ANSWERS TO TEST NUMBER 2

Question 1: (20 points)

Make yourself a rectangular diagram with “first toss” across the top and “second toss” down the side.

		First Toss					
		1	2	3	4	5	6
	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
Second	3	1,3	2,3	3,3	4,3	5,3	6,3
Toss	4	1,4	2,4	3,4	4,4	5,3	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6

The 6x6 rectangle of elementary events represents the sample space. Since the dice are fair the probabilities of individual basic events occurring are the same for all basic events in the sample space. We want the event

$$x < y$$

where x is the number on the first toss and y is the number on the second toss. You can read the sample points for which this will occur off the diagram. You should count 15 of the 36 sample points for which the event occurs. The probability of the event occurring is thus $15/36 = .416667$

> [Give yourself 7 points for setting up the grid correctly, 7 points for picking the correct points in it for which $x < y$ and 6 points for calculating the probability.]

Question 2: (10 points)

The probability of obtaining at least 1 head in n tosses of the coin is equal to 1 minus the probability of obtaining no heads in n tosses. So when the probability of obtaining no heads in n tosses is less than .10, n is the number of tosses that will lead to a 90% chance of obtaining at least 1 head.

Since the probability of a tail on any toss is .5, the probability of tails in n tosses is .5 to the power n. We thus want the smallest n that satisfies the condition

$$.5^n < .1$$

A value of $n = 4$ will do the trick.

> [Give yourself 5 points for recognizing that you need to calculate 1 minus the probability of obtaining no heads, and 5 points for realizing that $.5^n$ is the probability of getting no heads in n tosses.]

Question 3: (10 points)

If two events A and B are mutually exclusive then

$$P(A \cap B) = 0$$

Independence requires that

$$P(A \cap B) = P(A)P(B)$$

which is obviously not true. If two events are mutually exclusive the occurrance of one of the events means that the other event cannot occur. For the events to be independent each should occur independently of the other.

[Give yourself 4 points for recognizing what it means for two events to be mutually exclusive, 4 points for recognizing what independence implies and 2 points for drawing the correct conclusion.]

Question 4: (40 points)

You are given the following probabilities:
$P(D|C) = .6$ (Dies conditional upon eating cherries containing rat poison)
$P(D|M) = .9$ (Dies conditional upon eating Mousse containing cyanide)
$P(C) = .6$ (Probability of Choosing the Cherries)
$P(M) = .4$ (Probability of Choosing the Mousse)

[Give yourself 3 points for recognizing each of the above.]

The last two can be entered in the bottom row of the table below.

Calculate
$P(D \cap C) = P(D|C)P(C) = (.6)(.6) = .36$
$P(D \cap M) = P(D|M)P(M) = (.9)(.4) = .36$
and enter them in the appropriate cells in the table below.

[Give yourself 3 points for correctly doing each of the above two calculations.]

You can then fill in the rest of the table.

		Desert		
		Cherries	Mousse	
Result	Dies	.36	.36	.72
	Lives	.24	.04	.28
		.60	.40	1.00

[Give yourself 2 points for getting each of the numbers in the table correct and in the right place (18 points in total).]

The probability of Basil being poisoned by the chocolate mouse, given that he dies is

$P(M|D) = P(M \cap D)/P(D) = .36/.72 = .5$

[Give yourself 4 points for recognizing that the above calculation is required and doing it correctly.]

Question 5: (20 points)

The sample points in S and their probabilities are:

$$\begin{array}{l} s_1 \text{ — } .22 \\ s_2 \text{ — } .31 \\ s_3 \text{ — } .22 \\ s_4 \text{ — } .15 \\ s_5 \text{ — } .10 \end{array}$$

The following events have been defined

- $E_1 = \{s_1, s_3\}$.
- $E_2 = \{s_2, s_3, s_4\}$.
- $E_3 = \{s_1, s_5\}$.

The requested probabilities are calculated as follows:

a) $P(E_1) = P(s_1) + P(s_3) = .22 + .22 = .44$.

b) $P(E_2) = P(s_2) + P(s_3) + P(s_4) = .31 + .22 + .15 = .68$.

c) $P(E_1 \cap E_2) = P(s_3) = .22$.

d) $P(E_1|E_2) = P(E_1 \cap E_2)/P(E_2) = .22/.68 = .323$.

e) $P(E_2 \cap E_3) = 0$.

f) $P(E_3|E_2) = P(E_3 \cap E_2)/P(E_2) = 0/.68 = 0$.

[Give yourself 3 points for getting each of the above calculations correct and 2 bonus points if you did all six of them correctly.]