World Monetary Equilibrium under Alternative Exchange Rate Regimes and Assumptions about International Capital Mobility

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1 Introduction

The purpose here is to explore the way in which real and monetary equilibrium in the world economy is established under alternative international monetary regimes and alternative assumptions about international capital mobility. The international monetary arrangements or "system" in existence at any point in time will be some hybrid of three different types of regimes.

- A common currency regime in which countries or regions adopt a common currency or common currency standard. An example is the pre-1914 international gold standard under which a majority of industrialized nations pegged their currencies to gold. More strict examples are the common currencies adopted by regions within single countries and the common currency currently being established in the European Union.

- A key-currency regime in which countries peg their currencies that of a central "key-currency" country which may or may not tie its currency to gold. An example here is the Bretton Woods system between the end of World War II and the early 1970s where the majority of industrialized countries pegged their currencies to the U.S. dollar. For

*The basic purpose of this note is to rework and modernize a major theme from my book *World Monetary Equilibrium*, University of Pennsylvania Press, 1985, for use in my classes.

most of this period a fixed U.S. dollar price of gold was maintained although this did not constitute a gold standard because there was no relationship between the U.S. money supply and the country's stock of gold.

- A flexible exchange rate regime in which exchange rates are determined by the forces of demand and supply with or without occasional specific interventions by countries' authorities. Three types of intervention are possible:

  - Direct foreign exchange market intervention under which a country's authorities buy or sell particular foreign currencies for domestic currencies on the market.
  - Manipulation of the domestic money supply to influence or control the time path of the exchange rate.
  - Direct tariff, subsidy, or fiscal intervention in commodity and capital markets to influence otherwise market determined levels of exchange rates.

  The term *managed floating* is used to describe any interference in the free functioning of foreign exchange markets. It turns out that managed floating is inevitable under flexible exchange rates. There is really no such thing as a free float because the government of every country must decide on a time-path of domestic monetary growth, and whatever time-path it decides upon will influence the exchange rate. One might define a *free float* as one where governments conduct domestic policy without regard to the exchange rate, but such a definition is not particularly useful because government "intentions" are difficult to decipher and quantify.

There are also three alternative situations with respect to international capital mobility.

- Zero international capital mobility, which requires that the government restrict all private movements of capital between the domestic and rest-of-world economies but allows for government sponsored or controlled movements of capital. Essentially this means that capital cannot move at all in response to market forces.

- Perfect international capital mobility, which usually means two things:

  - Private individuals are free to buy and sell domestic and foreign assets across international boundaries.

  - Domestic and foreign assets are perfect substitutes in the portfolios of a sufficient number of individuals to equalize the real interest rates on all countries' assets.

- Imperfect international capital mobility, which implies that one or both of two conditions hold:
  - The assets of different countries are imperfect substitutes in portfolios so that real interest rates differ by risk premia.
  - There are restrictions on transactions in particular types of assets, which also lead to imperfect substitutability of the countries' assets.

The third of these conditions, of course, is the one that applies in reality. Nevertheless, it is useful to impose an assumption of zero or perfect capital mobility to simplify the analysis in its beginning stages and to explore the implications of international mobility of capital for the functioning of the international monetary system. All conclusions reached must ultimately take into account, or be insensitive to, imperfections in the international mobility of capital.

2 Some Preliminary Modeling Issues

The next step is to construct a model in which the above combinations of type of international monetary system and degree of international capital mobility can be analyzed. An older-style income-expenditure model will be used. One might envisage, instead, a dynamic, intertemporal, stochastic, optimizing approach that is able to transcend the well-known inadequacies of income-expenditure models. While income-expenditure models indeed have major weaknesses, these models are also much simpler than high-tech models and this simplicity can be used to advantage as long as we do not try to analyze issues that these simpler models cannot correctly address. Because of their simplicity and ability to unify many strands of (often incorrect) theoretical argument, these models remain the *lingua franca* of business macroeconomic analysis and forecasting. For this reason it is important that we outline their weakness as we construct an appropriate income-expenditure framework to address the problems at hand. In constructing our model we must proceed very carefully, paying particular attention to what assumptions have to be made at each stage to proceed. This means that the income-expenditure model we finally settle upon will be a truncated and modified version of traditional ones.

To pave the way, let us look at the standard Keynesian textbook income-expenditure model:

$$Y = C + I + G + B_T \tag{1}$$
$$C = C(Y, r, T) \tag{2}$$
$$I = I(r, Y) \tag{3}$$
$$B_T = B_T(Y, Y^*, P/(\pi P^*)) \tag{4}$$

where C is private consumption, I is private investment, T is government tax revenues, G is government expenditure, B_T is the balance of trade, P and P^* are the domestic and foreign price levels and π is the nominal exchange rate (domestic currency price of foreign currency). The ratio $P/(\pi P^*)$ is the real exchange rate which we will denote by q,

$$q = \frac{P}{\pi P^*}. \tag{5}$$

Substitution of (2), (3), and (4) into (1) yields the equation of the standard Keynesian IS curve,

$$Y = C(Y, r, T) + I(r, Y) + G + B_T(Y, Y^*, P/(\pi P^*)). \tag{6}$$

When the functions in (2), (3), and (4) are written in linear form, as is usually done in simple treatments, we can move all the terms involving Y to the left side and obtain the standard multipliers. The crude Keynesian analysis treats output on the left side of (6), aggregate supply, as a single good that is perfectly elastic in supply at the fixed domestic price level P. Somewhat more sophisticated versions treat aggregate output supplied as a function of the real wage rate

$$w = \frac{W}{P}$$

where W is the nominal wage rate, so

$$Y^S = Y(W/P). \tag{7}$$

The components C, I, G and B_T on the right side of the equality in (6) represent the demand for the the aggregate output good for purposes of private consumption and investment, government expenditure and net purchases by foreigners,

$$Y^D \quad = \quad C(Y,r,T) + I(r,Y) + G + B_T(Y,Y^*,P/(\pi P^*)). \tag{8}$$

All magnitudes except W, P and P^* and π are real magnitudes, with the two ratios of nominal magnitudes, w and q, also being real.

There are a number of serious problems with basing our model on this formulation as it stands. First, to correctly determine C we should be taking into account future as well as present levels of Y, r, and T. The problems resulting from failure to do this are particularly evident with respect to the effects of current taxes. If the government lowers current taxes, financing the resulting short-fall of revenue from expenditure by printing money or selling government bonds, and then satisfies its intertemporal budget constraint by raising future taxes (either on goods, incomes or, in the case of inflationary finance, on money holdings), consumption will increase only if the private sector sees these actions as an improvement in its wealth. Clearly, such a tax cut will not necessarily always increase wealth, and surely not, as crude Keynesian models assume, increase consumption the same amount as would an exogenous increase in Y. And, of course, an increase in Y will increase wealth and overall consumption (which will be distributed over the current and all future periods and not concentrated in the current one) only if it is a permanent increase not to be offset by an equivalent decline in subsequent periods. Second, in addition to neglect of intertemporal maximization considerations, the model clearly misrepresents the role of government expenditure. An increase in government expenditure G may well induce a reduction in private consumption or investment since the resources it requires will necessarily be drawn from the private sector by taxation or borrowing. It is thus not clear what the magnitude of the effect of an increase in G will be—it will depend on what the government is doing and how it finances the change in its expenditure. Because we are not interested in fiscal policy issues we can avoid these problems by lumping government in with the private sector and then aggregating consumption and investment on private and public account into single function of Y and r, putting in a shift variable which we will call Ψ_D to allow for all exogenous shocks to current-period domestic expenditure on domestic goods, whatever the source. We thus have

$$C(Y,r,\Psi_D) + I(r,Y,\Psi_D)) \quad = \quad Z(Y,r,\Psi_D) \tag{9}$$

which can be substituted into (8). After adding a further shift variable Ψ_F to the balance of trade function $B_T(\ldots)$ we obtain the following expression for the aggregate demand for domestic output.

$$Y^D = Z(Y, r, \Psi_D) + B_T(Y, Y^*, P/(\pi P^*), \Psi_F). \qquad (10)$$

Also, we are ignoring depreciation here and assuming that net repatriated earnings from foreign to domestic residents are initially zero so that we can treat gross and net national and domestic products as a single magnitude.

This aggregate demand function for the domestic output flow is presented as GG in Figure 1. The vertical line Y_0Y_0 gives the aggregate supply of domestic output. The aggregate demand for domestic output is negatively related to the real interest rate and shifts to the right with a fall in (devaluation of) the domestic real exchange rate. The aggregate supply of domestic output is insensitive to the real interest rate and shifts to the right with a fall in the real wage rate.

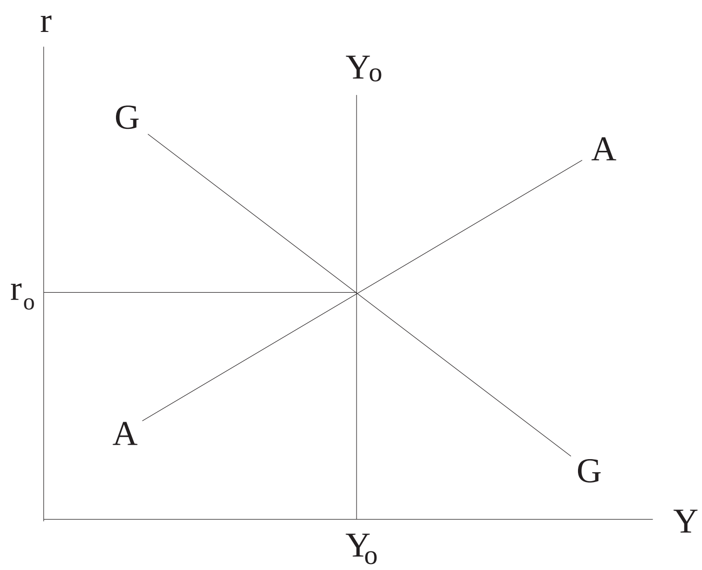

Figure 1: Current period stock and flow equilibrium in an income-expenditure style model.

The implications of technological change also present problems in these models. Elementary textbooks typically treat improvements of technology solely as rightward shifts in the full-employment level of output. This is insufficient because technological improvements also affect investment, shifting the aggregate demand for output curve rightward at the same time. The effects then depend upon whether aggregate demand increases more or less than full-employment output. Since technological change is not of interest here, we can avoid these complications by holding technology constant.

The equality of Y_D and Y_S together with equations (7) and (10) yield the condition of flow equilibrium in a single economy—the condition that the aggregate demand for the flow of output equal the aggregate supply—at the intersection of GG and Y_0Y_0 in Figure 1. Before using those equations, however, we must face one further problem. No micro-foundations underlying these curves are presented. How do we know that if such foundations were properly developed (which would require an entire paper much longer than this present one), the aggregate demand curve would be negatively sloped with respect to the real interest rate and would shift positively in response to a devaluation of the real exchange rate?

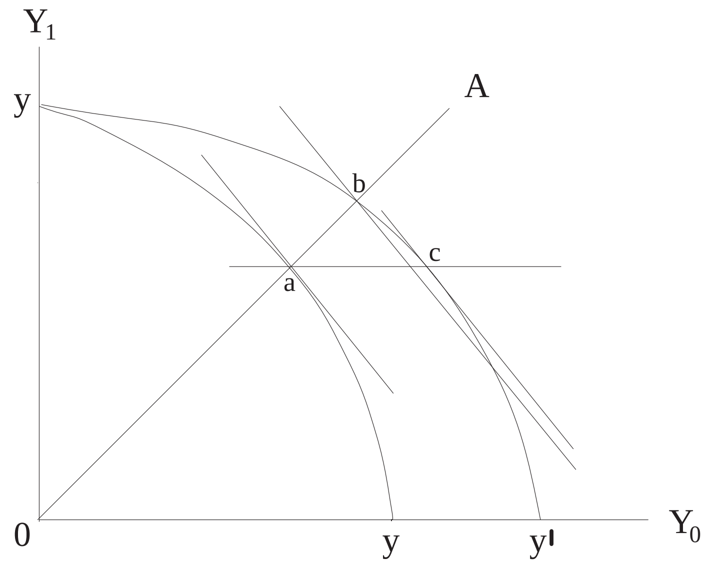

Figure 2: The effects of current changes in employment and aggregate output on the equilibrium domestic real interest rate.

An intuitive positive answer to the first part of this question is presented in Figure 2. Output the current period is on the horizontal axis and future output, collapsed into output in period 2, is on the vertical axis. The curve yy gives the community's ability to transform current into future output. An exogenous fall in the real wage rate will clearly cause firms to employ more labour and result in an increase in current output. For simplicity we hold net foreign expenditure on domestic goods (given by $B_T(\ldots)$) constant. Since the wage change is temporary and will be reversed as soon as wage and price setters realize what is going on, future output opportunities will be not much affected. The curve yy thus shifts proportionally to the right to yy'. This means that the slope of yy' at point c is the same as the slope of yy at point a. If the underlying indifference curves are homothetic,[1] the slope of the indifference curve passing through point b will be the same as the slope of both the indifference curve and opportunity locus at point a. The indifference curve passing through point b will thus be steeper than the opportunity locus at that point and the new equilibrium will have to be along that opportunity locus part way between points b and c. At this new equilibrium the common slope of the indifference curve and opportunity locus will necessarily be lower than the slope of the opportunity locus and indifference curve at point a. The slope of the tangency of the opportunity locus with the indifference curve equals $-(1 + r)$, so clearly r must fall as we move to the new equilibrium.[2] Even if the indifference curves are not homothetic the interest rate must fall as long as the new equilibrium is to the left of point c. For it to be to the right of that point, future consumption would have to be an inferior good. The intuition here is straight-forward. A rise in current income above its permanent level induces people to smooth consumption by purchasing assets to be redeemed for consumption later. This bids up asset prices and bids down the rate of interest.

The effect of a temporary increase in income on the real exchange rate, holding aggregate domestic expenditure (given by $Z(\ldots)$) constant is shown in Figure 3. The argument is the same as in the case of Figure 2, except

[1] This means that the indifference curves have the same slope along any ray from the origin. It implies that the ratio of present to future consumption is the same at all levels of wealth when the interest rate is constant.

[2] This can be seen from the fact that a reduction of current consumption will increase future consumption by the proportion $(1 + r)$—that is

$$\Delta C_1 = -(1 + r)\Delta C_0.$$

The common slope of the indifference curve and the opportunity locus at a point of equilibrium is therefore

$$\frac{\Delta C_1}{\Delta C_0} = -(1 + r).$$

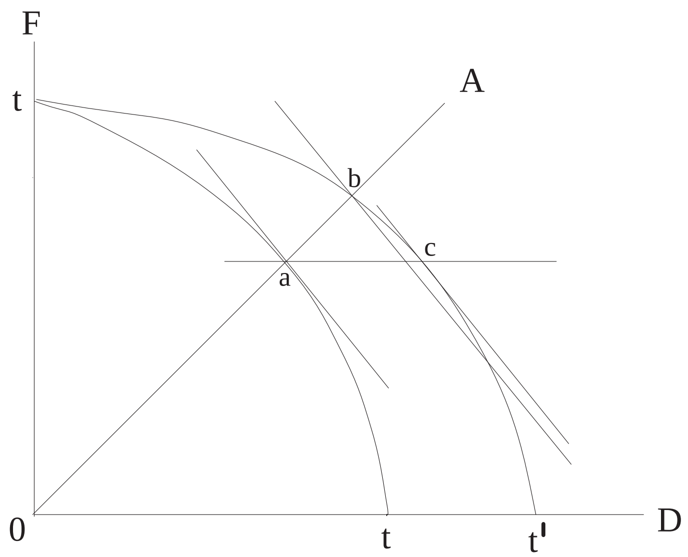

Figure 3: The effects of current changes in employment and aggregate output on the equilibrium domestic real exchange rate.

that different variables are on the axes. The quantity of domestic goods produced both at home and abroad is represented on the horizontal axis and the quantity of foreign output goods produced in both the domestic and rest-of-world economies is represented on the vertical one. The ability of the world to transform domestic output goods into foreign output goods, where at least some of each country's goods can be produced in the other country, is given by the transformation curve tt. An exogenous fall in the real wage rate and expansion of domestic output shifts tt proportionally more to the right than up, to tt', and the equilibrium point moves from a to somewhere between points b and c on tt'. The relative price of domestic goods in terms of foreign goods—that is, the real exchange rate—is represented by the slope of the common tangent of the opportunity locus with the indifference curve in equilibrium. It clearly falls as long as the foreign good is not inferior (which would lead to an equilibrium to the right of point c). Again, the intuition is straight-forward. An expansion of output in one part of the world increases the world supply of a particular set of goods, some traded and some nontraded. If preferences are convex to the origin, the relative prices of the expanding goods must fall. The resulting fall in the real exchange rate at a given level of the real interest rate will thus shift the (severely truncated) IS curve, represented by GG in Figure 1, to the right.

Next we must deal with the question of how to model aggregate supply. Ideally we would want to use a form of Lucas supply curve which would express output relative to its (here constant) full employment level as a function of unanticipated shocks to the price level or, in a more Keynesian-style approach, as a response to unanticipated aggregate demand shocks. For the purpose at hand, however, we can adopt the very crude assumption that output is either fixed at full employment (i.e., at its normal or natural level) and that wages and prices are therefore flexible, or that output is in perfectly elastic supply at a fixed price level. This latter assumption violates our equation (7) but it buys us simplicity at no cost in terms of subsequent results. In the presentation in Figure 4, this assumption means that domestic output will be determined by the levels of the real interest rate and real exchange rate under less-than-full-employment conditions and is at its exogenous full-employment level under conditions of wage and price flexibility and full employment.

The above methodology will enable us to adequately portray individual country and world flow equilibrium. The condition of flow equilibrium is that the aggregate world demand for each country's output flow equal the magnitude of that flow determined by conditions of supply. An additional condition of flow equilibrium will appear when we assume that capital is not internationally mobile, but discussion of it can be postponed to a later section.

The other basic conditions of individual country and world equilibrium relate to the markets for assets. World stock equilibrium requires that every unit of capital, wherever employed, must be willingly held by someone at the existing world market price of that asset. Since asset yields and asset prices are inversely related, this implies that there must be an equilibrium market yield on every asset. Human capital has returns that are market determined but implicit to the person in which the capital is embodied.

Standard Keynesian closed economy macroeconomics specifies asset equilibrium as equilibrium in two markets—the money market and the bond market. These models tend to confuse stocks and flows, arguing that one of the three markets—money, bond, and goods (or output-flow in the above analysis)—can be ignored, since agents in the aggregate are wealth constrained as to their total holdings of all these items. That is, if the money and goods markets are in equilibrium at any given level of aggregate wealth

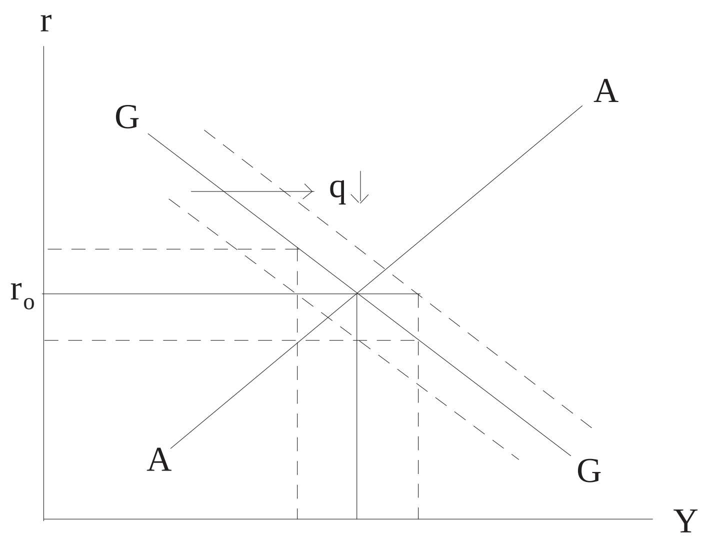

Figure 4: The response of current aggregate output and employment to changes in the real interest rate and real exchange rate.

the bond market will automatically be in equilibrium. On this basis the bond market is usually ignored.

Here we take a more sophisticated view of assets, realizing that the world stock of assets will consist of money, ownership claims to capital goods, equities, bonds, human capital, and so forth. We divide these assets in to two classes, monetary and non-monetary. Human capital is assumed to be embodied in the person who owns it and is therefore non-tradeable. Since the total stock of tradeable assets in the world is fixed, equality of the desired and actual stocks of money will imply that the desired and actual stocks of the aggregate of non-monetary assets will also be equal. Although we will follow tradition and think of there being a single yield or rate of interest on each country's non-monetary assets, a more sophisticated analysis, unnecessary for what we are trying to do, would recognize that there will be a vector of equilibrium interest rates on non-monetary assets at which someone in the world will be willing to hold each and every unit of every asset. If one non-monetary asset is more desirable than another at a particular vector of interest rates, agents will try to buy it and sell the less valuable assets. Since the total stock of every asset is given at each moment of time the prices of the assets and their relative yields must instantaneously adjust to reflect the relative values the market attaches to them. We will keep most of these details in the background allowing, for the most part, only one non-monetary asset per country.

It is important here to distinguish between the aggregate stock constraint and the aggregate flow constraint. As noted, older Keynesian treatments ignore the bond market on the grounds that if the public holds its equilibrium stock of money and is absorbing its equilibrium amount of goods per period, it will necessarily hold the equilibrium stock of bonds. The treatment here omits the equilibrium condition for aggregate non-monetary assets on the grounds that, given the stock of wealth at each point in time, when people are holding their desired stock of money they must also be satisfied with their holdings of non-monetary assets. The goods or commodity market is a market for the flows of output produced by the stock of assets and should not be lumped in with money and bonds as part of an overall stock-flow constraint. One cannot add stocks and flows—stocks are the quantities of assets in existence at a point in time and flows are the goods and services produced by those assets at a point in time. Stocks exist at a point in time while flows occur at a rate through time. Corresponding to the stock or wealth constraint whereby the market for non-monetary assets is in equilibrium if the demand for money equals the existing stock of money, there is a flow constraint that says that if both income and consumption are determined then savings cannot be separately determined—the total income flow must all be used for either consumption or savings, so we do not need separate equilibrium conditions determining savings and consumption.

In a world of zero capital mobility we can thus define asset equilibrium in each country as the condition of equality of demand and supply for real money balances

$$\frac{M}{P} = L(i, Y) \tag{11}$$

where i is the nominal interest rate which, according to the standard Fisher condition

$$i = r + E_p, \tag{12}$$

equals the real interest rate plus the expected rate of domestic inflation, here denoted by E_p.

When there is capital mobility, either perfect or imperfect, an additional condition is involved. Equality of the demand and supply of money in every country is sufficient to ensure that the world aggregate supply of non-monetary assets equals the demand. But it does not ensure that the division of this aggregate world demand into domestic and foreign assets equals the division of the existing aggregate world stock of assets into its domestic and foreign components. World residents could have their desired aggregate stock of non-monetary assets yet nevertheless prefer to hold a larger fraction of that stock in domestic assets than the existing fraction of domestic assets in the total. This would result in a rise in the prices of domestic assets and fall in domestic interest rates until the desired ratio of domestic to foreign assets in portfolios is brought into equality with the actual ratio of domestic to foreign assets in existence. This equilibrium condition follows naturally from the conditions of efficient markets and interest rate parity. Efficient markets requires that

$$\xi \quad = \quad E_\pi + \rho_T, \tag{13}$$

where ξ is the forward discount on the domestic currency, E_π is the expected rate of increase in the domestic currency price of foreign currency in terms of domestic currency (expected rate of depreciation of the domestic currency) and ρ_T is a foreign exchange risk premium. The intuition behind this is that risk-neutral traders can earn expected profits by taking uncovered short or long positions in domestic currency when it is expected to depreciate by more or less than the current forward discount. If traders are not risk-neutral they will require an additional return, represented by a greater or smaller expected depreciation than implied by the forward discount, to compensate for the risk of taking an uncovered short or long position. The interest rate parity condition is

$$i - i^* \quad = \quad \xi + \rho_X, \tag{14}$$

where a * superscript indicates that the variable applies to the foreign economy. The intuition here is that arbitrage profits can be obtained whenever the domestic/foreign interest differential differs from the risk-adjusted forward discount. Equations (13) and (14) are Euler equations that arise from the solution of appropriate intertemporal optimization models. Combining the two conditions by substituting (13) into (14), we obtain

$$i - i^* \quad = \quad E_\pi + \rho_T + \rho_X \quad = \quad E_\pi + \rho \tag{15}$$

where $\rho = \rho_T + \rho_X$. This says that the domestic interest rate must equal the foreign interest rate plus a risk premium plus the expected rate of nominal devaluation of the domestic currency.

$$i \quad = \quad i^* + \rho + E_\pi \tag{16}$$

This equation can be further modified by substituting into it the Fisher equation (12) for each country to obtain

$$r \quad = \quad r^* + \rho - (E_p - E_\pi - E_{p^*}) \quad = \quad r^* + \rho - E_q \tag{17}$$

where $E_q = (E_p - E_\pi - E_{p^*})$, the expected rate of increase (appreciation) of the real exchange rate. This equation says that the domestic real interest rate must equal the foreign real interest rate plus a risk premium minus the expected rate of appreciation of the domestic real exchange rate.

For any individual economy, the demand for money condition together with the the Fisher equation yields

$$\frac{M}{P} \quad = \quad L(r + E_p, Y) \tag{18}$$

which can be portrayed in Figures 1 and 4 as the curve AA which gives the combinations of income and the real interest rate for which the demand for money equals the existing stock. This curve is upward sloping because a rise in the real interest rate reduces the quantity of money demanded, requiring a rise in income to equivalently increase the quantity of money demanded to maintain equality with the unchanged money supply. This curve shifts to the left with a fall in the expected rate of inflation E_p.

Equation (17) along with (18) and an equivalent foreign real demand for money equation,

$$\frac{M^*}{P^*} \quad = \quad L^*(r^* + Ep^*, Y^*), \tag{19}$$

form the three conditions of world stock (or asset or portfolio) equilibrium when there is some degree of international capital mobility. These equations clearly have adequate micro-foundations. The demand for money can be derived from shopping technology models as well as standard asset preference theory, and the real interest rate equation (17) emerges directly from the Euler conditions of intertemporal optimization.

We can now proceed to set up our model.

3 The Model with International Capital Mobility

Let us begin with the situation where capital is internationally mobile, though not necessarily perfectly so—that is, where people are free to trade a wide variety of domestic and foreign assets across the international border without restriction. These assets need not be perfect substitutes in portfolios and not all assets need be freely traded.

The conditions of flow equilibrium are

$$
\begin{aligned}
Y &= D(Y, r, \Psi_D) + B_T(Y, Y^*, P/(\pi P^*), \Psi_F). && (20)\\
Y^* &= D^*(Y^*, r^*, \Psi_D^*) - [P/(\pi P^*)][B_T(Y, Y^*, P/(\pi P^*), \Psi_F)]. && (21)
\end{aligned}
$$

The foreign balance of trade measured in nominal units of domestic currency is identical to the negative the domestic nominal trade balance measured in domestic currency. As a result the foreign real trade balance equals minus $P/(\pi P^*)$ times the domestic real trade balance, as shown in (21).

The stock equilibrium conditions are

$$
\begin{aligned}
\frac{M}{P} &= L(r + E_p, Y, \Psi_M) && (22)\\
\frac{M^*}{P^*} &= L^*(r^* + E_p^*, Y^*, \Psi_M^*), && (23)
\end{aligned}
$$

and

$$
r = r^* + \rho - E_q. \qquad (24)
$$

Exogenous shift variables Ψ_M and Ψ_M^* have been added to the two demand for money equations.

When the exchange rate is flexible, the above five equations solve for the five endogenous variables Y, Y^*, r, r^* and π under less-than-full employment conditions, given the exogenously determined levels of M, M^*, P, P^*, E_p, E_{p^*}, E_q, and the shift variables Ψ_D, Ψ_D^*, Ψ_F, Ψ_M and Ψ_M^*. When there is full employment, P and P^* become endogenous and Y and Y^* exogenous. When the exchange rate is fixed, π becomes exogenous, requiring that some previously exogenous variable become endogenous. In the key-currency case, the peripheral or non-key-currency country's nominal money supply becomes endogenous. In the common currency case an additional world money supply equation must be added along with an additional exogenous variable, the world nominal money stock, in which case both countries' nominal money supplies become endogenous.

The system can be reduced to four equations in the four variables Y or P, Y^* or P^*, r^* and π or M by substituting (24) into the other four equations.

$$
\begin{aligned}
Y &= Z(Y, r^* + \rho - E_q, \Psi_D) + B_T(Y, Y^*, P/(\pi P^*), \Psi_F) && (25)\\
Y^* &= Z^*(Y^*, r^*, \Psi_D^*) - [P/(\pi P^*)][B_T(Y, Y^*, P/(\pi P^*), \Psi_F)] && (26)\\
\frac{M}{P} &= L(r^* + \rho - E_q + E_p, Y, \Psi_M) && (27)\\
\frac{M^*}{P^*} &= L^*(r^* + E_p^*, Y^*, \Psi_M^*) && (28)
\end{aligned}
$$

Under a common currency system we would add a fifth equation specifying the level of the world money stock and M^* would then also become endogenous.

To extract information about the effects of exogenous shocks on the equilibrium levels of the endogenous variables, we differentiate the above four equations totally with respect to all variables. It is useful to simplify the results by choosing the units in which outputs are measured to make the initial levels of P, P^* and π equal to unity, and setting r initially equal to r^* (which implies that ρ and E_q are initially zero). The results are as follows

$$
\begin{aligned}
dY &= \frac{\partial Z}{\partial Y}\,dY + \frac{\partial Z}{\partial r}\,dr^* + \frac{\partial Z}{\partial r}\,d\rho + d\Psi_D + \frac{\partial B_T}{\partial Y}\,dY\\
&\quad + \frac{\partial B_T}{\partial Y^*}\,dY^* + \frac{\partial B_T}{\partial q}\,dP - \frac{\partial B_T}{\partial q}\,d\pi - \frac{\partial B_T}{\partial q}\,dP^* + d\Psi_F && (29)\\
dY^* &= \frac{\partial Z^*}{\partial Y^*}\,dY^* + \frac{\partial Z^*}{\partial r^*}\,dr^* + d\Psi_D^* + \frac{\partial B_T}{\partial Y}\,dY\\
&\quad - \frac{\partial B_T}{\partial Y^*}\,dY^* - \frac{\partial B_T}{\partial q}\,dP + \frac{\partial B_T}{\partial q}\,d\pi + \frac{\partial B_T}{\partial q}\,dP^* + d\Psi_F && (30)\\
dM + M\,dP &= \frac{\partial L}{\partial r}\,dr^* + \frac{\partial L}{\partial r}\,d\rho + \frac{\partial L}{\partial Y}\,dY + d\Psi_M && (31)\\
dM^* + M^* dP^* &= \frac{\partial L^*}{\partial r^*}\,dr^* + \frac{\partial L^*}{\partial Y^*}\,dY^* + d\Psi_M^* && (32)
\end{aligned}
$$

Since shocks to E_q have the same qualitative effects as shocks to ρ, and shocks to E_p and E_p^* have the same effects the respective countries' demand for money shocks, Ψ_M and Ψ_M^*, these variables are ignored. To analyze shocks to them we simply look at the effects of shocks to ρ, Ψ_M and Ψ_M^*.

The shock to the world real interest rate can be extracted by summing equations (31) and (32).

$$
\begin{aligned}
dM + dM^* \quad &= \quad \left[\frac{\partial L}{\partial r} + \frac{\partial L^*}{\partial r^*}\right] dr^* + \frac{\partial L}{\partial r} d\rho - M dP - M^* dP^* \\
&\quad + \frac{\partial L}{\partial Y} dY + \frac{\partial L^*}{\partial Y^*} dY^* + d\Psi_M + d\Psi_M^* \qquad (33)
\end{aligned}
$$

Dividing both sides of this expression by $M^W = M + M^*$, noting that $L(\ldots) = L = M$ and $L^*(\ldots) = L^* = M^*$, and multiplying and dividing terms by M, M^*, r, r^*, Y and Y^* as appropriate, we can manipulate this expression to yield

$$
\begin{aligned}
\frac{M}{M^W}\frac{dM}{M} + \frac{M^*}{M^W}\frac{dM^*}{M^*} \quad &= \quad \left[\frac{M}{M^W}\frac{\partial L}{\partial r}\frac{r}{L} + \frac{M^*}{M^W}\frac{\partial L^*}{\partial r^*}\frac{r^*}{L^*}\right]\frac{dr^*}{r^*} \\
&\quad + \frac{M}{M^W}\frac{\partial L}{\partial r}\frac{r}{L}\frac{d\rho}{r} + \frac{M}{M^W}\frac{Y}{L}\frac{\partial L}{\partial Y}\frac{dY}{Y} + \frac{M^*}{M^W}\frac{Y^*}{L^*}\frac{\partial L^*}{\partial Y^*}\frac{dY^*}{Y^*} \\
&\quad + \frac{M}{M^W}\frac{dP}{P} + \frac{M^*}{M^W}\frac{dP^*}{P^*} + \frac{M}{M^W}\frac{d\Psi_M}{M} + \frac{M^*}{M^W}\frac{d\Psi_M^*}{M^*} \qquad (34)
\end{aligned}
$$

This can be simplified by substituting in $\delta = M/M^W$, the share of domestic money in the world money supply, and the interest and income elasticities of demand for money

$$
\eta = -\frac{\partial L}{\partial r}\frac{r}{L} \quad \eta^* = -\frac{\partial L^*}{\partial r^*}\frac{r^*}{L^*} \quad \epsilon = \frac{\partial L}{\partial Y}\frac{Y}{L} \quad \epsilon^* = \frac{\partial L^*}{\partial Y^*}\frac{Y^*}{L^*}
$$

to obtain

$$
\begin{aligned}
\delta\frac{dM}{M} + (1-\delta)\frac{dM^*}{M^*} \quad &= \quad -[\delta\,\eta + (1-\delta)\,\eta^*]\frac{dr^*}{r^*} \\
&\quad - \delta\,\eta\,\frac{d\rho}{r} + \delta\,\epsilon\,\frac{dY}{Y} + (1-\delta)\,\epsilon^*\,\frac{dY^*}{Y^*} \\
&\quad + \delta\,\frac{dP}{P} + (1-\delta)\,\frac{dP^*}{P^*} + \delta\,\frac{d\Psi_M}{M} + (1-\delta)\,\frac{d\Psi_M^*}{M^*}. \qquad (35)
\end{aligned}
$$

When we move dr^*/r^* to the left side, this becomes

$$
\begin{aligned}
\frac{dr^*}{r^*} \quad &= \quad -\frac{\delta}{\eta_w}\frac{dM}{M} - \frac{1-\delta}{\eta_w}\frac{dM^*}{M^*} + \frac{\delta}{\eta_w}\frac{dP}{P} + \frac{1-\delta}{\eta_w}\frac{dP^*}{P^*} - \frac{\delta\,\eta}{\eta_w}\frac{d\rho}{r} \\
&\quad + \frac{\delta}{\eta_w}\,\epsilon\,\frac{dY}{Y} + \frac{1-\delta}{\eta_w}\,\epsilon^*\,\frac{dY^*}{Y^*} + \frac{\delta}{\eta_w}\frac{d\Psi_M}{M} + \frac{1-\delta}{\eta_w}\frac{d\Psi_M^*}{M^*}, \qquad (36)
\end{aligned}
$$

where $\eta_w = \delta\,(r^*/r)\,\eta + (1-\delta)\,\eta^*$. The immediate effect of a money supply shock in a country on the domestic and world interest rate is proportional to the share of that country's real money stock in the world real money stock—i.e., to the size of the country. Small countries have little effect on the world interest rate.

Summing (31) and (32) to produce (36) does not eliminate an equation from the system. Full specification of asset equilibrium now requires (36) plus either one of the two demand for money equations or a single equation obtained by substituting (31) into (32) to eliminate r^*,

$$\begin{aligned}
&\frac{1}{\eta}\left[\frac{dM}{M}-\frac{dP}{P}-\frac{d\Psi_M}{\Psi_M}\right]+\frac{d\rho}{r}-\frac{\epsilon}{\eta}\frac{dY}{Y} \\
&= \frac{1}{\eta^*}\left[\frac{dM^*}{M^*}-\frac{dP^*}{P^*}-\frac{d\Psi_M^*}{\Psi_M^*}\right]-\frac{\epsilon^*}{\eta^*}\frac{dY^*}{Y^*}. \qquad (37)
\end{aligned}$$

The two real goods market equations (29) and (30) can be simplified in the same way. Assuming that the trade balance is initially zero, that the risk premium on domestic assets is initially zero, and that the initial ratio of domestic to foreign output equals the ratio of the domestic to foreign money stock—i.e., that initially

$$Y = Z(\ldots) = Z, \qquad Y^* = Z^*(\ldots) = Z^*,$$

$$r = r^*,$$

$$\frac{Y}{Y^*} = \frac{\delta}{1-\delta}$$

—we have

$$\begin{aligned}
\frac{dY}{Y} &= \vartheta\frac{dY}{Y}-\gamma\frac{dr^*}{r^*}-\gamma\frac{d\rho}{r}+\frac{d\Psi_D}{Y} \\
&\quad -\mu\frac{dY}{Y}+\frac{1-\delta}{\delta}\mu^*\frac{dY^*}{Y^*}-\sigma\frac{dP}{P}+\sigma\frac{d\pi}{\pi}+\sigma\frac{dP^*}{P^*}+\frac{d\Psi_F}{Y} \qquad (38) \\
\frac{dY^*}{Y^*} &= \vartheta^*\frac{dY^*}{Y^*}-\gamma^*\frac{dr^*}{r^*}+\frac{d\Psi_D^*}{Y^*}+\frac{\delta}{1-\delta}\mu\frac{dY}{Y}-\mu^*\frac{dY^*}{Y^*} \\
&\quad +\frac{\sigma\delta}{1-\delta}\frac{dP}{P}-\frac{\sigma\delta}{1-\delta}\frac{d\pi}{\pi}-\frac{\sigma\delta}{1-\delta}\frac{dP^*}{P^*}+\frac{d\Psi_F}{Y^*}. \qquad (39)
\end{aligned}$$

Here

$$\vartheta = \frac{\partial Z}{\partial Y} = \frac{\partial Z}{\partial Y}\frac{Y}{Z}$$

and

$$\vartheta = \frac{\partial Z^*}{\partial Y^*} = \frac{\partial Z^*}{\partial Y^*}\frac{Y^*}{Z^*}$$

are the home-country income elasticities of aggregate demand for home goods,

$$\gamma = -\frac{\partial Z \, r}{\partial r \, Z}$$

and

$$\gamma^* = -\frac{\partial Z^* \, r^*}{\partial r^* \, Z^*}$$

are the interest elasticities of demand for home goods,

$$\mu = -\frac{\partial B_T}{\partial Y} = \frac{\partial B_T \, Y}{\partial Y \, Z}$$

and

$$\mu^* = -\frac{\partial B_T}{\partial Y^*} - \frac{\partial B_T \, Y^*}{\partial Y^* \, Z^*}$$

are the domestic and foreign income elasticities of demand for imports, and

$$\sigma = -\frac{1}{Y}\frac{\partial B_T}{\partial q}$$

is the elasticity of the domestic trade balance with respect to the real exchange rate.

These equations can be simplified in different ways for each type of international monetary system and depending upon whether there is full employment or less than full employment.

4 Flexible Exchange Rates

Under flexible exchange rates the relevant equations above apply with no modification other than the imposition of a zero change in the price levels under less-than-full-employment conditions and a zero change in incomes under full employment.

4.1 Less Than Full Employment

$$\begin{aligned} \frac{dY}{Y} &= \vartheta \frac{dY}{Y} - \gamma \frac{dr^*}{r^*} - \gamma \frac{d\rho}{r} + \frac{d\Psi_D}{Y} \\ &\quad - \mu \frac{dY}{Y} + \frac{1-\delta}{\delta} \mu^* \frac{dY^*}{Y^*} + \sigma \frac{d\pi}{\pi} + \frac{d\Psi_F}{Y} \end{aligned} \tag{40}$$

$$\begin{aligned}
\frac{dY^*}{Y^*} &= \vartheta^* \frac{dY^*}{Y^*} - \gamma^* \frac{dr^*}{r^*} + \frac{d\Psi_D^*}{Y^*} + \frac{\delta}{1-\delta}\mu \frac{dY}{Y} \\
&\quad - \mu^* \frac{dY^*}{Y^*} - \frac{\sigma\delta}{1-\delta}\frac{d\pi}{\pi} + \frac{d\Psi_F}{Y^*}. \qquad (41)\\
\frac{dr^*}{r^*} &= -\frac{\delta}{\eta_w}\frac{dM}{M} - \frac{1-\delta}{\eta_w}\frac{dM^*}{M^*} - \frac{\delta\,\eta}{\eta_w}\frac{d\rho}{r} + \frac{\delta}{\eta_w}\epsilon \frac{dY}{Y} \\
&\quad + \frac{1-\delta}{\eta_w}\epsilon^* \frac{dY^*}{Y^*} + \frac{\delta}{\eta_w}\frac{d\Psi_M}{M} + \frac{1-\delta}{\eta_w}\frac{d\Psi_M^*}{M^*}, \qquad (42)\\
\frac{1}{\eta}\left[\frac{dM}{M} - \frac{d\Psi_M}{\Psi_M}\right] + \frac{d\rho}{r} - \frac{\epsilon}{\eta}\frac{dY}{Y} &= \frac{1}{\eta^*}\left[\frac{dM^*}{M^*} - \frac{d\Psi_M^*}{\Psi_M^*}\right] - \frac{\epsilon^*}{\eta^*}\frac{dY^*}{Y^*} \qquad (43)
\end{aligned}$$

These four equations solve for the equilibrium relative changes in Y, Y^*, r^*, and π, given exogenous shocks to the domestic and foreign supplies and demands for money, the risk premium on domestic securities, the aggregate demands for domestic and foreign goods and the balance of trade.

4.2 Full Employment

$$\begin{aligned}
0 &= -\gamma \frac{dr^*}{r^*} - \gamma \frac{d\rho}{r} + \frac{d\Psi_D}{Y} - \sigma \frac{dP}{P} + \sigma \frac{d\pi}{\pi} + \sigma \frac{dP^*}{P^*} + \frac{d\Psi_F}{Y} \qquad (44)\\
0 &= -\gamma^* \frac{dr^*}{r^*} + \frac{d\Psi_D^*}{Y^*} + \frac{\sigma\delta}{-\delta}\frac{dP}{P} - \frac{\sigma\delta}{1-\delta}\frac{d\pi}{\pi} - \frac{\sigma\delta}{1-\delta}\frac{dP^*}{P^*} + \frac{d\Psi_F}{Y^*} \qquad (45)\\
\frac{dr^*}{r^*} &= -\frac{\delta}{\eta_w}\frac{dM}{M} - \frac{1-\delta}{\eta_w}\frac{dM^*}{M^*} + \frac{\delta}{\eta_w}\frac{dP}{P} + \frac{1-\delta}{\eta_w}\frac{dP^*}{P^*} - \frac{\delta\,\eta}{\eta_w}\frac{d\rho}{r} \\
&\quad + \frac{\delta}{\eta_w}\frac{d\Psi_M}{M} + \frac{1-\delta}{\eta_w}\frac{d\Psi_M^*}{M^*} \qquad (46)\\
\frac{1}{\eta}\left[\frac{dM}{M} - \frac{dP}{P} - \frac{d\Psi_M}{\Psi_M}\right] + \frac{d\rho}{r} &= \frac{1}{\eta^*}\left[\frac{dM^*}{M^*} - \frac{dP^*}{P^*} - \frac{d\Psi_M^*}{\Psi_M^*}\right] \qquad (47)
\end{aligned}$$

These four equations solve for the equilibrium relative changes in the domestic and foreign price levels, the world real interest rate r^*, and the nominal exchange rate, given exogenous real and monetary shocks noted in the less-than-full-employment case.

5 Fixed Exchange Rates with a Key Currency

Under a key currency system the flow equations remain essentially the same as above except for the imposition of a zero change in the nominal exchange rate. Shocks to stock equilibrium are best portrayed in this case by working with the demand for money conditions directly.

The stock of money in the peripheral country, which we will assume to be the domestic (unstarred) country will equal the sum of the domestic source component D and the stock of official reserves R. That is,

$$M \quad = \quad R + D. \tag{48}$$

By fixing its exchange rate, the domestic economy renders its money supply endogenous, so we can replace dM with

$$dR + dD,$$

whence

$$\frac{dM}{M} \quad = \quad \frac{dD}{M} + \frac{dR}{M} \tag{49}$$

To the extent that the domestic authorities hold foreign cash as reserves, an increase in domestic official reserves will reduce the nominal money supply abroad. But official reserves may also be held in securities rather than cash, in which case changes in domestic reserve holdings will not put foreign money into and out of circulation. Letting θ be the share of domestic foreign exchange reserves held as cash, we can express the foreign money stock as

$$M^* \quad = \quad D^* - \theta R \tag{50}$$

so that

$$\frac{dM^*}{M^*} \quad = \quad \frac{dD^*}{D^*} - \theta \, \frac{dR}{D^*} \tag{51}$$

The most convenient representation of the conditions of world stock equilibrium in this case is simply the domestic and foreign equations of monetary equilibrium with (49), (51) and (24) substituted into them.

The system of equations relevant to the fixed exchange rate, key-currency case are then as follows.

5.1 Less Than Full Employment

$$
\begin{aligned}
\frac{dY}{Y} &= \vartheta\,\frac{dY}{Y} - \gamma\,\frac{dr^*}{r^*} - \gamma\,\frac{d\rho}{r} + \frac{d\Psi_D}{Y} - \mu\frac{dY}{Y} \\
&\qquad + \frac{1-\delta}{\delta}\,\mu^*\,\frac{dY^*}{Y^*} + \frac{d\Psi_F}{Y} && (52)\\
\frac{dY^*}{Y^*} &= \vartheta^*\,\frac{dY^*}{Y^*} - \gamma^*\frac{dr^*}{r^*} + \frac{d\Psi_D^*}{Y^*} + \frac{\delta}{1-\delta}\,\mu\frac{dY}{Y} - \mu^*\,\frac{dY^*}{Y^*} + \frac{d\Psi_F}{Y^*} && (53)\\
\frac{dr^*}{r^*} &= -\frac{1}{\eta^*}\left[\frac{dD^*}{M^*} - \theta\,\frac{dR}{dM^*} - \frac{d\Psi_M^*}{M^*}\right] + \frac{\epsilon^*}{\eta^*}\,\frac{dY*}{Y*} && (54)\\
\frac{dR}{M} &= -\eta\,\frac{dr^*}{r^*} - \eta\frac{d\rho}{r} + \epsilon\,\frac{dY}{Y} + \frac{d\Psi_M}{M} - \frac{dD}{M} && (55)
\end{aligned}
$$

Note here that if the peripheral country holds all its reserves in short-term key-currency non-monetary assets, θ equals zero and the three equations (52), (53) and (54) solve for the equilibrium relative changes in domestic and foreign outputs and the world interest rate independently of what happens in the domestic economy. These rest-of-world solutions then plug into (55) to obtain the equilibrium change in domestic foreign exchange reserves as a fraction of the money stock. Any independent domestic monetary policy, represented by a shock to D will be neutralized by an automatic reduction in official reserves R.

5.2 Full Employment

$$
\begin{aligned}
0 &= -\gamma\,\frac{dr^*}{r^*} - \gamma\,\frac{d\rho}{r} + \frac{d\Psi_D}{Y} - \sigma\,\frac{dP}{P} + \sigma\,\frac{dP^*}{P^*} + \frac{d\Psi_F}{Y} && (56)\\
0 &= -\gamma^*\frac{dr^*}{r^*} + \frac{d\Psi_D^*}{Y^*} + \frac{\sigma\delta}{1-\delta}\,\frac{dP}{P} - \frac{\sigma\delta}{1-\delta}\,\frac{dP^*}{P^*} + \frac{d\Psi_F}{Y^*} && (57)\\
\frac{dr^*}{r^*} &= -\frac{1}{\eta^*}\left[\frac{dD^*}{M^*} - \theta\,\frac{dR}{dM^*} - \frac{dP^*}{P^*} - \frac{d\Psi_M^*}{M^*}\right] && (58)\\
\frac{dR}{M} &= \frac{dP}{M} - \eta\,\frac{dr^*}{r^*} - \eta\frac{d\rho}{r} + \frac{d\Psi_M}{M} - \frac{dD}{M} && (59)
\end{aligned}
$$

Note again that if the peripheral country holds all its reserves in short-term key-currency non-monetary assets, θ equals zero and the three equations (56), (57) and (58) solve for the equilibrium relative changes in the domestic and foreign price levels and the world interest rate independently of what happens in the domestic economy. These rest-of-world solutions can then be substituted into (59) to obtain the equilibrium change in domestic foreign exchange reserves as a fraction of initial money holdings. Any attempt at independent domestic monetary policy will be neutralized by an automatic reduction in official reserves.

6 Fixed Exchange Rates with a Common Currency

The real sector equations remain the same when we have a fixed exchange rate with a common currency but again small adjustments have to be made to the asset equations. We now have a world money supply equation

$$G_W = M + M^* \tag{60}$$

where G_W is the world money (e.g., gold) stock. This means that

$$\frac{\delta}{\eta_w}\frac{dM}{M} + \frac{1-\delta}{\eta_w}\frac{dM^*}{M^*} = \frac{dG_W}{G_W}. \tag{61}$$

This expression can be substituted into equation (36) to obtain one of the now three equations determining asset or stock equilibrium. The other two equations are simply the equations of the individual countries' now endogenously determined money supply shocks, represented by their respective demand for money shocks. Thus we have the following equations for the less-than-full-employment and full-employment cases respectively.

6.1 Less Than Full Employment

$$\begin{aligned}
\frac{dY}{Y} &= \vartheta\frac{dY}{Y} - \gamma\frac{dr^*}{r^*} - \gamma\frac{d\rho}{r} + \frac{d\Psi_D}{Y} - \mu\frac{dY}{Y} \\
&\qquad + \frac{1-\delta}{\delta}\mu^*\frac{dY^*}{Y^*} + \frac{d\Psi_F}{Y} && (62) \\
\frac{dY^*}{Y^*} &= \vartheta^*\frac{dY^*}{Y^*} - \gamma^*\frac{dr^*}{r^*} + \frac{d\Psi^*_D}{Y^*} + \frac{\delta}{1-\delta}\mu\frac{dY}{Y} \\
&\qquad - \mu^*\frac{dY^*}{Y^*} + \frac{d\Psi_F}{Y^*} && (63) \\
\frac{dr^*}{r^*} &= -\frac{dG_W}{G_W} - \frac{\delta\,\eta}{\eta_w}\frac{d\rho}{r} + \frac{\delta}{\eta_w}\epsilon\frac{dY}{Y} \\
&\qquad + \frac{1-\delta}{\eta_w}\epsilon^*\frac{dY^*}{Y^*} + \frac{\delta}{\eta_w}\frac{d\Psi_M}{M} + \frac{1-\delta}{\eta_w}\frac{d\Psi^*_M}{M^*}, && (64) \\
\frac{dM^*}{M^*} &= -\eta^*\frac{dr^*}{r^*} + \epsilon^*\frac{dY^*}{Y^*} + \frac{d\Psi^*_M}{\Psi^*_M} && (65)
\end{aligned}$$

$$\frac{dM}{M} = -\eta\,\frac{dr^*}{r^*} + \epsilon\,\frac{dY}{Y} + \frac{d\Psi_M}{\Psi_M} \tag{66}$$

The top three equations, (62), (63) and (64) solve for the equilibrium relative changes in Y, Y^* and r^*, given changes in the world stock of money (or gold) and the other exogenous shocks. These solutions then plug into (64) and (66) to yield the endogenous relative changes in the domestic and foreign nominal money stocks. Notice here that the adjustment of the separate countries' money stocks occurs instantaneously as the consequence of the continued maintenance of portfolio equilibrium by their residents through international exchanges of money and non-monetary assets.

6.2 Full Employment

$$\begin{aligned}
0 &= -\,\gamma\,\frac{dr^*}{r^*} - \gamma\,\frac{d\rho}{r} + \frac{d\Psi_D}{Y} - \sigma\,\frac{dP}{P} + \sigma\,\frac{dP^*}{P^*} + \frac{d\Psi_F}{Y} && (67)\\
0 &= -\,\gamma^*\frac{dr^*}{r^*} + \frac{d\Psi_D^*}{Y^*} + \frac{\delta}{1-\delta}\,\frac{dP}{P} - \frac{\delta}{1-\delta}\,\frac{dP^*}{P^*} + \frac{d\Psi_F}{Y^*} && (68)\\
\frac{dr^*}{r^*} &= -\frac{dG^w}{G^w} + \frac{\delta}{\eta_w}\,\frac{dP}{P} + \frac{1-\delta}{\eta_w}\,\frac{dP^*}{P^*} - \frac{\delta\,\eta}{\eta_w}\,\frac{d\rho}{r}\\
&\qquad + \frac{\delta}{\eta_w}\,\frac{d\Psi_M}{M} + \frac{1-\delta}{\eta_w}\,\frac{d\Psi_M^*}{M^*} && (69)\\
\frac{dM^*}{M^*} &= \frac{dP^*}{P^*} - \eta^*\,\frac{dr^*}{r^*} + \frac{d\Psi_M^*}{\Psi_M^*} && (70)\\
\frac{dM}{M} &= \frac{dP}{P} - \eta\,\frac{dr^*}{r^*} + \frac{d\Psi_M}{\Psi_M} && (71)
\end{aligned}$$

Again, the top three equations, solve for the equilibrium relative changes in P, P^* and r^*, given changes in the world stock of money (or gold) and the other exogenous shocks. These solutions then plug into the bottom two equations to yield the endogenous relative changes in the domestic and foreign nominal money stocks. Note again that the adjustment of the separate countries' money stocks occurs instantaneously as the consequence of the continued maintenance of portfolio equilibrium by their residents.

7 Equilibrium With Zero International Capital Mobility

The preceding model assumed that people are free to trade assets across the international boundary. It allowed for the fact that domestic and foreign assets are typically imperfect substitutes in portfolios by including exogenous changes in the risk premium on domestic assets ρ. It was also consistent with the possibility that there may be restrictions on the sale abroad of particular types of assets such as certain non-renewable natural resources, equity shares in commercial banks and other financial institutions and so forth.

Now we turn to a more traditional model which allows for no international mobility of capital in response to market forces—to make things simple it will be assumed that there can be no international transactions in capital assets other than official foreign exchange reserves.

These new assumptions leave the flow equilibrium conditions essentially unaltered but require a substantial change in the way we view asset equilibrium. The connection between the two countries' interest rates imposed by equation (24) now disappears. Each country's real interest rate is now determined by domestic asset conditions alone. World asset equilibrium now consists of the two money market equilibrium equations which can be written as follows.

$$\frac{dM^*}{M^*} = \frac{dP^*}{P^*} - \eta^* \frac{dr^*}{r^*} + \epsilon^* \frac{dY^*}{Y^*} + \frac{dPsi^*_M}{\Psi^*_M} \tag{72}$$

$$\frac{dM}{M} = \frac{dP}{P} - \eta \frac{dr}{r} + \epsilon \frac{dY}{Y} + \frac{dPsi_M}{\Psi_M} \tag{73}$$

The flow equilibrium conditions are as before except that equation (24) can no longer be used to eliminate the domestic interest rate.

$$\begin{aligned} \frac{dY}{Y} &= \vartheta \frac{dY}{Y} - \gamma \frac{dr}{r} - \gamma \frac{d\rho}{r} + \frac{d\Psi_D}{Y} \\ &\quad - \mu \frac{dY}{Y} + \frac{1-\delta}{\delta} \mu^* \frac{dY^*}{Y^*} - \sigma \frac{dP}{P} + \sigma \frac{d\pi}{\pi} + \sigma \frac{dP^*}{P^*} + \frac{d\Psi_F}{Y} \end{aligned} \tag{74}$$

$$\begin{aligned} \frac{dY^*}{Y^*} &= \vartheta^* \frac{dY^*}{Y^*} - \gamma^* \frac{dr^*}{r^*} + \frac{d\Psi^*_D}{Y^*} + \frac{\delta}{1-\delta} \mu \frac{dY}{Y} - \mu^* \frac{dY^*}{Y^*} \\ &\quad + \frac{\sigma\delta}{1-\delta} \frac{dP}{P} - \frac{\sigma\delta}{1-\delta} \frac{d\pi}{\pi} - \frac{\sigma\delta}{1-\delta} \frac{dP^*}{P^*} + \frac{d\Psi_F}{Y^*} \end{aligned} \tag{75}$$

Under a fixed exchange rate regime, these equations solve for the equilibrium relative changes in r and r^* and either Y and Y^* or P and P^*, depending upon whether there is full employment, given the exogenously determined level of π and the underlying exogenous real and monetary shocks in the two countries. When the exchange rate is flexible, however, we need another equation to determine the equilibrium relative change in π. This is taken care of by adding the standard balance of payments equation

$$B \;=\; \frac{dR}{dt} \;=\; B_T(Y, Y^*, \frac{P}{\pi P^*}, \Psi_F) \tag{76}$$

where B is the balance of payments surplus, representing the flow of additions to the stock of foreign exchange reserves. When we take the differential of this equation at a point in time we obtain

$$\frac{1}{Y}\,d\left(\frac{dR}{dt}\right) \;=\; -\mu\frac{dY}{Y} + \frac{1-\delta}{\delta}\,\mu^*\,\frac{dY^*}{Y^*} - \sigma\,\frac{dP}{P} + \sigma\,\frac{d\pi}{\pi} + \sigma\,\frac{dP^*}{P^*}. \tag{77}$$

Under a flexible exchange rate the stock of official reserves is constant, so

$$-\mu\frac{dY}{Y} + \frac{1-\delta}{\delta}\,\mu^*\,\frac{dY^*}{Y^*} - \sigma\,\frac{dP}{P} + \sigma\,\frac{d\pi}{\pi} + \sigma\,\frac{dP^*}{P^*} \;=\; 0. \tag{78}$$

This reduces the flow equilibrium equations to

$$\frac{dY}{Y} \;=\; \vartheta\,\frac{dY}{Y} - \gamma\,\frac{dr}{r} - \gamma\,\frac{d\rho}{r} + \frac{d\Psi_D}{Y} \tag{79}$$

$$\frac{dY^*}{Y^*} \;=\; \vartheta^*\,\frac{dY^*}{Y^*} - \gamma^*\frac{dr^*}{r^*} + \frac{d\Psi_D^*}{Y^*}. \tag{80}$$

In this case equations (73) and (79) solve for the equilibrium relative changes in r and P or Y, and (72) and (80) solve for the equilibrium relative changes in r^* and P^* or Y^* with the solutions for the relative changes in the price and/or output levels then plugging into (78) to yield the equilibrium relative change in π. When the exchange rate is fixed the solution is a bit more complicated. The four equations (72), (73), (74) and (75) solve for the relative changes to the two interest rates and the two countries' output or price levels, and the solutions for the relative changes in outputs or prices can then be substituted into the balance of payments equation (77) to obtain the change the balance of payments surplus—that is, in the rate of growth of official foreign exchange reserve holdings.

8 An Error to Avoid

Back in the 1960s, before the stock-flow nature of equilibrium in the presence of international capital mobility was fully understood, international capital movements were incorporated by simply modifying the balance of payments equilibrium equation (76) to add market determined net capital flows as a function of the ratio of domestic to foreign interest rates.

$$B \;=\; \frac{dR}{dt} \;=\; B_T(Y, Y^*, \frac{P}{\pi P^*}, \Psi_F) + N\left(\frac{r}{r^*}\right) \tag{81}$$

This formulation generates a curve, usually called BB or BP, which is added to the standard IS/LM model. It is tempting to do this. What could be more reasonable than assuming that capital flows to the country where interest rates are highest at a rate depending on the magnitude of the difference in interest rates?

This illustrates a common feature of economics—what seems obvious is often wrong. The formulation contains a fallacy of composition. Any individual who observes a higher interest rate in, say, the domestic economy and whose attitude to risk remains unchanged will surely shift some of his/her portfolio to the securities with the higher interest rate. But what happens when everyone tries to do this? Asset prices in the low interest rate country will be bid down, and the interest rate in that country up, while asset prices in the high interest rate country will be bid up and the interest rate in that country down. This process will stop when the relative interest rates in the two countries reflect the risk premium on the domestic asset, as indicated by equation (24). When world asset holders can freely exchange assets across international boundaries at every point in time, the interest rates on those assets will immediately adjust to reflect their relative risks. Capital will not flow in response to interest differentials because at every point in time those interest rate differentials will adjust to render such flows unprofitable. At every moment in time, interest differentials will be such as to make the existing stocks of capital employed in every country willingly held by world asset holders. Capital will then flow in response to international differences in savings relative to investment across countries. The equality of these flows with the current account balance is a condition of flow equilibrium—that is, in equilibrium desired savings minus investment in every country must be equal to the desired net capital outflow.

Unfortunately, the BB curve still appears in too many textbooks, though no longer in the best ones. A similar formulation also appears in many elementary texts in economic principles and some not-so-elementary texts in economic history in describing the so-called price-specie-flow mechanism of gold standard adjustment. Suppose that there is an increase, say through gold discoveries, in the stock of monetary gold in a particular country. The result, it is claimed, will be a bidding up of the price level in that country relative to the price level in the rest of the world. This will make the country's goods more expensive, causing an increase in its imports and decline in its exports and thereby resulting in an unfavorable balance of payments (that is, a balance of payments deficit). Gold will then flow out of the country in which it is in surplus to finance that country's unfavorable payments balance, increasing the money supply in the rest of the world and causing prices to rise there while at the same time moderating the rise in prices in the gold-surplus country. The process stops when an equilibrium relationship between the two countries' price levels has been reestablished and, as a result, the balance of payments disequilibrium has been eliminated.

The above gold-standard adjustment argument correctly describes the situation when there is zero capital mobility. When non-monetary assets cannot be exchanged across international boundaries, the only way that residents of a country with too much gold can convert that gold into other assets is through a deficit in the balance of trade and current account balance. Foreign goods, ultimately capital, are purchased with the excess money balances. International exchanges of assets for money being prohibited, capital can only be moved between countries by changing their levels of savings and investment and, concomitantly, changing exports relative to imports.

When people are free to exchange assets across international boundaries at any time, as indeed was the case in the hey day of the gold standard, residents of the country with excess gold will simply sell their surplus gold to people in the rest of the world for part of the existing world stock of capital assets—end of story! No change in exports or imports is required.