ANSWERS TO TEST NUMBER 9

Question 1: (20 points)

a) We are estimating the mean grade of a group so we need a confidence interval for the mean.

b) We are estimating the quantity for a particular element whose income happens to be $40,000 so we need a confidence interval for a single element or 'new' observation.

c) We are estimating the mean expenditure for a group of companies here and so we need a confidence interval for the mean.

[Give yourself 6 points for each one of the above you get right plus 2 additional points if you correctly answered all of them.]

Question 2: (80 points)

The regression results, as stated in the question, are as follows:

Regression Output (excl. Dummy and Trend)	
Constant	-62.28
Mean Squared Error	2154.9
R Squared	0.93
No. of Observations	41
Degrees of Freedom	38

	USRGNP	UKSTINT
X Coefficient(s)	0.190	-7.26
Std Err of Coef.	0.008	6.57

Regression Output (incl. TREND)	
Constant	-58.66
Mean Squared Error	1937.4
R Squared	0.94
No. of Observations	41
Degrees of Freedom	37

	USRGNP	UKSTINT	TREND
X Coefficient(s)	0.28	-13.38	-7.24
Std Err of Coef.	0.04	6.75	3.15

Regression Output (incl. Dummy and Trend)	
Constant	-43.97
Mean Squared Error	1966.7
R Squared	0.943273
No. of Observations	41
Degrees of Freedom	36

	USRGNP	UKSTINT	TREND	DUMMY
X Coefficient(s)	0.31	-14.30	-9.79	-22.32
Std Err of Coef.	0.06	6.94	4.96	33.30

a) The three equations are

```
(1) USRMON = - 62.28 + .19 USRGNP - 7.26 UKSTINT

(2) USRMON = - 58.66 + .28 USRGNP - 13.38 UKSTINT - 7.24 TREND

(3) USRMON = - 43.97 + .31 USRGNP - 14.30 UKSTING - 9.59 TREND - 22.32 DUMMY
```

`USRMON` (the U.S. real money stock) is the dependent variable and all the remaining variables are independent variables.

[Give yourself 3 points for correctly setting down each equation and 1 point for recognizing which are the dependent and independent variables.]

b) The relevant null hypothesis are that $\beta_{USRGNP} \leq 0$ and $\beta_{UKSTINT} \geq 0$. The corresponding t-ratios for `USRGNP` for the three regressions are

$$\begin{aligned} t_1 &= \frac{.19}{.008} = 23.75 \\ t_2 &= \frac{.28}{.04} = 6.5 \\ t_3 &= \frac{.31}{.06} = 5.17 \end{aligned}$$

which indicates clearly that we can reject the null hypothesis of a negative relationship between real income and the quantity of real money balances and conclude that there is a significant positive relationship between these two variables. The t-ratios for `UKSTINT` for the respective regressions are

$$\begin{aligned} t_1 &= \frac{-7.26}{6.57} &= -1.105 \\ t_2 &= \frac{-13.38}{6.75} &= -1.98 \\ t_3 &= \frac{-14.30}{6.94}, &= -2.06 \end{aligned}$$

Using Xlispstat we calculate the 5% critical values of the t distribution for a one-tailed test with 38, 37, and 36 degrees of freedom, respectively, as

```
> (t-quant .95 38)
1.685954460163816
> (t-quant .95 37)
1.687093619592898
> (t-quant .95 36)
1.6882977141129205
```

This tells us that we can reject the null hypothesis of a positive or zero relationship between the real quantity of money and the short-term interest rate at the 5% level when trend is included in the regression but not when trend is excluded. The critical value of t at 10% level is

```
> (t-quant .90 38)
1.3042302038897926
```

which indicates that we could not reject the null hypothesis at the 10% level either in the case where trend is excluded.

[Give yourself 3 points for correctly setting up the null hyptheses, a half-point for correctly calculating each of the six t-ratios, and 4 points for understand how to interpret the results and reach the correct conclusion.]

c) The t-statistics for the null hypothesis that the coefficient of trend is zero in the two regressions in which it is included are

$$
\begin{aligned}
t_2 &= \frac{-7.24}{3.15} = -2.3 \\
t_3 &= \frac{-9.79}{4.96} = -1.97
\end{aligned}
$$

and the corresponding 5% critical values are

```
> (t-quant .975 37)
2.026192463019134
> (t-quant .975 36)
2.0280940009689217
```

We cannot reject at the 5% level the null hypothesis of no trend effect when the dummy variable is included in the regression but we can reject it when the dummy variable is excluded. We could reject the null at the 10% level in both cases.

[Give yourself 2 points for choosing the correct null hypothesis, a half-point for correctly calculating each t-ratio and 2 points for interpreting the results correctly.]

d) The t-statistics for the null hypothesis that the coefficient of the dummy variable is zero is

$$
t_3 = \frac{-22.32}{33.30} = -0.67
$$

which is well below (in absolute value) the critical value for rejection of the null at the 10% level. We can reject the hypothesis that the demand for real money balances depends upon whether or not the observation in question comes from the set of years delineated by the dummy variable.

[Give yourself 2 points for choosing the correct null hypothesis, 1 point for correctly calculating the t-ratio and 2 points for interpreting the results correctly.]

e) The sum of squared residuals is equal to the mean squared error times the number of degrees of freedom. When the two regressors are included the sum of squared residuals is therefore (1966.7)(36) = 70801.2 which is the unrestricted sum of squares. When the two regressors are excluded we obtain the restricted sum of squares, which equals (2154.9)(38) = 81886.2. The difference between the restricted and unrestricted sum of squares is (81886.2 - 70801.2) = 11085.0 which when divided by the number of restrictions (= 2) becomes 5542.5. This number, divided by the mean squared error of the unrestricted regression (the sum of squared errors divided by the degrees of freedom of that regression) is distributed as the F(2,36). The test statistic is 5542.5/1966.7 = 2.82. Using Xlispstat, we calculate the 5% and 10% critical values of F as

```
> (F-quant .95 2 38)
3.24481836073279
> (F-quant .90 2 38)
2.4479199420090834
```

We can reject the null hypothesis that the coefficients of both variables are zero at the 10% level, but not the 5% level.

[Give yourself 8 points for setting up the test correctly, 3 points for understanding what the null hypothesis is and 4 points for correctly interpreting the results.]

f) To determine whether the regression as a whole is statistically significant, we need the total sum of squares. We are given the R^2 from which it follows that

$$\begin{aligned}
R^2 &= \frac{SSTO - SSE}{SSTO} \\
SSTO\,R^2 &= SSTO - SSE \\
SSTO\,R^2 - SSTO &= -SSE \\
(1 - R^2)\,SSTO &= SSE \\
SSTO &= \frac{(n-k-1)MSE}{1-R^2} = \frac{(37)(1937.4}{1-.943273} = \frac{70801.2}{0.056727} = 1248104.08
\end{aligned}$$

We could have obtained the same result by applying the R^2 and mean squared error from any of the three regressions. The sum of squares due to regression for the second regression, the one that excludes the dummy variable (which is insignificant when included), is thus equal to due to regression for the second regression,

$$SSR = R^2\, SSTO = (.94)(1248104.08) = 1173217.83$$

This divided by the number of restrictions imposed on the dependent variable by leaving out all the regressors is $1173217.83/3 = 391072.61$ which when divided by the mean squared error of the regression yields the test statistic $391072.61/1937.4 = 201.85$. This test statistic is distributed according to the F distribution with 3 degrees of freedom in the numerator and 37 degrees of freedom in the denominator. The 1% critical value for F(3,37) is

```
> (F-quant .99 3 37)
4.359539785848906
```

which clearly indicates rejection of the null hypothesis that the coefficients of all the independent variables are zero. The regression is statistically significant—that is, there is a statistically significant relationship between the dependent variable and the three independent variables.

[Give yourself 8 points for setting up the test correctly, 3 points for understanding what the null hypothesis is and 4 points for correctly interpreting the results.]

g) The variance of the dependent variable is the total sum of squares divided by $n - 1$, or $195409.6/40 = 4885.24$.

[Give yourself 5 points for understanding how to do this calculation.]

h) The observed residuals have two disturbing characteristics. First, their variance appears to increase with time, implying that σ^2 is not constant. Secondly, they appear to be serially correlated—that is, each residual's size seems partly dependent upon the size of the previous residual. In fact, the Durbin-Watson statistic for the regression that includes trend but not the dummy variable is 1.5 which is below the critical value for concluding that there is no serial correlation in the residuals (1.66) but above the critical value for concluding that there is serial correlation in the residuals (1.34).

[Give yourself 5 points for recognizing that the variance of the error term is not constant through time and 5 points for recognizing the possibility that serial correlation is present and understanding what it is.]