf}) > (\theta_{sb} / \theta_{ub})$, $(\theta_{sb} / \theta_{lb}) > (\theta_{sf} / \theta_{lf})$.

The first assumption states that food is skilled labor-unskilled labor intensive relative to tobacco. The second states that tobacco is skilled labor-land intensive relative to food. Note also that these assumptions ensure that the first two terms in the determinant are negative while the final term is positive. We now introduce a third assumption. We assume that the cost share for skilled labor in coffee is sufficiently small that the sign of the determinant is strictly negative. Algebraically, this results in the restriction that

$$\frac{\theta_{lc}(\theta_{uf}\theta_{sb} - \theta_{ub}\theta_{sf}) + \theta_{uc}(\theta_{lb}\theta_{sf} - \theta_{lf}\theta_{sb})}{\theta_{lf}\theta_{ub} - \theta_{lb}\theta_{uf}} > \theta_{sc}\,. \tag{A2}$$

Consequently, the shadow wage of unskilled labor and the return to land rise, while the skilled wage falls.

Uniqueness and Stability of a Stationary Coercive Equilibrium

Equilibrium will be determined by the four following expressions in conjunction with the definition of a stationary steady state:

$$\pi'(V_t)w_u U_{1,t} = f'(k_t)(E_{1,t}/E_{0,t})$$

$$f(k_t) = [(1+\beta)/\beta]*\{[(1-\pi(V_t)]*(w_u/w_s)]$$

$$K_t + V_t = \tau r L$$

$$E_{1,t} + U_{1,t} = N_{1,t}$$

$$E_{1,t} = E_{0,t}$$

We assume that there is an interior solution with positive amounts of educational and coercive capital. While we do not pursue an existence proof, It can be shown easily using the functional forms $f(k) = (k)^{\gamma}$, where $0 < \gamma < 1$, and $\pi(V_t) = [V_t/(1 + V_t)]$.

The system has four equations and five unknowns. We pursue a strategy where we show that an expression with the two unknowns $E_{1,t}$ and $E_{0,t}$ cuts through the "45 degree line" that defines $E_{1,t} = E_{0,t}$ "from above" unambiguously. This allows us to state that a stationary equilibrium is unique and stable. To do this, we show that taking a total derivative of this expression yields a slope strictly less than 1 in the neighborhood where $E_{1,t} = E_{0,t}$. To do so take total derivatives of the first four equations above in their endogenous variables (assuming that factor prices have been pinned down by international goods prices):

$$E_{0,t}\pi'(V_t)w_u dU_{1,t} + E_{0,t}\pi''(V_t)\mathrm{w_u}U_{1,t}dV_t + \pi'(V_t)w_u U_{1,t}dE_{0,t}$$
$$= f''(k_t)*(E_{1,t})k_t - f''(k_t)*(E_{1t}/(E_{0,t})^2)dE_{0,t} + f'(k_t)dE_{1,t}$$

$$f'(k_t)(1/E_{0,t})dK_t - f'(k_t)(K_t/(E_{0,t})^2)dE_{0,t} = [(1+\beta)/\beta][(1-\pi(V_t)]\mathrm{d}(w_u/w_s)] - [(1+\beta)/\beta](w_u/w_s)\pi(V_t)]dV_t$$

$$dK_t = -dV_t$$

$$dE_{1,t} = -dU_{1,t}$$

Substituting out expressions for $dU_{t,1}$ and dK and simplifying gives

$$\left[\frac{\pi''(V_t)}{\pi'(V_t)}E_{0,t} + \frac{f''(k_t)}{f'(k_t)}\right]dV_t = -\frac{f''(k_t)}{f'(k_t)}\frac{1}{E_{0,t}}dE_{0,t} - dE_{0,t} + \frac{E_{0,t}}{E_{1,t}}dE_{1,t} + \frac{E_{0,t}}{U_{1,t}}dE_{1,t}$$

$$\frac{V_t}{K_t}\left[\frac{f(k_t)\pi'(V_t)E_{0,t}}{f'(k_t)[1-\pi(V_t)]}-1\right]dV_t=\frac{V_t}{E_{0,t}}dE_{0,t}\,.$$

Combining these two expressions and evaluating them in the neighborhood where $E_{1,t} = E_{0,t}$ allows the following expression:

$$\frac{dE_{1,t}}{dE_{0,t}}=\left[\frac{U_{1,t}}{U_{1,t}+E_{0,t}}\right]\left[1+\frac{f''(k_t)}{f'(k_t)}\frac{1}{E_{0,t}}+\frac{\left[\frac{\pi''(V_t)}{\pi'(V_t)}E_{0,t}+\frac{f''(k_t)}{f'(k_t)}\right]k_t}{\left[\frac{f(k_t)\pi'(V_t)E_{0,t}}{f'(k_t)[1-\pi(V_t)]}-1\right]}\right].$$

Note that the first expression is strictly less than one. The second expression is also strictly less than one – it is the sum of one and two negative numbers. This is correct given our parameter restrictions such that the coercion avoidance effect dominates as we discuss in Appendix B.3. See Appendix B.3. for details. Consequently, an expression in $E_{1,t} = E_{0,t}$ cuts the line $E_{1,t} = E_{0,t}$ from above which proves uniqueness and stability of the stationary equilibrium.

Calibration of the Response of Education to a Change in the Price of Coffee with Coercion

To predict the response of educational attainment to a change in coffee prices with coercion, express the education indifference condition (equation (5)) and the budget constraint in proportional changes and substitute to yield

$$\widehat{E}=\left(\frac{\tau rL}{K}\right)\widehat{r}-\frac{1}{\varepsilon_{f(k),k}}\left(\widehat{w}_u-\widehat{w}_s\right)+\frac{V}{K}\left[\left(\frac{K}{V}\right)\left(\frac{-\varepsilon_{(1-\pi(V)),V}}{\varepsilon_{f(k),k}}\right)-1\right]\widehat{V}, \tag{A3}$$

where $\varepsilon_{(1-\pi(V)),V}$ represents the elasticity of post-coercion wages with respect to coercion and $\varepsilon_{f(k),k}$ the elasticity of skills per worker with respect to educational capital per worker.

We calibrate the elasticity of human capital accumulation with respect to coercive capital based on parameter values and Puerto Rican data during the period, to develop a prior about the net effect of increased coercion on educational attainment. Using data on the ratio of public school expenditures to rural/urban police expenditures, we obtain a value of V/K of 0.091. Note that for the coercion avoidance effect to dominate we need for the expression in brackets to be greater than zero.

Given that K/V is approximately 11, and we assume that $\varepsilon_{f(k),k} = 0.5$, the coercion avoidance effect will dominate if $|\varepsilon_{(1-\pi(V)),V}| > 0.045$. Even if we allowed for constant returns to scale in the education production function ($\varepsilon_{f(k),k} = 1$), we would only need that the elasticity of post-coercion wages with respect to coercion to be greater than 0.09 in absolute terms. We consider this to be an extremely small number and not a binding constraint for purposes of our analysis. Consider the case where workers keep 50 percent of their income. This would imply that a 10 percent increase in military expenditure decreases post-coercion wages from 50 to 49.1 percent. Given the effectiveness of anti-vagrancy restrictions, the elasticity of coercion with respect to coercive expenditures would be greater than this minimal magnitude.

Education Response to a Change in the Price of Coffee with Coercion: Structural Parameters

Equation (9) yields an expression for changes in educational attainment as a function of shadow factor prices:

$$\hat{E}=\frac{1}{\kappa}\left[\left(\frac{\tau rL}{K}\right)\hat{r}-\frac{1}{\varepsilon_{f(k),k}}\left(\hat{w}_u-\hat{w}_s\right)\right]+\alpha_0\hat{r}+\alpha_1\left(\hat{w}_u-\hat{w}_s\right) \tag{A4}$$

where κ, α_1, and α_2 are all positive constants with $\kappa > 1$. The detailed structural composition of these coefficients is

$$\kappa = 1 + \left(\frac{V}{K}\right)\left[\frac{-\varepsilon_{(1-\pi'(V)),V}}{\varepsilon_{f'(k),k}(V/K)} - 1\right]\left[\frac{\varepsilon_{f'(k),k} - (E/U)}{\varepsilon_{\pi'(V),V} + \varepsilon_{f'(k),k}(V/K)}\right] > 1,$$

$$\alpha_0 = \left(\frac{V}{\kappa K}\right)\left[\frac{-\varepsilon_{(1-\pi'(V)),V}}{\varepsilon_{f'(k),k}(V/K)} - 1\right]\left[\frac{\varepsilon_{f'(k),k}(\tau rL/K)}{\varepsilon_{\pi'(V),V} + \varepsilon_{f'(k),k}(V/K)}\right] > 0,$$

and

$$\alpha_1 = \left(\frac{V}{\kappa K}\right)\left[\frac{\varepsilon_{(1-\pi'(V)),V}}{\varepsilon_{f'(k),k}(V/K)} - 1\right]\left[\frac{1}{\varepsilon_{\pi'(V),V} + \varepsilon_{f'(k),k}(V/K)}\right] > 0.$$

The Response of Public Education Provision to a Change in the Price of Coffee with Coercion

Having developed a prediction on how E responds to a change in the exogenous price of coffee, we now examine the response of educational capital (K) to higher coffee prices. Noting that $\tau rL = V + K$, taking proportional changes, substituting into equation (8), and rearranging, we obtain the following:

$$\hat{K} = \left[\frac{-\varepsilon_{\pi'(V),V}\left(\tau rL/K\right)}{D}\right]\hat{r} + \frac{V}{K}\left[\frac{\left(E/U\right) - \varepsilon_{\pi f(k),k}}{D}\right]\hat{E} - \frac{V}{KD}\left(\hat{w}_u - \hat{w}_s\right)$$

where $D = -[\varepsilon_{\pi'(V),V} + \varepsilon_{f'(k),k}(V/K)] > 0$. Whether K rises or falls in response to higher coffee prices is generally ambiguous (recall that $\varepsilon_{\pi'(V),V}$, $\varepsilon_{f'(k),k} < 0$). Therefore, *ceteris paribus*, K will correlate positively with r and E but negatively with ($\hat{w}_u - \hat{w}_s$). Increases in r have a positive income effect via the government's budget constraint. In addition, the government targets spending per student (k) according to the first order condition (7) leading K and E to move in the same direction. K correlates negatively with ($\hat{w}_u - \hat{w}_s$) through a price effect in which increased unskilled wages increase the incentive for coercion and dampens the government's desire to supply K. Whether K rises or falls in response to these forces is determined by the sum of these effects and is ambiguous. However it increases less than in the case without coercion in which all additional tax revenue is spent on educational capital.

Calibration of the Response of School Provision to a Change in the Coffee Price with Coercion

An obvious question is if we need pathological parameter values to obtain an absolute fall in educational expenditures in response to an increase in coffee prices. To assess this, we simulate values of $\hat{E}$ using equation (9) and all possible combinations of factor-good cost shares that satisfy the assumptions in Proposition 1. We then generate the associated values of $\hat{K}$ using equation (10) and the same changes in factor prices. We can then estimate a relationship between $\hat{E}$ and $\hat{K}$ to observe $\hat{K}$ in the neighborhood of $\hat{E} = 0$ for differing values of $\varepsilon_{\pi'(V),V}$ and given the parameter values discussed in Appendix B.3. We find that if the coercion function is sufficiently close to constant returns, K will fall in response to the coffee price shock when E is constant. Specifically, if $\varepsilon_{\pi'(V),V} = 0$, we observe constant returns, and K will fall when E is constant if $\varepsilon_{\pi'(V),V} \geq -0.20$. While we do not have a strong prior on what the actual value of this parameter is, we believe that this shows that we do not need pathological beliefs to general a fall in K when E is constant in equilibrium