Open-Economy Macroeconomic Theory: A Basic Framework

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1 Introduction

The goal here is to develop a general equilibrium framework for analysing the effects of domestic and rest-of-world real and monetary shocks on interest rates, real and nominal exchange rates, output, income and prices from the point of view of an open economy. Our focus is on short-term rather than long-term steady state effects. The framework developed will be similar in structure to the standard IS-LM model which has a long history and is still used in intermediate textbooks.[1]

Although the standard IS-LM framework can be used to analyse the issues of concern here provided that the scope of application is limited to avoid serious errors that can result from interpretations of the role of government, it suffers from the fact that it is neither conceptually based on nor rigorously derived from intertemporal utility maximisation. The purpose here is to drive a model from proper theoretical foundations that will enable

*I would like to thank Allan Hynes for helpful discussions of the issues covered in this paper.

[1]The classic model whose central features will be reproduced here can be found in R. A. Mundell's famous paper "Capital Mobility and Stabilisation Policy Under Fixed and Flexible Exchange Rates," *Canadian Journal of Economics and Political Science*, Vol. 29, No. 4 (November), 1963, 475-85. The basic framework of this model is much more broadly applicable—for example to imperfect capital mobility—than was thought at the time of its conception. Subsequent refinements to handle imperfect capital mobility, both by Mundell and others, involving the introduction of what came to be known as the BB curve, turn out in retrospect to have been incorrect and unnecessary. One needs only to set the domestic real interest rate equal to the foreign rate plus a risk premium plus the expected rate of change of the domestic real exchange rate.

us to analyse broad issues that cannot be analysed with conventional narrowly focused mathematical models because of the complexities that would have to be introduced into those models.[2]

Short-term monetary and real shocks in our model will result in deviations of the variables of interest from their long-term equilibrium growth paths. Although changes in these steady-state growth paths will not be of concern, but will necessarily be occurring in the background, it is useful to embed these growth paths in the broadest possible framework and thereby make the effects of our short-term shocks independent of restrictive assumptions about the growth process. Accordingly, it is assumed that output is a function of many kinds of capital, among them knowledge, technology, and human capital, all of which are produced at constant cost in terms of each other and in terms of consumer goods.[3] Natural resources can be thought of as present, in different forms, in all countries and ongoing investment in technology will be expected to lead to changes through time in countries' output growth and the relative prices of their outputs in terms of that of the rest of the world. It is assumed that long-term diminishing returns to the natural resource base can be prevented by appropriate investment in knowledge and technology. Rates of time preference can be thought of as constants which may change from time to time. And the allocation of investment among the various types of capital may be subject to institutional distortions that can also change upon occasion. The result will be an AK type of framework with output flows represented by constant, though possibly changing from time to time, marginal products of the aggregate capital stock.[4]

It is assumed that in the absence of money a significant fraction of the output from the aggregate capital stock will be used up in the process of making transactions. The introduction of a monetary asset handed out by government will reduce these transaction costs, thereby increasing the output available for consumption and investment and, hence, the level of income. Representative agents are assumed to have linearly additive intertemporal utility functions.

[2]The best conventional presentation of the wide range of contemporary models is probably the well-known textbook by Maurice Obstfeld and Kenneth Rogoff, *Foundations of International Macroeconomics*, MIT Press, 1996.

[3]This constant cost assumption is a necessary requirement for aggregation of output and capital into clearly definable units.

[4]For a general discussion of AK models and criticisms of them in the literature see Ellen R. McGrattan, "A Defence of AK Growth Models," *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 22, No. 4 (Fall), 1998, 13-27.

Very simple specific functional forms are chosen to establish a structure that will produce a steady state around which short-term monetary and real shocks can generate variations in the relevant variables. Readers interested in details regarding long-term growth will need to modify and refine the model appropriately.

An intuitive representation of the basic model is given in the next section and the model is formally developed in section 3. These two sections can be read in either order. In section 4 the problem of exchange rate overshooting is addressed and some basics with regard to balance of payments equilibrium and the respective roles of the current and capital accounts are reviewed in Section 5. A final section concludes.

2 An Intuitive Representation

2.1 The Basic Model

Our short-term model has a very simply intuitive representation. There are three elements: First, along some steady state consumption path, temporary changes in income will be channelled into investment in order to intertemporally smooth consumption. Given adjustment costs of adapting newly produced capital goods to the existing capital stock that increase with the level of investment, an expansion of investment increases its marginal cost without changing expected future returns. The rate of interest will thus be inversely related to income as it deviates around its steady-state full-employment level. This gives us a negatively sloped curve in Figures 1 and 2 with the real interest rate on the vertical axis and the level of real income on the horizontal one. This curve, similar to the standard IS curve but here called GG denoting short-term goods market equilibrium, will shift in response to broadly defined short-term real shocks based on expectations about the future income from the existing capital stock that may be related under some circumstances to what is happening in an imagined public sector. Any statements about the effects of fiscal policy, however, would have to be based on issue-by-issue analyses that will not be pursued here. Although it is convenient to draw the graph with the origin at a zero interest rate and zero income, we are only interested in deviations of the variables around a defined full-employment income level represented by a vertical line $Y_f Y_f$.[5]

[5] An alternative graphical portrayal of the equilibrium that would be more realistic but less IS-LM-like would be to set the origin at the full-employment levels of the real interest rate and real income.

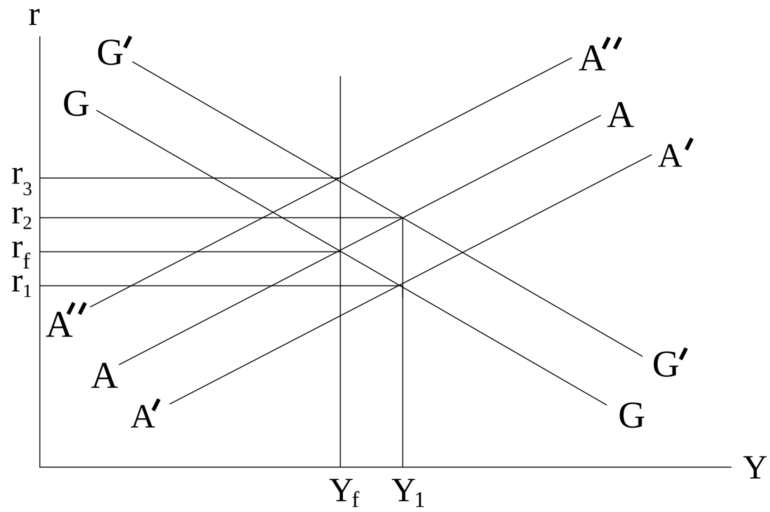

Figure 1: Deviations of income from its full-employment level in a large open economy.

The second element of our intuitive representation is the understanding that short-term increases (decreases) in a country's real exchange rate, here defined as the relative price of its output in terms of rest-of-world output, will shift its GG curve downward (upward). The intuition here is simply that a temporary increase in the a country's output relative to rest-of-world output will cause, or result from, a temporary decline in the world price of that output.

An appropriate inclusion of asset equilibrium is the third element of our model. If the demand for a country's monetary asset by its residents equals the supply, the demand of those residents for the assets represented by capital goods employed at home and abroad must also equal the supply. When assets cannot be owned outside the country in which their income-source resides, asset equilibrium is represented solely by the country's demand function for money. The combinations of the real interest rate and real income for which the demand for money equals any given supply can be represented in Figures 1 and 2 by an upward sloping curve, called AA to represent asset equilibrium. It is the counterpart to the conventional LM curve. The curve is upward sloping because an increase in the real interest rate will increase the opportunity cost of holding the non-interest-bearing monetary asset and reduce the quantity of it demanded, requiring an increase in the volume of transactions associated with an increase in income to make asset holders willing to hold the existing quantity. This curve will shift downward with an increase in the supply of the monetary asset and upward in response to factors that will increase the demand for it.

The AA and GG curves together with the vertical $Y_f Y_f$ line are sufficient for the analysis of short-term monetary and real shocks to income in a large open rest-of-world economy portrayed in Figure 1—changes in a small open economy's real exchange rate with respect to the rest of the world will result in an insufficient relative change in the demand for rest-of-world output to shift the GG curve appreciably.

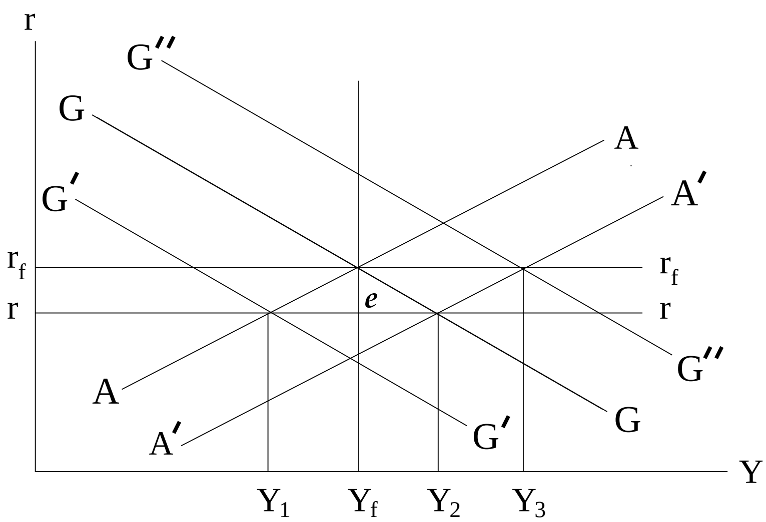

Figure 2: Response of a small open economy to short-term monetary and real shocks.

When world residents own claims to capital stock employed in both a small open economy and the rest of the world, the small country's interest rate must adjust relative to the rest-of-world interest rate so that world residents will be willing to hold the existing mix of small country and rest-of-world employed capital stock. Because of its size, conditions in the small country will have no effect on the rest-of-world interest rate. The result will be a horizontal line on Figure 2, which is a small-country version of Figure 1, at an interest rate the small-country takes as given. The level of this interest rate is imposed by asset and goods market conditions in the rest of the world, by the risk of holding small-country as opposed to rest-of-world capital, and by the expected rate of change in the small country's real exchange rate. An expected increase in the real exchange rate will create an expected future capital gain on investment in the home asset, making the real interest rate required to get world residents to hold it lower, shifting the horizontal interest rate line downward.

2.2 Comparative Statics Implications

The general effects of short-term monetary and real shocks are intuitively straight-forward. Consider first the large rest-of-world economy. An increase in the stock of the monetary asset results in an attempt to convert this excess supply into claims on capital stock, reducing the real interest rate at which this larger stock of money will be willingly held. The AA curve in Figure 1 shifts downward to $A'A'$ and, assuming the nominal price of output is fixed, income rises above its full-employment level to Y_1 and the real interest rate falls to r_1. Income can rise above its full-employment level here because we interpret the latter as the 'normal' level of income rather than the maximum that could possibly be produced. Were the nominal price level flexible, an immediate rise in the price of output would maintain real money holdings at the desired level, thereby keeping the AA curve at its initial level.

An increase in the expected future output from the capital stock, in the absence of any change in current output, will shift the GG curve in Figure 1 upward to $G'G'$ as the result of its effect on the desired level of investment. If nominal prices are constant, income will expand to Y_1 and the real interest rate will rise to r_2. Were prices completely flexible they would rise, reducing the real stock of monetary assets at the unchanged nominal stock sufficiently to drive the AA curve upward to $A''A''$, at which point the real interest rate will have risen to r_3 where the representative agent will willingly hold the lower real money stock at the full-employment level of income. Nothing here is inconsistent with the representative agent reallocating some portion of the full-employment level of output between current consumption and current investment in response to a change in the desired steady-state rate of consumption growth. The point of intersection of $G'G'$ and Y_fY_f will depend on the direction of such a change in investment and the adjustment costs thereof.

Now consider the small open economy represented in Figure 2. The consequences of monetary and real shocks depend critically on whether the exchange rate is fixed or flexible. Start with the case where the nominal exchange rate is fixed and suppose that there is an decline in the demand for liquidity that would shift the AA curve to the right to $A'A'$. The representative agent will attempt to re-establish asset equilibrium by selling the excess holdings of the monetary asset to the rest-of-world agent in return for claims on capital goods. To eliminate downward pressure on the nominal value of the small country's currency arising from the resulting excess supply of it on the foreign exchange market, the authorities are forced to sell foreign exchange reserves, which can be viewed simply as government-held claims on foreign-employed capital, in return for the domestic monetary asset. These claims on foreign-employed capital are, of course, owned indirectly by the domestic representative agent on public account. The result will be a decline in the supply of domestic liquidity equal to the reduction in the demand, causing the AA curve to remain at its initial level. Similarly, if the small country's authorities increase the supply of the monetary asset in the presence of unchanged demand, they will be forced to buy back that excess liquidity in return for foreign exchange reserves and no shift of AA will result. To the extent that foreign exchange reserve changes have a wealth effect induced by forced alterations of the mix of domestic- and foreign-employed capital held by domestic residents on combined private and public account, there may be shifts in the risk premium on domestic-employed capital and the interest rate line. Such shifts will be very small, however, since the change in the asset mix resulting from monetary shocks of the magnitude typically experienced will be a tiny fraction of the initial levels of domestic- and foreign-employed assets held.

Suppose now that there is a short-term real shock in the small economy that shifts its GG curve to the right to $G''G''$ in Figure 2. This real shock can take the form of either an increase the expected future return to domestic-employed capital or a shift in world demand toward the small country's output, with the current full-employment level of output being unaffected in both cases. As domestic income rises above the full-employment level, the demand for liquidity increases, prompting a sale of claims on capital to the rest of the world. To prevent the domestic currency from appreciating, the small country's authorities must supply that desired increase in liquidity by purchasing foreign exchange reserves. The AA curve will thus automatically shift to the right along with the GG curve, ending up at $A'A'$, and income will rise to Y_3. This assumes, of course, that the small country's nominal output price is fixed. Were it completely flexible, it would rise in response to the increased demand for domestic output, raising the small country's real exchange rate and reducing the demand for its output, thereby shifting the GG curve back to its initial position and the AA curve along with it.

Finally, consider the case where the world interest rate falls due to monetary expansion in the rest of the world, reducing the interest rate in the small open economy to r from r_f in Figure 2. Under our assumption that the nominal exchange rate is fixed, the small country's income will rise to

Y_2 and the domestic authorities, to maintain the fixed exchange rate, will increase the stock of monetary assets through the purchase of foreign exchange reserves, driving the AA curve to $A'A'$. The rest-of-world monetary shock is thereby transmitted directly to output in the domestic economy. Were the small country's output price perfectly flexible it would rise to keep output from moving above the full-employment level. This higher price level in the small country will increase its real exchange rate, reducing world demand for its goods and shifting GG to the left. A new full-employment equilibrium will be established at point e where the new AA and GG curves will ultimately intersect. The domestic authorities will have to increase the stock of the monetary asset sufficiently to move AA to this new position, supplying whatever nominal money holdings the representative agent desires at the interest rate r, the income level Y_f and the domestic price level that will produce the real exchange rate required to make the GG curve pass through point e. If the price level in the rest of the world were also to immediately adjust, the small-country's price level and the rest-of-world price level will rise in proportion, maintaining both the AA and GG curves, as well as world interest rates, at their initial levels with output remaining at its full-employment levels in both parts of the world.

We can conclude that when the small country fixes its nominal exchange rate with respect to the rest of the world it's authorities lose any monetary control over the levels of domestic income and prices—they are forced to create whatever stock of liquidity the domestic representative agent chooses to hold. The only avenue of short-term influence over domestic output is the possibility of trying to engineer real shocks by some sort of fiscal policy action. As noted above, the effects of specific fiscal policies can only be determined by a detailed modelling of each individual policy action, which will be impossible within the simple framework being developed here. This confirms the fixed exchange rate part of the the well-known principle attributed to Fleming and Mundell—a fixed exchange rate neutralises domestic monetary shocks and transmits domestic real shocks to income and prices.[6] Indeed, even under continuous full-employment conditions, any changes in the real exchange rate are translated to the domestic price level when the nominal

[6] The principles developed in the previously cited Mundell paper were enunciated one year earlier in a paper by J. Marcus Fleming, "Domestic Financial Policies Under Fixed and Under Floating Exchange Rates," *International Monetary Fund Staff Papers*, Vol. 9, No. 3 (November), 1962, 369-79.

exchange rate is fixed.[7]

An important additional conclusion is that the small country's authorities need never run out of foreign exchange reserves—should reserves become low they can simply sell non-monetary assets in return for the monetary asset or otherwise reduce the stock of liquidity. Then, in response to the resulting upward pressure on the domestic currency in the foreign exchange market, they will be forced to sell the monetary assets so acquired for the desired increase in the stock of foreign exchange reserves. Indeed, there is no reason to have a major balance of payments deficit or surplus in the first place—once the nominal exchange rate has been fixed, the authorities are required to provide the community with its desired stock of liquidity and cannot influence output in the short run or the price level in the long run by trying to do otherwise.[8] The function of the stock of foreign exchange reserves is simply to act as a buffer that provides an automatic mechanism for day-to-day adjustment of domestic agents' money holdings to the desired level. The function of monetary policy is reduced to maintaining an appropriate average level of the stock of foreign exchange reserves.

Now let suppose that the small open economy's nominal exchange rate is flexible. Consider the decline in the demand for liquidity that would shift AA curve to the right to $A'A'$. The attempt of the domestic representative agent to sell monetary assets to foreigners in return for claims on real capital will, assuming a fixed price level, cause the nominal and real exchange rates to devalue, shifting world demand onto domestic output and thereby increasing income to Y_3—the devaluation shifts the GG curve to $G''G''$. An appropriate expansion of the stock of the monetary asset by the small country's authorities, in the absence of any change in the demand for liquidity, would have the same effect. If the small country's price level happens to be

[7] This follows from the definition of the real exchange rate

$$Q = \frac{\Pi P_d}{P_f}$$

where Q is the real exchange rate, Π is the nominal exchange rate defined as the foreign currency price of the domestic currency, and P_d and P_f are the respective domestic and foreign price levels.

[8] One could extend our basic model to introduce the possibility that the authorities could, by changing the stock of foreign exchange reserves, allow (force) domestic residents to hold a more (less) desirable mix of domestic and foreign assets on combined public and private account, thereby changing the level of domestic wealth and the level of real economic activity. It would be difficult to establish, however, that the empirical magnitude of such wealth effects would be of any significance in an economy in which the stock of liquidity is a small fraction, and any changes in it a tiny fraction, of wealth.

completely flexible the excess supply of liquidity resulting either from a reduction of demand for or an increase in the stock of the domestic monetary asset will simply cause the price level to rise until the real stock of the monetary asset is reduced to its desired level. The AA curve is thereby kept at its original position. The nominal exchange rate devalues as the price level rises with the real exchange rate and the GG curve remaining unchanged.

Next, consider a real shock in the small economy that would shift the GG curve to the right to $G''G''$. Any increase in income increases the demand for liquidity which will cause the domestic representative agent to sell claims on real capital to the rest-of-world representative agent to acquire the necessary money holdings. When the price level is fixed, the result is an appreciation of the domestic nominal and real exchange rates. This appreciation will be sufficient to maintain the GG curve at its original position. The small country's income will remain at its initial level. When the small country's price level is flexible there will be no reason for it to change. A net increase in the demand for domestic output is prevented by the appreciation of the nominal and real exchange rates.

Finally, consider the situation where a monetary expansion in the rest of the world reduces the small country's interest rate to r in Figure 2. This decline in the interest rate will increase the demand for the domestic monetary asset at the initial full-employment level of income. Assume that the small country's authorities maintain the stock of the monetary asset constant. Attempts by the domestic representative agent to acquire the desired increase in liquidity by selling claims on real capital to the foreign representative agent will cause the small country's nominal and real exchange rates to appreciate, switching world demand off domestic output and shifting the GG curve to the left to $G'G'$. Given a fixed price level, domestic income will fall to Y_1. A short-term expansion of income in the rest of the world will be accompanied by a fall in the small country's income. If the domestic price level is flexible it will fall, driving the AA curve to the right until it crosses the vertical full-employment income line at the point e. The now smaller appreciation of the small country's real exchange rate will moderate the leftward shift of the GG curve so that it too will pass through point e.

Alternatively, suppose that the decline in the world interest rate was due to a negative real shock in the rest of the world. The effects on the small country's income and price levels will be the same as in case where the interest rate falls on account of a rest-of-world monetary shock. The only difference is that income in the rest of the world will decline rather than increase, moving in the same direction as income in the small country.

To conclude, it is clear that by allowing its exchange rate with respect to the rest of the world to float the small economy renders its monetary policy effective in controlling the level of domestic income in the short run and the price level in the long run. And it neutralises the effects of domestic real shocks on output, income and the price level. This verifies the second half of the Fleming-Mundell principle—that monetary policy is effective under flexible exchange rates while fiscal induced domestic real shocks will be ineffective in changing domestic income and prices.

Allowing exchange rate flexibility does not, however, insulate the small open economy from short-term monetary or real shocks originating in the rest of the world. Holding the domestic supply of liquidity constant, domestic output and prices will move in the opposite direction to rest-of-world output and prices in the case of a rest-of-world monetary shock and in the same direction as rest-of-world output and prices in the case of a rest-of-world real shock.

2.3 Changes in the Equilibrium Full-Employment Income Path

To this point we have been treating real shocks as changes in expected future income or temporary shifts in the rest-of-world demand for a small country's output under conditions where the actual forces determining full-employment output have not changed. The analysis must now be extended to incorporate temporary short-term shocks to employment and income arising from changes in the steady-state growth path of full-employment income. It must be kept in mind that a change in the equilibrium steady-state growth path may, in the short-run, result in a gradual movement of full-employment income to that new equilibrium path.

Continuous full-employment will require constant adjustment of the level of liquidity to 'finance' the changes in full-employment income along the steady-state growth path—if the supply of the monetary asset does not adjust to meet the demand, short-term changes in employment and deviations of income from its full-employment level will result. The steady-state adjustment path is shown in Figure 3. In a situation where the authorities hold the nominal stock of liquidity constant, the rightward adjustment of the AA curve will occur as a result of a negative inflation rate sufficient to increase the real stock of liquidity at the desired rate. To the extent that the authorities increase the stock of the monetary asset at a positive rate, this negative rate of inflation will be moderated or reversed. In the large rest-of-world economy or a small open economy with a flexible exchange rate, stochastic variations of the demand for liquidity that are unmatched by offsetting changes in the supply of the monetary asset will result in corresponding variations of the levels of income and prices. In a small open economy with a fixed exchange rate, stochastic shocks to the demand for liquidity will lead to automatic offsetting changes in the supply as the domestic authorities adjust the stock of foreign exchange reserves as required to maintain the fixed exchange rate.

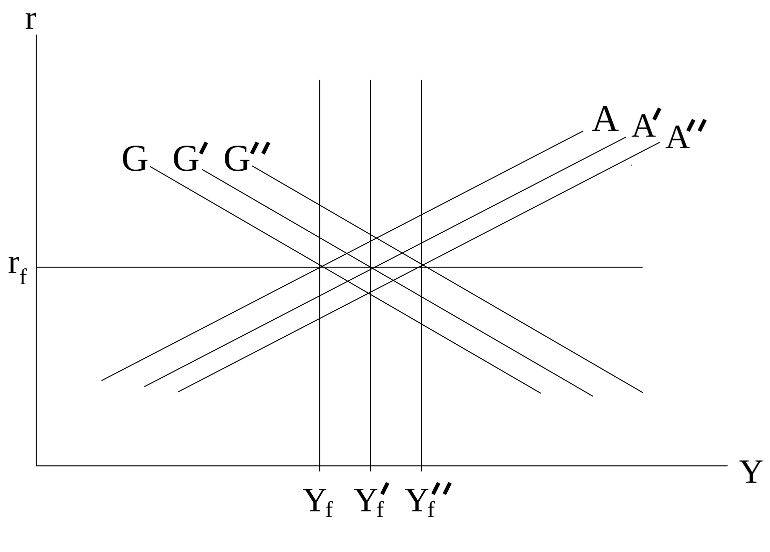

Figure 3: Adjustments along the steady-state growth path.

Now consider changes in the full-employment real interest rate and income in response to the adjustment of the economy to a new steady-state growth rate. The situation with respect to a rest-of-world economy that is too large to be affected by what happens abroad is shown in Figure 4. The current level of full-employment income, which is also the flow of income from the current capital stock, increases from $Y_{\mathtt{f}}$ to $Y_{\mathtt{f}}'$. Since the increase in income known to be permanent, there will be an increase in the full-employment real interest rate equal to the increased flow of real income from from the current capital stock minus the increase in the adjustment costs of adding capital associated with whatever expansion of the level of investment is required to produce the new current rate of income growth. Suppose that the full-employment real interest rate rises from $r_{\mathtt{f}}$ to $r_{\mathtt{f}}'$ in Figure 4. Whether full employment will be maintained in the face of this shock will then depend on the slope of the AA curve. As the curve is drawn in Figure 4, full employment will be maintained. Were that curve steeper, current income would rise by less than the increase in full-employment income and the interest rate would rise above $r'_{\mathtt{f}}$ in the short run. A flatter AA curve than that shown would result in a short-run increase in current income in excess of the rise in its full-employment level and an interest rate between $r_{\mathtt{f}}$ and $r'_{\mathtt{f}}$. Quite clearly, a detailed knowledge of the structure of the economy would be necessary to enable one to predict the direction and magnitude of any short-term deviation of actual income and interest rates from their full-employment levels in response to a permanent increase in the income generated by the existing capital stock.

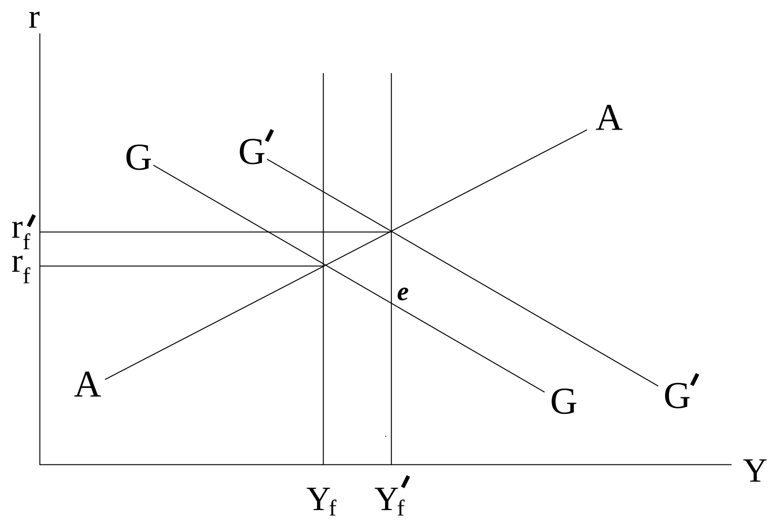

Figure 4: Real adjustments around or toward a new steady-state growth path.

In the case where the change in the full-employment level of income generated by the current stock of capital is consequent on a purely stochastic shock with no anticipation of permanency, the vertical $Y_{\mathtt{f}}$ line will shift as in Figure 4 but the GG curve will remain unchanged. In the absence of an increase in the stock of liquidity, the current level of income will also remain unchanged. For current income to rise in step with it's full-employment level, the authorities will have to increase the stock of the monetary asset sufficiently to shift the AA curve to the right to pass through point e. This will temporarily reduce the full-employment real interest rate as the adjustment costs of adding the entire increase in full-employment income to the capital stock are absorbed. Since the underlying stochastic shock is temporary, permanent income and consumption will be unchanged.

Shocks in a small open economy that are symmetric with those occurring in a rest-of-world economy that is identical except for scale can also be analysed using Figure 4, now interpreted as representing the small economy. The shifts of the curves, adjusted for scale, will be the same for both economies. Nevertheless, the deviations of income from its full-employment level will differ in the small economy as compared to the rest of the world depending upon the exchange rate regime the small country adopts. When the exchange rate is fixed, equilibrium will be determined by the intersection of the *GG* curve and the interest rate line generated by conditions in the rest-of-world economy, with the small-country's authorities forced to supply the stock of liquidity that will drive the *AA* curve through that intersection. When the exchange rate is flexible the short-term equilibrium will be determined by the intersection of the *AA* curve with the same interest rate line. In this case the real exchange rate will adjust to drive the *GG* curve through that intersection.

The effects on the small open economy of shocks in the rest of the world in the absence of domestic shocks can be seen with reference to Figure 5. The shock will be transmitted to the small economy solely through the real interest rate which, let us assume, rises to $\tilde{r}_1$.[9]

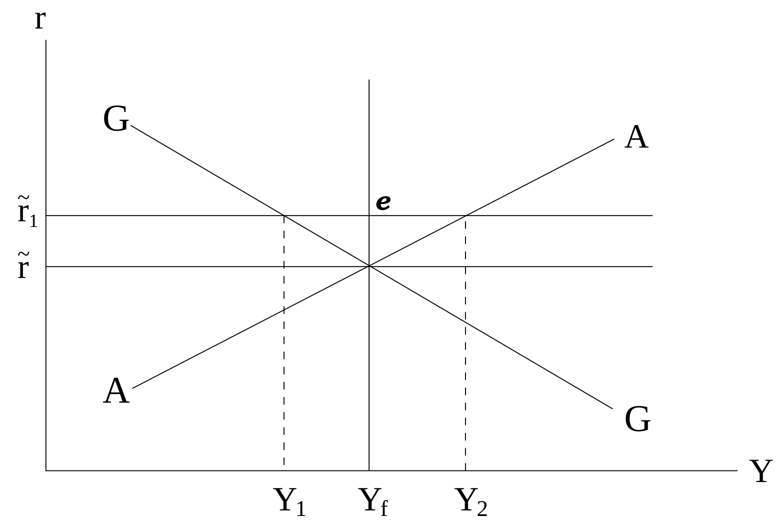

Figure 5: Response of small open economy to a rest-of-world growth shock.

[9]It is possible that the rest-of-world shocks could result in a change in the equilibrium real exchange rate in which case the *GG* curve would also shift. In this event, however, the principles governing the adjustment would be the same as those that follow.

Two cases arise, depending upon whether the exchange rate is fixed or flexible. If it is fixed, domestic income will be determined by the intersection of the GG curve and the new interest rate line, falling to Y_1, with the AA curve shifting to the left as the domestic authorities purchase the domestic monetary asset in return for foreign exchange reserves to keep the home currency from depreciating. If the exchange rate is flexible, income will be determined by the intersection of the AA curve and the interest rate line, at Y_2, with the GG curve shifting to the right in response to a devaluation of the domestic currency that will result from a desire of the domestic representative agent to buy capital assets from the foreign representative agent in an attempt to decumulate the domestic monetary asset. The authorities could prevent the short-term increase in income in the flexible exchange rate case by reducing the stock of liquidity by an amount sufficient to shift the AA curve to the left to pass through the point e, thereby moderating appropriately the devaluation of the domestic currency.

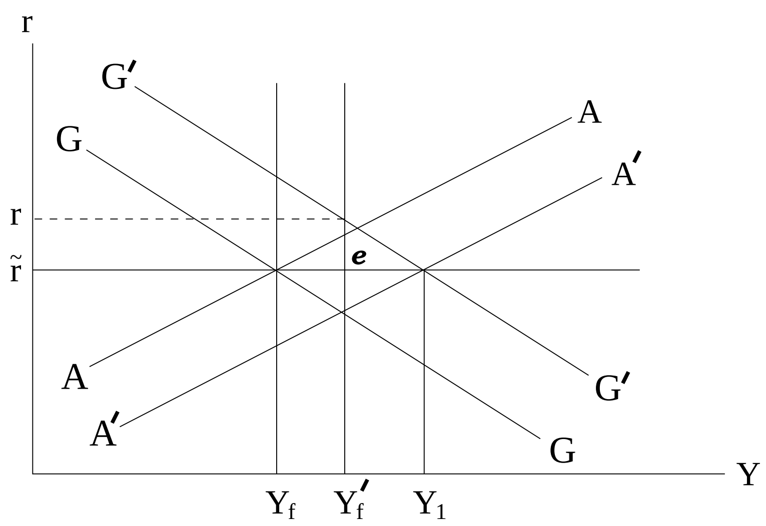

Figure 6: Response of small open economy to a domestic growth shock.

When the price level is flexible, with the exchange rate fixed, both it and the real exchange rate will fall, shifting the GG curve to the right to pass through point e. To keep the nominal exchange rate from depreciating, the authorities are forced to sell foreign exchange reserves in return for the domestic monetary asset, thereby shifting the AA curve to the left to pass through point e. In the case where the price level and the exchange rate are both flexible, the price level will rise sufficiently to shift the AA curve to the left to pass through point e, and the nominal exchange rate will devalue sufficiently to reduce the real exchange rate to the point where the GG curve has shifted to the right to pass through point e.

The effect of a permanent increase in the full-employment level of income generated by the small country's existing capital stock is analysed using Figure 6. The level of income consistent with full employment rises from $Y_{\mathtt{f}}$ to $Y'_{\mathtt{f}}$ and the GG curve shifts to the right to $G'G'$—the world real interest rate at which the new full-employment level of output would be produced without any change in the real exchange rate is given by the dashed line at r. As in previous cases, the short-term effect on the level of real income will depend upon whether the exchange rate is fixed or flexible. Under fixed exchange rates, equilibrium is determined by the intersection of the $G'G'$ curve and the world interest rate line at $\tilde{r}_{\mathtt{f}}$ with the AA curve shifting endogenously to $A'A'$ as a result of the purchase of foreign exchange reserves by the authorities, and equivalent expansion of the stock of liquidity, required to prevent the home currency from appreciating. When the exchange rate is flexible, equilibrium is determined by the intersection of the curve AA with the rest-of-world determined interest rate line and the level of income will remain unchanged at $Y_{\mathtt{f}}$ in the short-run. The GG curve will be maintained at its original pre-shock level by an endogenous appreciation of the home currency. The authorities can engineer an increase in the actual income level equivalent to the increase in the full-employment level by increasing the stock of liquidity and thereby shifting the AA curve to the right sufficiently to intersect the interest rate line at point e. The appreciation of the domestic currency will thereby be moderated so that the GG curve will also pass through that point.

Were the small country's price level perfectly flexible, the new equilibrium would be at point e regardless of the exchange rate regime adopted. In the fixed exchange rate case, the price level will rise, increasing the real exchange rate sufficiently to shift $G'G'$ to the left until it passes through e. And the authorities will be forced to increase the stock of liquidity by purchasing foreign exchange reserves until the AA curve has shifted to the right to pass through e. When the exchange rate is flexible, the domestic price level will fall to shift AA to the right, and the domestic nominal exchange rate will appreciate to drive $G'G'$ to the left, until both curves pass through point e.

2.4 Country Size and Key-Currencies

The analysis must now be extended to examine the situation where the world consists of two big countries. Two principles of interest emerge. The first is that when the exchange rate is flexible the world real interest rate, and output and employment in both countries, will be influenced in the short run by monetary and real shocks in both countries, with the influence of each country depending on its relative size. The second is that when one country, henceforth called the peripheral country, fixes its nominal exchange rate with respect to the other country, henceforth called the key-currency country, and holds its foreign exchange reserves in the form of claims on capital employed in the key-currency country, it loses its ability to influence domestic output and employment by monetary policy. This turns out to be the case regardless of the sizes of the two countries. That is, if the key-currency country is small and the peripheral country is large, the small key-currency country's authorities will end up running world monetary policy. An important exception arises where the peripheral country holds its foreign exchange reserves in the monetary asset of the key-currency country. In this event, the peripheral country can change the stock of liquidity in the key-currency country simply by changing its stock of foreign exchange reserves.

These principles can be illustrated with reference to Figures 7 and 8 which portray a world consisting of two countries of roughly equal size. The

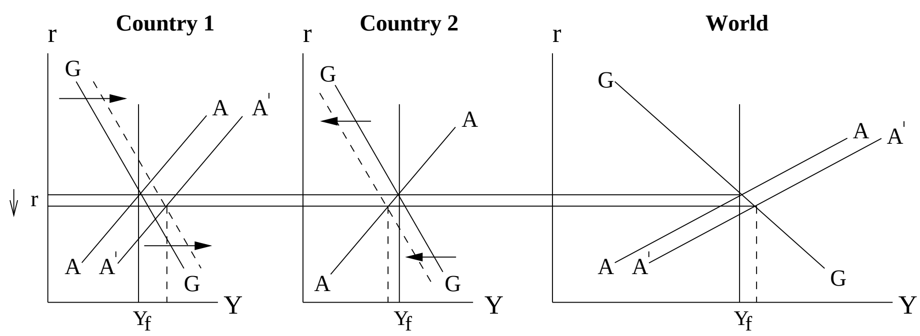

Figure 7: World response to monetary shock in country 1 with two countries of similar size and with a flexible exchange rate.

horizontal distance of each curve from the vertical axis in the right-most panels, representing the entire world, is the sum of the corresponding distances for the panels representing the respective countries. To visualise why this is the case, note first that the sum of the two countries' full-employment

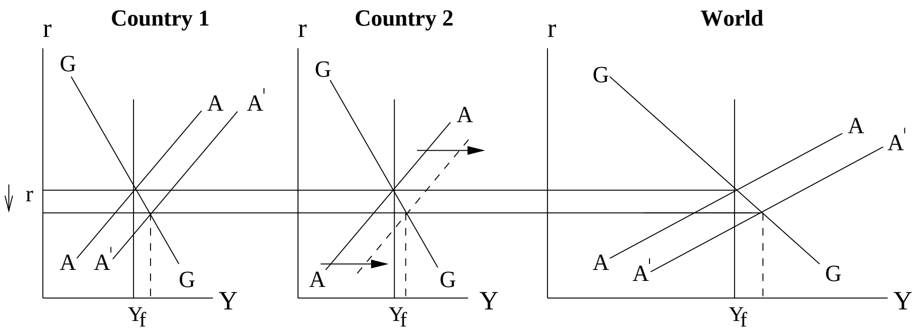

Figure 8: World response to monetary shock in country 1 with two countries of similar size where country 2 fixes the nominal exchange rate.

incomes must equal the world full-employment income. Then start with a given full-employment world real interest rate and imagine that interest rate increasing and decreasing relative to its full-employment position. World income will always be the sum of the two countries' incomes. Now imagine that the stock of liquidity in each country is consistent with full-employment at the world full-employment interest rate. The world income consistent with that level of world liquidity will be the sum of the two countries' full-employment incomes. And as the world interest rate varies around its full-employment level, the level of world income consistent with asset equilibrium will be the sum of the incomes consistent with asset equilibrium in the two countries.

Suppose that there is a positive monetary shock in Country 1 that shifts its AA curve to the right to $A'A'$ in the two figures. This will cause the world AA curve to shift to the right by the same amount, as indicated by $A'A'$ in the third panel on the right in Figure 7. The representative agent in Country 1 tries to purchase real capital assets with her excess holdings of the monetary asset. When the exchange rate is flexible, the resulting excess supply of Country 1's currency in the foreign exchange market will cause it to devalue, shifting world demand to domestic output from the output of Country 2. Country 1's GG curve shifts to the right and Country 2's GG curve to the left. In the short run, income rises in Country 1 at the expense of income in Country 2. At the same time, since Country 1 is large its excess supply of liquidity represents significant excess supply of liquidity in the world as a whole with the result that the world real interest rate falls. In the short-run, therefore, Country 1 gains income both as a result of the fall in the world interest rate and as a result of the shift of world demand to its output from that of Country 2 caused by the decline in the real exchange rate as the nominal exchange rate falls with price levels fixed in the two countries. In the long-run, assuming complete price level flexibility, Country 1's price level will rise, returning its AA curve and the world AA curve to their original levels. Real incomes in both countries will return to their full-employment levels. Country 1's nominal exchange rate with respect to Country 2 will depreciate sufficiently to offset the rise in its price level with the real exchange rate returning to its initial level.

Clearly, the magnitude of the fall in the world interest rate as a result of a positive monetary shock in Country 1 will be bigger the larger is Country 1 in relation to the rest of the world. Obviously, although the analysis is not pursued here, the effects of real shocks in one country on the world interest rate will also depend directly on the size of that country.

Now suppose that, in the face of the above monetary shock, Country 2 fixes its nominal exchange rate with respect to Country 1. The excess supply of liquidity in Country 1 and the corresponding excess demand for real assets held by the representative agent in Country 2 now leads to a surplus in Country 2's balance of payments which its authorities have to finance by accumulating foreign exchange reserves. The representative agent of Country 1 buys claims on capital goods from the representative agent of Country 2 using currency supplied by Country 2's authorities in return for the currency of Country 1, which those authorities immediately exchange for claims on capital employed in Country 1. The net effect is an increase in the monetary asset in Country 2 in proportion to the excess supply of the monetary asset in Country 1. Country 2's government now owns an amount of foreign-employed capital equal in value to the increase in the quantity of the domestic monetary asset, and Country 2's representative agent's private holdings of domestically-employed capital is less by the same amount. The representative agent in Country 2 is not worse off, of course, because the future returns from the government held foreign-employed capital will be eventually accrue to that agent. The conclusion is that any increase in the excess supply of liquidity in Country 1 must necessarily be matched by an increase in the supply of liquidity in Country 2 of equal proportion—otherwise, Country 2's nominal exchange rate with respect to Country 1 would appreciate. Country 2's AA curve thus shifts to the right by the same amount as Country 1's in Figure 8, with the world AA curve shifting to the right by the sum of these two amounts. The world real interest rate falls and income rises in both countries in the short-run. In the long run when prices are flexible, they rise in the same proportion in both countries with the nominal and real exchange rates remaining unchanged.

It is clear that Country 1's authorities run world monetary policy. And Country 2's authorities, to maintain the nominal exchange rate fixed, must continually supply whatever stock of liquidity its residents demand. Moreover, the effects on the world real interest rate and the world price level of monetary shocks in Country 1 are completely independent of the size of Country 1 in relation to the rest of the world. When the rest of the world (Country 2 in the above example) consists of a number of countries, all of which hold their exchange rates fixed with respect to Country 1, that country can be referred to as the key-currency country and the other countries referred to as peripheral countries.

This conclusion that even a small key-currency country runs world monetary policy depends critically on the assumption that the peripheral countries hold their foreign exchange reserves in ownership claims to capital employed in the key-currency country and not in the key-currency country's monetary asset. Otherwise, a large peripheral country could change the supply of liquidity in the key-currency country, and thereby shift its AA curve, by shifting foreign exchange reserves between claims on real capital and monetary asset holdings. Peripheral-country holdings of the key-currency country's monetary asset will not be part of that country's money supply because they cannot be used by private agents in making transactions.

It turns out that one country (say Country 1) can be, in essence, a key-currency country and thereby run world monetary policy even under a system of flexible exchange rates if the other countries' authorities routinely adjust their home money supplies to maintain their exchange rates with respect to the key-currency country more or less constant, even though no official exchange rate parities have been declared.

3 Formal Development of the Model

3.1 Aggregate Production and Income

Assume that a single good is produced that can either be consumed or added to the capital stock. Although we visualise there being many types of capital—structures, machines, inventories, human skills, basic knowledge, technological knowledge, institutions, etc.—it is convenient, and sufficient, at this point to assume that there are only two types, the quantities of which can be represented by K_1 and K_2, where a unit of each type can be obtained by sacrificing one unit of consumption. Let output X be produced solely by these two types of capital according to the aggregate production function

$$X = \mu K_1^{1/2} K_2^{1/2} = \mu K_1 k^{1/2} \tag{1}$$

where $k = K_2/K_1$ and μ is a scale constant. Note that labour—that is, human capital—is embedded in the capital stock as defined.

Let aggregate income, denoted by Y, be equal to output minus depreciation,

$$Y = X - \delta K \tag{2}$$

where $K = K_1 + K_2$ and δ is the depreciation rate.

To maximise the output from any given K, k must take a particular value, call it k^*. This level of k can be found by taking the total differential of (1) and finding the value of k that will reduce it to zero.

$$dX = \frac{\mu}{2} K_1^{-1/2} K_2^{1/2} dK_1 + \frac{\mu}{2} K_1^{1/2} K_2^{-1/2} dK_2 \;=\; 0 \tag{3}$$

Given that a change in k holding the capital stock constant implies that $dK_2 = -dK_1$, this expression reduces to

$$\frac{\mu}{2}\left[\left(\frac{K_2}{K_1}\right)^{1/2} - \left(\frac{K_1}{K_2}\right)^{1/2}\right] dK_1 = 0 \tag{4}$$

which will hold whenever $K_1 = K_2$, which occurs when $k = k^* = 1$.

The fact that

$$K = K_1 + K_2 = K_1\left(1 + \frac{K_2}{K_1}\right) = K_1(1 + k) \tag{5}$$

implies

$$K_1 = \frac{K}{1 + k} \tag{6}$$

which can be substituted into (1) to yield

$$X = \mu \frac{k^{1/2}}{1 + k} K = mK \tag{7}$$

where

$$m = \mu \frac{k^{1/2}}{1 + k}$$

is the marginal product of capital, which is at a maximum when $k = k^* = 1$. Note that, given the state of resource allocation, m is a constant that is independent of the levels of the capital stock and output. Its maximum possible level, denoted by $\hat{m}$, is $\mu/2$. The resulting expression for aggregate income is

$$Y = (m - \delta)\,K. \tag{8}$$

Allowing for the possibility that the stock of capital may be misallocated among its two forms, we can rewrite the above equation as

$$Y = [\hat{m}(1-\gamma) - \delta]\,K. \tag{9}$$

where γ, which must take a value between zero and unity, measures the fraction of output lost as a result of misallocation of capital, and m takes its optimum value where $k = 1$. When the cost of converting consumable output into each form of capital is unity, the expression $[\hat{m}(1-\gamma) - \delta]$ will equal the real rate of interest. If we assume that $\gamma = 0$, $\delta = .05$ and the scale parameter μ equals .2, $\hat{m}$ will equal 0.1 and the real interest rate will equal 5%. A reduction of k to 0.5 or increase in it to 2.0 will reduce this interest rate to 4.43%. This would imply a value of γ equal to 0.057.

3.2 Inter-temporal Optimisation: The Equilibrium Growth Rate

Utility depends on the entire time path of consumption. A convenient representation is as follows:

$$U = U(C_0) + \beta\,U(C_1) + \beta^2\,U(C_2) + \beta^3\,U(C_3) + \beta^4\,U(C_4)\,\ldots\ldots\ldots \tag{10}$$

where $\beta = 1/(1+\rho)$ with ρ being the rate of time preference, and the time-horizon is infinite. Each period's budget constraint is as follows:

$$\begin{aligned} C_t &= [\hat{m}(1-\gamma) - \delta]\,K_t - (K_{t+1} - K_t) - \alpha\,K_t \left(\frac{K_{t+1} - K_t}{K_t}\right)^2 \\ &= [\hat{m}(1-\gamma) - \delta][1 - g_k - \alpha\,g_k^2]\,K_t \end{aligned} \tag{11}$$

where

$$g_k = \frac{K_{t+1} - K_t}{K_t}$$

is the growth rate of the capital stock—that is, the ratio of investment to the stock of capital. The term multiplied by α expresses the adjustment costs of putting new capital into productive form—that is, adapting it to the already existing capital stock—as a constant fraction α of the capital stock multiplied by the square of its current-period growth rate. While this form of the adjustment cost function is convenient for the general framework being developed here, one might want to modify it in more detailed analyses. It is assumed that a permanent reduction of the capital stock requires the same adjustment costs as a current addition.

Since two variables, C_t and K_{t+1}, are subject to choice in each period, optimisation requires that, together with the intertemporal budget constraint, two conditions hold. First, it should be impossible to increase utility by shifting a unit of consumption between any two periods. This requires that the value of a unit of consumption in period t equal to the present value of that same unit of consumption in any adjacent period

$$U'(C_t)\, dC_t = \beta\, U'(C_{t+1})\, (1 + r_t)\, dC_t \tag{12}$$

where r_t is the rate of interest in period t and $(1 + r_t)\, dC_t$ represents the amount of consumption obtained in period $t+1$ as a result of the sacrifice of dC_t units of consumption in year t, $U'(C_{t+1})$ is the utility of that additional consumption, and β discounts that next-period consumption back to the current period. This expression can be reorganised to yield

$$\frac{U'(C_t)}{U'(C_{t+1})} = \frac{1 + r_t}{1 + \rho}. \tag{13}$$

Under the convenient assumption that the felicity function $U(C) = log(C)$, $U'(C) = 1/C$ and the above expression becomes

$$\frac{C_{t+1}}{C_t} = \frac{1 + r_t}{1 + \rho}. \tag{14}$$

At this point we can introduce the possibility of resource misallocation through interference with the rate of consumption growth by introducing an implicit tax τ on savings as follows:

$$\frac{C_{t+1}}{C_t} = \frac{(1 + r_t)(1 - \tau)}{1 + \rho}. \tag{15}$$

The second condition of optimisation is that the present value of a unit of capital added to the capital stock and brought into production in any period be equal to its marginal cost. The latter equals unity (the amount of output that must be sacrificed to produce a unit of capital) plus the marginal adjustment cost, which equals the derivative of

$$\alpha K_t \left(\frac{K_{t+1} - K_t}{K_t} \right)^2$$

with respect to K_{t+1}, that is

$$2\alpha \left(\frac{|K_{t+1} - K_t|}{K_t} \right) = 2\alpha\, |g_t|$$

where g_t is the growth rate of the capital stock in period t. The second condition of optimisation is thus

$$1 + 2\alpha\, |g_t| = \frac{\hat{m}(1-\gamma) - \delta}{r_t}, \tag{16}$$

which says that the marginal cost of a unit of capital, measured in output units, must equal the present value of that unit of capital, given by the perpetual per-period return to it divided by the interest rate. An important assumption here is that the representative agent expects the currently observed values of m, γ and δ to remain constant in all future periods. The period t interest rate can thus be expressed

$$r_t = \frac{\hat{m}(1-\gamma) - \delta}{1 + 2\alpha\, |g_t|} \tag{17}$$

The steady state growth rate can now be obtained by rearranging the first condition of optimisation (15) to yield

$$r_t = \left(\frac{(1+\rho)}{(1-\tau)} \frac{C_{t+1}}{C_t} \right) - 1 \tag{18}$$

and substituting into it the second condition (17) to yield

$$\frac{\hat{m}(1-\gamma) - \delta}{1 + 2\alpha\, |g|} = \frac{1+\rho}{1-\tau}\,(1+g) - 1 \tag{19}$$

where, from the budget constraint and the definitions of X and Y,

$$g = \frac{K_{t+1} - K_t}{K_t} = \frac{C_{t+1} - C_t}{C_t} = \frac{X_{t+1} - X_t}{X_t} = \frac{Y_{t+1} - Y_t}{Y_t}$$

is the economy's steady-state growth rate. Equation (19) can be solved numerically for the equilibrium steady state growth rate arising from any given set of parameters m, γ, δ, α, ρ and τ.[10]

[10] This can be done in XlispStat using the batch file `ssgrth.lsp`. For example, the values $\hat{m} = .16$, $\delta = .05$, $\rho = .015$, $\alpha = 20.00$, $\gamma = .10$ and $\tau = .01$ yield a growth rate of 2.3 %. Increasing γ to .4 and τ to .03, with the other parameters unchanged, will reduce the equilibrium growth rate to zero.

3.3 Money and Output

In the absence of money, a significant fraction of the economy's resources will be used up in the process of making transactions, reducing the output available for consumption and investment. These transactions costs, and the role of money in reducing them, must now be incorporated into the analysis. Suppose that there exists a government that can eject liquidity into the economy by issuing a monetary asset. Denote the level of real liquidity by L.[11] Assume for the moment that this government issues an amount of the monetary asset that will maximise the output that the economy can produce, paying interest on that asset at the market rate and thereby following a Friedman rule.[12] If the optimum amount of liquidity is present at all times, the analysis of the previous two sections will go through without modification. The entire flow of output from the capital stock, as previously defined, will represent final output.

The assumption that interest is paid on monetary assets at the market rate is, of course, unrealistic and must be relaxed. The consequence of insufficient liquidity is a reduction of the income associated with any given capital stock. We can thus modify equation (9), again simply imposing a convenient functional form, as follows

$$\frac{Y}{K} = [\hat{m}(1-\gamma) - \delta] \left[1 - \frac{1}{3\lambda}\left(\phi - \lambda\frac{L}{K}\right)^3\right] \tag{20}$$

where

$$\frac{L}{K} \leq \frac{\phi}{\lambda}.$$

In the case where the above condition holds with equality, (20) reduces to (9). When there is no liquidity in the system (20) reduces to

$$\frac{Y}{K} = [\hat{m}(1-\gamma) - \delta]\,\Upsilon \tag{21}$$

where Υ,

$$0 \leq \Upsilon = \left[1 - \frac{\phi^3}{3\lambda}\right] \leq 1,$$

[11] In general, it is useful to think of the stock of liquidity as different from the stock of any particular monetary asset because in real economies there are many types of monetary asset—cash, demand deposits, time deposits, etc.—as well as some non-monetary assets that also provide some liquidity.

[12] Milton Friedman, "The Optimum Quantity of Money," *The Optimum Quantity of Money and Other Essays*, Chicago: Aldine Publishing, 1969, 1-50.

is the fraction of output remaining when there is no liquidity in the economy.

The real interest rate, previously given by equation (17), now becomes

$$r = \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,|g_t|}\left[1-\frac{1}{3\lambda}\left(\phi-\lambda\frac{L}{K}\right)^3\right] \tag{22}$$

and the steady-state growth rate must now satisfy

$$\frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,|g|}\left[1-\frac{1}{3\lambda}\left(\phi-\lambda\frac{L}{K}\right)^3\right] = \frac{1+\rho}{1-\tau}(1+g)-1 \tag{23}$$

instead of (19).

The demand function for money can be obtained by setting the derivative of (20) with respect to L equal to the real interest rate plus the expected inflation rate and other institutional costs of holding money minus any interest paid on the monetary asset.

$$r+\psi = [\hat{m}(1-\gamma)-\delta]\left(\phi-\lambda\frac{L}{K}\right)^2 \tag{24}$$

where ψ represents any additional costs of holding money minus interest paid on it. This demand function can be represented by a parabola which crosses the vertical (interest rate) axis at $[\hat{m}(1-\gamma)-\delta]\phi^2$ and becomes tangent and equal to the horizontal (L/K) axis at liquidity/capital ratio of $L/K = \phi/\lambda$ as shown in Figure 9. The graph can be refined by putting L, which represents real liquidity, on the horizontal axis and letting K be a constant shift factor with the curve shifting proportionally to the right with increases in Y which varies proportionally with K as indicated in equation (19).

3.4 Variations in Employment

Standard real business cycle theory would postulate variations in the level of output resulting from effects of ongoing investment in the technological portion of the aggregate capital stock which can now be thought of as consisting of many types of capital rather than merely the two types postulated in equation (1).[13] These variations would usually be accompanied

[13] We would then write the production function as

$$X = \mu K_1^{1/n} K_2^{1/n} K_3^{1/n} \ldots\ldots K_n^{1/n}$$

where half of these capital types were initially aggregated into previously defined K_1 and the other half into K_2. All the previous results would continue to hold.

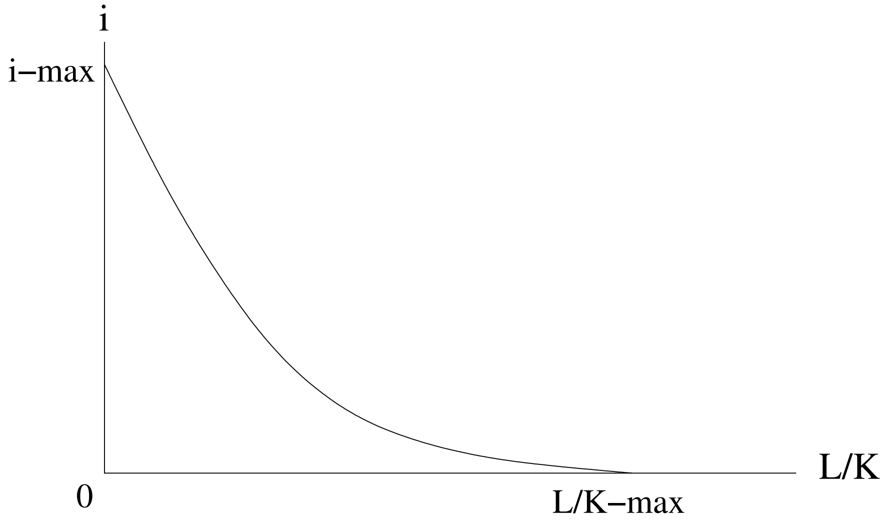

Figure 9: Demand function for money (liquidity): L/K-max $= \phi/\lambda$
i-max $= [\hat{m}(1 - \gamma_{-}\delta]\phi^2$.

by changes in labour-leisure choice by those in whom the human capital portion of the aggregate capital stock is embodied. Our interest here is in more traditional changes in the level of employment that involve involuntary changes in the degree of utilisation of the human and non-human portions of the aggregate capital stock.

Since such employment and output changes are temporary, they can be viewed as changes in the level of income Y independent of changes in the level of K with the income changes coming from changes in the fraction of K utilised. An alternative form of output variation would be temporary changes in m around $\hat{m}$ in equation (20). These changes in income, being temporary, will have no effects on the expected future income flow from capital or on current consumption, which will depend on the average future, or permanent, level of income. The variations in current income around its permanent full-employment level will thus be channelled into temporary increases and decreases in the stock of capital K.

Asset equilibrium holds whenever the real quantity of liquidity is at its equilibrium level—otherwise, there would be excess demand or supply of the monetary asset and corresponding excess supply or demand for the other capital assets K_i. Equation (24) can thus be viewed as the condition of short-term asset equilibrium when K is replaced with the quantity of capital actually utilised. The capital stock will be over-utilised or under-utilised to the extent that income is above or below its full-employment level. Letting ΔY represent the excess of the current level of income over its full-employment level, we can express the quantity of capital utilised as

$$K_U = \frac{Y_f + \Delta Y}{Y_f} K \tag{25}$$

and express (24) in the form

$$r + \psi = [\hat{m}(1-\gamma) - \delta] \left[\phi - \lambda \frac{L}{Y_f + \Delta Y} \frac{Y_f}{K} \right]^2 \tag{26}$$

where Y_f is the full-employment level of income. This equation appears in Figures 1 through 8 as the AA curve which is upward sloped because a larger ΔY is associated with a higher real interest rate. The full-employment real interest rate is the level of r when ΔY is zero. As noted in the previous section, this curve is a counterpart to the Hicks-Hansen LM curve.

Equilibrium in the market for domestic output can be obtained by writing equation (22) as

$$r = \frac{\hat{m}(1-\gamma) - \delta}{1 + 2\alpha\,(g + \Delta Y/K)} \left[1 - \frac{1}{3\lambda} \left(\phi - \lambda \frac{L}{Y_f + \Delta Y} \frac{Y_f}{K} \right)^3 \right] \tag{27}$$

with g, the current steady-state growth rate, always positive. The real rate of interest is affected in two ways by an expansion of income relative to its full-employment level. First, it falls as a result of the increased in adjustment costs of putting new capital in place as the level of investment increases by an amount equal to the increase in income. Second, it falls as a result of the fact that an expansion of income increases the volume of transactions, increasing the amount of output and income used up in transactions-making activity, given the unchanged stock of liquidity L, and hence reducing the flow of returns from the capital stock. Since the real interest rate rate clearly falls (rises) relative to its full-employment level as income rises (falls) relative to its full-employment level, (27) can be represented by a downward-sloping curve similar to GG in Figures 1 through 8. Unfortunately, because the variable L appears in both (26) and (27), the resulting GG curve will shift whenever a shift of or movement along the AA curve occurs.

3.5 Solution for Full-Employment Levels

The full-employment equilibrium levels of the variables are determined by the following three equations:

$$r = \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,|g|}\left[1-\frac{1}{3\lambda}\left(\phi-\lambda\frac{L}{K}\right)^3\right] \tag{28}$$

$$\frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,|g|}\left[1-\frac{1}{3\lambda}\left(\phi-\lambda\frac{L}{K}\right)^3\right] = \frac{1+\rho}{1-\tau}\,(1+g)-1 \tag{29}$$

$$r+\psi = [\hat{m}(1-\gamma)-\delta]\left(\phi-\lambda\frac{L}{K}\right)^2 \tag{30}$$

If we normalise the initial stock of capital at unity, there are three variables that will be determined by these equations—r, L, and g.

This requires that, combining (28) and (30),

$$(\phi-\lambda L)^2-\frac{\psi}{\hat{m}(1-\gamma)-\delta} = \frac{1}{1+2\alpha\,|g|}\left[1-\frac{1}{3\lambda}\,(\phi-\lambda L)^3\right] \tag{31}$$

In the special case where the optimum stock of the monetary asset is in place, the term

$$\phi-\lambda L=0$$

and the above expression reduces to

$$-\psi = \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,|g_t|}=r$$

The optimum quantity of money holdings is

$$\frac{\phi}{\lambda}=\omega. \tag{32}$$

In the opposite case, where there is zero liquidity, (31) reduces to

$$\psi = [\hat{m}(1-\gamma)-\delta]\,\phi^2-\frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,|g|}\left[1-\frac{\omega\,\phi^2}{3}\right]$$

The term

$$1-\frac{\omega\,\phi^2}{3}=\Upsilon$$

gives the fraction of income that would remain if the stock of liquidity were reduced to zero. Were this expression equal to zero, the shadow return to liquidity or nominal interest rate would become equal to

$$\psi = [\hat{m}(1-\gamma) - \delta]\,\phi^2$$

and, as can be seen from (28), the real interest rate will be reduced to zero. By specifying the level of real liquidity at which usable output is maximised—ω—and the fraction of income that would remain at a zero stock of liquidity—Υ—we can calculate the values of ϕ and λ.

$$\phi = \sqrt{\frac{3(1-\Upsilon)}{\omega}} \tag{33}$$

$$\lambda = \frac{\phi}{\omega} \tag{34}$$

Normalising the initial value of the capital stock at unity and calibrating the other parameters according to the table below, and measuring the levels of the other variables in units of capital stock, the model generates an optimum stock of liquidity of .03 and a steady-state growth rate with optimum liquidity of 2.307%. Where the costs of holding money are equal to to the real interest rate, the equilibrium real stock of liquidity is .02755, the steady state growth rate is 2.306%, the fraction of income lost as a result of not having the optimum stock of money is .00044, the real interest rate is 4.9% and the level of income, measured in units of capital stock is .094. These results are purely illustrative, of course, as a different calibration and selection of functional forms will produce different results—they are calculated to show that the model structure can generate equilibrium values of the variables that are in the range of plausibility.

3.6 The Shapes of the GG and AA Curves

The equation of the AA curve is obtained by substituting (25) into (26) and setting $K = 1$ and $\psi = 0$ to yield

$$r = [\hat{m}(1-\gamma) - \delta]\left[\phi - \lambda\frac{L}{K_U}\right]^2. \tag{35}$$

To obtain the equation of the GG curve, we substitute (25) into (27) and set $K = 1$, yielding

$$r = \frac{\hat{m}(1-\gamma) - \delta}{1 + 2\alpha\left[g + (K_U - 1)\,Y_f\right]}\left[1 - \frac{1}{3\lambda}\left(\phi - \lambda\frac{L}{K_U}\right)^3\right]. \tag{36}$$

Table 2: Calibration of the Model

Parameter	Description	Value
$\hat{m}$	Maximum Marginal Prod of Capital	0.16
δ	Depreciation Rate	0.05
ρ	Rate of Time Preference	0.015
α	Adjustment Cost Parameter	20
γ	Misallocation of Capital Parameter	0.10
τ	Implicit Tax on Saving	0.01
ω	Optimum Ratio of Liquidity to Capital	.03
Υ	Fraction of Output Produced Under Zero Liquidity	0.2

The resulting curves are calculated numerically in Xlispstat using the batch file `theory.lsp` and displayed in Figure 1a.[14] With one exception, these calculations are based on the parameters in Table 2. The exception is necessary because the adjustment costs of adapting newly produced capital goods to the original capital stock are likely to be much more affected by a short-run change in the level of investment relative to its steady-state than by a long-run change in the steady state level of investment. Accordingly, the parameter α is increased to 100 in equation (36). Y_f is assigned the value produced by the calculations in the previous sub-section but then normalised at unity in presenting the results. The real stock of liquidity is then increased by 1% and a new level of AA calculated—as expected, an increase in liquidity shifts the curve to the right in Figure 1a. As can be seen from equation (36), this increase in L will also shift the GG curve to the left. Fortunately, however, as evident from the calculations in the previous subsection, the effect of this increase in L on the fraction of output used up by the process of making transactions is trivial and this shift in GG can be ignored—indeed, it is probably too small to even show up on the graph.

Now consider the case where, due to an improvement in resource alloca-

[14] In making these calculations the following definitions were adopted for programming purposes:

$$\texttt{mterm} = \hat{m}(1-\gamma) - \delta$$

$$\texttt{tterm} = 1 - (1/3\lambda)[\phi - \lambda\, L/K_U]^3$$

$$\texttt{dmterm} = [\phi - \lambda\, L/K_U]^2$$

$$\texttt{acterm} = 1 + 2\alpha\,[g + (K_U - 1)\,Y_f]$$

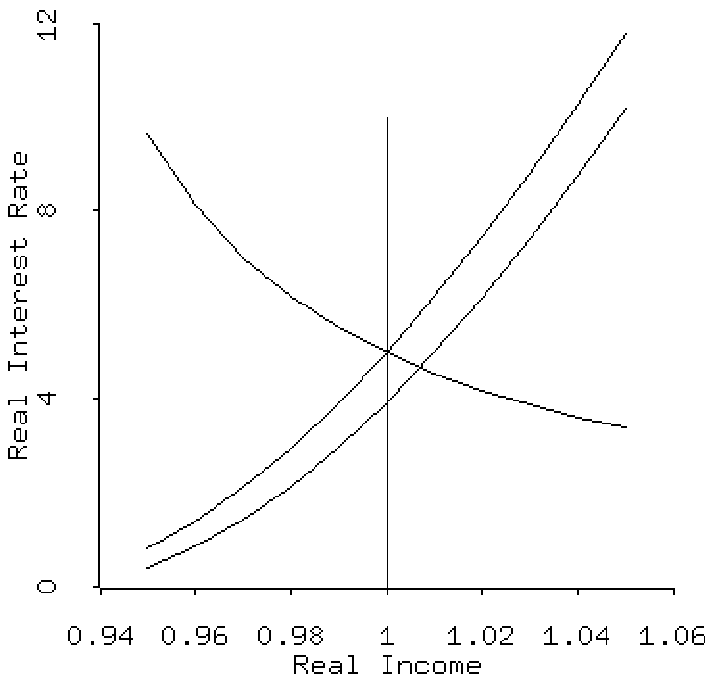

Figure 1a: Monetary shocks to the full-employment income level in a large open economy.

tion, the parameter γ falls, increasing the flow of income from the capital stock. Using the same values of the other parameters, and reducing γ from .1 to .082, which is equivalent to raising $\hat{m}$ by 2%, we numerically calculate a new GG curve and new full-employment levels of interest rates and income as shown in Figure 4a. The result is the same as in the case of Figure 4 except for the fact that here the slope of the AA curve is much steeper. As a result, the effect on income and employment in the short-run is negative, given by the intersection of the LM curve with the new (right-most) GG curve. The interest rate rises in the short-run and the level of utilisation of the capital stock declines although output actually increases.

Suppose that the above shift of γ was expected by the representative agent to occur in subsequent periods but does not occur in the current period. In this case, the full-employment level of income will remain unchanged. The plot in Figure 4a might suggest that the short-run change in

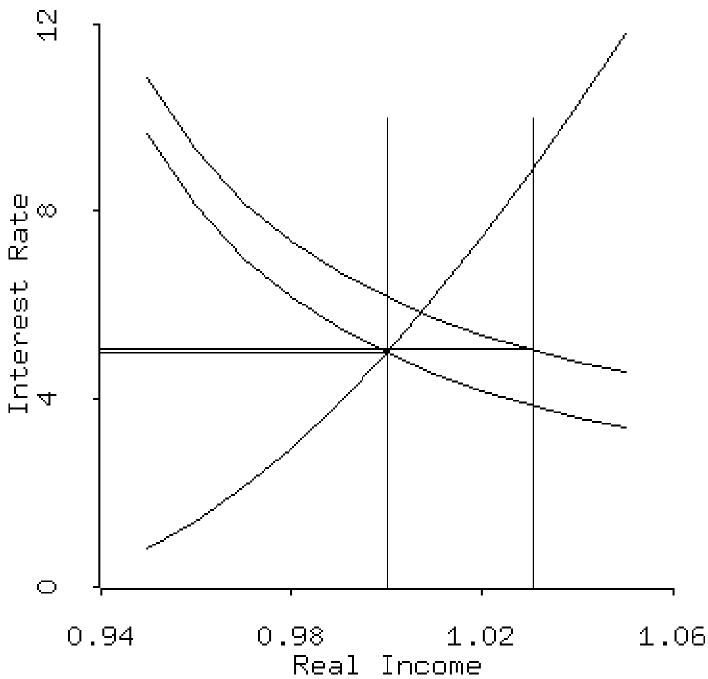

Figure 4a: Real adjustments around or toward a new steady-state growth path.

the level of current income will be the same as above, although the level of utilization of the capital stock will now increase in the current period, rather than decrease from a higher full-employment income level. But such and interpretation would be misleading. Imagine that the representative agent, though expecting the flow of income from the capital stock to increase in the future, decides to sit tight in the current period, investing and consuming the amounts consistent with previous steady-state growth. In this case, the interest rate will rise by the same amount as the expected future flow of output from the current capital stock. The exact increase will be .0029 or 29 basis points, which is about three times the the increase in the full-employment interest rate in the case where the increase in the output flow from the capital stock actually occurred and was permanent. As can be seen in Figure 4a, this rise in the interest rate will be a small fraction of the vertical distance between the two GG curves at the initial full-employment income level. Of course, it is reasonable to expect that the representative agent, expecting consumption to increase in the future, would decide to smooth it by increasing current-period consumption. Given the unchanged current level of output, this would lead to a reduction of the level of investment and a further rise in the market interest rate, making the actual upward shift of GG somewhat greater than a mere 29 basis points but still considerably less than the shift shown in Figure 4a. All we can say is that an increase in the expected future return from the capital stock in the face of an unchanged current return will shift the GG curve upward.

Another possible real shock is a random temporary shift of the full-employment output level as a result of a change in the weather or randomness in the occurrance of discoveries resulting from technological investment. If the shock has a life longer than one period the effect will be some fraction of the effect of a permanent shock to γ. If the increase in full-employment income is only expected to last one period there will be no effect on current income if appropriate finance is not provided to shift the AA curve to the right. The gain in the full-employment income level due to the shock will be offset by an equivalent reduction in the utilisation of the capital stock, maintaining actual income at its initial level. Since consumption will be unchanged, the associated reduction in the level and adjustment costs of investment may cause the real interest rate to rise, shifting GG upward to some degree. If complete monetary finance of the shock is provided the real interest rate will fall and real income will increase by the full amount of the increase in the full-employment income level—the equilibrium income-interest-rate combination will simply move downward to the right along the original GG curve.

3.7 Extension to the Small Open Economy Case

The analysis of the previous section applies to either a closed economy or an open one that represents virtually the entire world. We now need to examine the shape and adjustments of the GG and AA curves for a small open economy. Assume that the small country and the rest of the world produce separate goods, both of which enter into consumption and capital accumulation in both economies. Let consumption in the small country, C_t, be characterised as a constant-elasticity-of-substitution (CES) function of the quantities of the domestic and rest-of-world outputs consumed as follows:

$$C_t = \left[\xi \left(C_{Dt}\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)\left(\tilde{C}_{Dt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{37}$$

where C_{Dt} is domestic (small country) consumption of domestic output and $\tilde{C}_{Dt}$ is domestic consumption of rest-of-world output , ξ is the share of domestic consumption comprised of domestic output, and σ is the elasticity of substitution between the two outputs in domestic consumption. To keep the analysis simple, assume that domestic investment, $I_t = K_{t+1} - K_t$, is the same CES function of domestic and rest-of-world outputs:

$$I_t = \left[\xi \left(I_{Dt}\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)\left(\tilde{I}_{Dt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{38}$$

where I_{Dt} is domestic investment of domestic output and $\tilde{I}_{Dt}$ is domestic investment of rest-of-world output. Similarly, rest-of-world consumption and investment, $\tilde{C}_t$ and $\tilde{I}_t$ $(= \tilde{K}_{t+1} - \tilde{K}_t)$, are treated as CES functions of rest-of-world and domestic outputs:

$$\tilde{C}_t = \left[\tilde{\xi} \left(\tilde{C}_{Ft}\right)^{\frac{\sigma-1}{\sigma}} + (1-\tilde{\xi})\left(C_{Ft}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{39}$$

$$\tilde{I}_t = \left[\tilde{\xi} \left(\tilde{I}_{Ft}\right)^{\frac{\sigma-1}{\sigma}} + (1-\tilde{\xi})\left(I_{Ft}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{40}$$

where $\tilde{C}_{Ft}$ is rest-of-world consumption of its own output and C_{Ft} is rest-of-world consumption of domestic output, $\tilde{I}_{Ft}$ is rest-of-world investment of its own output and I_{Ft} is rest-of-world investment of domestic output, $\tilde{\xi}$ is the shares of rest-of-world consumption and investment comprised of its own output, and the elasticity of substitution is the same as in the domestic economy. It should be noted that, the domestic economy being very small in relation to the rest of the world, $\tilde{\xi}$ will be extremely close to unity while ξ would typically be substantially less than unity.[15] The marginal products of the domestic and rest-of-world outputs in domestic and rest-of-world consumption and investment are as follows:

$$\frac{\partial C_t}{\partial C_{Dt}} = \xi \left(\frac{C_t}{C_{Dt}}\right)^{\frac{1}{\sigma}} \qquad \frac{\partial C_t}{\partial \tilde{C}_{Dt}} = (1-\xi)\left(\frac{C_t}{\tilde{C}_{Dt}}\right)^{\frac{1}{\sigma}}$$

$$\frac{\partial \tilde{C}_t}{\partial \tilde{C}_{Ft}} = \tilde{\xi} \left(\frac{\tilde{C}_t}{\tilde{C}_{Ft}}\right)^{\frac{1}{\sigma}} \qquad \frac{\partial \tilde{C}_t}{\partial C_{Ft}} = (1-\tilde{\xi})\left(\frac{\tilde{C}_t}{C_{Ft}}\right)^{\frac{1}{\sigma}}$$

$$\frac{\partial I_t}{\partial I_{Dt}} = \xi \left(\frac{I_t}{I_{Dt}}\right)^{\frac{1}{\sigma}} \qquad \frac{\partial I_t}{\partial \tilde{I}_{Dt}} = (1-\xi)\left(\frac{I_t}{\tilde{I}_{Dt}}\right)^{\frac{1}{\sigma}}$$

[15] To avoid notational confusion, the reader should keep in mind that the presence or absence of a ˜ superscript denotes the country, foreign or domestic, whose intermediate or final good is being consumed or invested and the $_D$ and $_F$ subscripts denote the country whose residents are purchasing the intermediate consumption and investment goods.

$$\frac{\partial \tilde{I}_t}{\partial \tilde{I}_{Ft}} = \tilde{\xi}\left(\frac{\tilde{I}_t}{\tilde{I}_{Ft}}\right)^{\frac{1}{\sigma}} \qquad\qquad \frac{\partial \tilde{I}_t}{\partial I_{Ft}} = (1-\tilde{\xi})\left(\frac{\tilde{I}_t}{I_{Ft}}\right)^{\frac{1}{\sigma}}$$

Movements of the relative price of the two goods represent changes in the small country's real exchange rate with respect to the rest of the world, hereafter denoted by Q. This relative price must equate the marginal returns to domestic and rest-of-world output in generating domestic and rest-of-world consumption and investment, and will therefore equal the ratios of the relevant marginal products:

$$\begin{aligned} Q_t &= \frac{\xi}{1-\xi}\left(\frac{\tilde{C}_{Dt}}{C_{Dt}}\right)^{\frac{1}{\sigma}} = \frac{\xi}{1-\xi}\left(\frac{\tilde{I}_{Dt}}{I_{Dt}}\right)^{\frac{1}{\sigma}} \\ &= \frac{\tilde{\xi}}{1-\tilde{\xi}}\left(\frac{\tilde{C}_{Ft}}{C_{Ft}}\right)^{\frac{1}{\sigma}} = \frac{\tilde{\xi}}{1-\tilde{\xi}}\left(\frac{\tilde{I}_{Ft}}{I_{Ft}}\right)^{\frac{1}{\sigma}} \end{aligned} \tag{41}$$

It is easily shown that the elasticities of C_{Dt}, I_{Dt}, C_{Ft}, and I_{Ft} with respect to Q are equal to $-\sigma$ and the elasticities of $\tilde{C}_{Dt}$, $\tilde{I}_{Dt}$, $\tilde{C}_{Ft}$, and $\tilde{I}_{Ft}$ with respect to Q are equal to σ.

Given that the small country is too small to influence conditions in the rest of the world, the latter's situation can be correctly described by the analysis of the preceding sections. The real interest rate in the small country is determined entirely by conditions in the rest of the world and can be denoted denoted by $\tilde{r}$. This results in a complete separation of intertemporal consumption decisions in the small country from decisions about the rate of addition to that country's capital stock.

From equation (15) the equilibrium rate of growth of consumption in the small country becomes

$$g_c = \frac{C_{t+1}}{C_t} - 1 = \frac{(1+\tilde{r})(1-\tau)}{1+\rho} - 1. \tag{42}$$

Consumption will grow faster in the small country than in the rest of the world if its value of the ratio $(1-\tau)/(1+\rho)$ is larger than the corresponding value for the rest of the world—that is, if its rate of time preference and/or institutional impediments to future consumption are smaller.

Appropriate modification of (22) yields the equilibrium rate of growth of the capital stock invested in the small open economy. This capital stock, of course, may be owned by residents of both the small country and the rest of the world.

$$\frac{K_{t+1} - K_t}{K_t} = g_k = \frac{\hat{m}(1-\gamma) - \delta}{2\alpha\,\tilde{r}} \left[1 - \frac{1}{3\lambda}\left(\phi - \lambda\frac{L}{K}\right)^3\right] - \frac{1}{2\alpha}. \tag{43}$$

As long as

$$\hat{m}(1-\gamma) - \delta \left[1 - \frac{1}{3\lambda}\left(\phi - \lambda\frac{L}{K}\right)^3\right] > \tilde{r}$$

the capital stock invested in the small country will be increasing through time in equilibrium. And it will grow faster than that invested in the rest-of-the-world if the term to the left of the inequality exceeds the corresponding term for the rest of the world. Clearly, since both parts of the world have access to the same technology, capital stock in the part that has the most efficient allocation of capital, as evidenced by a smaller value of γ, will grow the fastest. It must nevertheless be the case, as required by (43) above, that the growth rates of domestic and foreign holdings of capital in each part of the world must be the same.

Obviously, the growth rate of consumption or capital in the small country can not exceed or fall short of that in the rest of the world in perpetuity or the country would eventually either become virtually the entire world or virtually disappear from it. Nevertheless, it is reasonable to expect the two parts of the world to have different rates of consumption growth for extended periods because of differences in time preference (different values of ρ) or in the institutional impediments to intertemporal saving (different values of τ). Also, the part of the world that suffers from greater distortions in the allocation of capital among its various types (higher values of γ) will experience lower output flows from capital stock employed within its borders and hence lower returns to, and smaller levels of, home investment.

While consumption and capital stock must grow at the same rates for the entire world, either the small economy or the rest of the world can have a higher rate of growth of consumption than of the capital stock located within its borders. In this case a part of the home consumption will utilise the output flow from home ownership of capital employed abroad. A portion of home savings will be channelled abroad rather than invested in home-employed capital. When access to world technology by both parts of the world is the same, such differences will arise from differences in the efficiency with which capital stock is utilised, as reflected in the parameter γ. The part of the world with lower values of this parameter will experience greater growth of home-employed capital stock.

Given stability of the underlying parameters, it must be true that along an equilibrium path the growth rates of consumption and capital stock must be constants. The difference between them, which will reflect either the purchase of foreign-employed capital by home residents or the sale of home-employed capital to foreigners, must therefore also be constant and sufficient to satisfy the intertemporal budget constraint.

Nothing here requires that Q be constant along the steady state equilibrium growth path. If the growth of capital employed in the small open domestic economy exceeds that in the rest of the world, domestic output will be growing relative to output abroad. Condition (41) will then require that Q be falling at some rate through time. This is not what one would expect on the basis of the Balassa-Samuelson hypothesis which argues that as technology expands labour intensive non-traded output components should increase in price relative to traded output components so that the country's real exchange rate should rise.[16] To allow for this we could pre-multiply ξ and $\tilde{\xi}$ by respective technological parameters ϑ and $\tilde{\vartheta}$ which could be assumed to vary through time as the world technological capital stock increases. Under Balassa-Samuelson assumptions they would both be increasing at appropriate rates through time. In situations where world technological growth increases the return to investment, say, in the domestic economy relative to the rest of the world, the resulting increased demand for domestic relative to foreign non-traded output components could be represented by increases in ϑ relative to $\tilde{\vartheta}$, (and $\vartheta\xi$ relative to $\tilde{\vartheta}\tilde{\xi}$), which will raise the equilibrium level of Q. The effect of an increase in world relative prices of goods the small open economy specialises in producing can also be imposed by increasing ϑ relative to $\tilde{\vartheta}$ in the model.

These technological issues are of no real concern here, however, as our goal is simply to analyse short-run deviations of variables from their full employment levels in the small open economy. To accomplish this we need three equations and an imposed constraint. The first two equations, representing the AA and GG curves respectively, are (35), reproduced below for convenience, and a modification of (36) to emphasise that the expansion of income relative to its full-employment level in the term

$$1 + 2\alpha\,[g + (K_U - 1)] = 1 + 2\alpha\,(g_k + \Delta Y/K) = 1 + 2\alpha\,[g_k + \Delta I/K]$$

results entirely from short-run deviations of real investment in the small open

[16]See Bela Balassa, "The Purchasing Power Parity Doctrine: A Reappraisal," *Journal of Political Economy*, Vol. 72, No. 6 (December), 1964, 584-96, and Paul A. Samuelson, (1964), "Theoretical Notes on Trade Problems," *Review of Economics and Statistics*, Vol. 46, No. 2 (May), 1964, 145-54.

economy from its equilibrium level under steady-state growth, denoted here by ΔI. And the growth rate g, which in the closed economy case applied to both consumption and capital stock growth, must now be replaced by g_k, the growth-rate of the the domestically employed capital stock. Note that the level of income involved here is the income produced by the domestic-employed capital stock rather than the income of the domestic representative agent—that is, GDP rather than GNP as they are conventionally defined. With the initial level of K normalised at unity, our first two equations are

$$
\begin{aligned}
I: \qquad r &= [\hat{m}(1-\gamma)-\delta]\left[\phi-\lambda\frac{L}{K_U}\right]^2 && (36)\\
II: \qquad r &= \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,[g_k+\,\Delta I]}\left[1-\frac{1}{3\lambda}\left(\phi-\lambda\frac{L}{K_U}\right)^3\right]. && (45)
\end{aligned}
$$

The steady-state growth rate of domestic investment in the above equation is obtained from equation (43), also reproduced below,

$$
g_k = \frac{\hat{m}(1-\gamma)-\delta}{2\alpha\,r}\left[1-\frac{1}{3\lambda}\left(\phi-\lambda\frac{L}{K}\right)^3\right]-\frac{1}{2\alpha}. \qquad (44)
$$

A companion equation that does not enter into the short-run analysis is equation (42)

$$
g_c = \frac{(1+r)(1-\tau)}{1+\rho} \qquad (43)
$$

which gives the steady-state growth rate of domestic consumption.

The third equation in our three-equations system is derived in part from any one of the four equations (41). Taking the total differentials of all of these yields

$$
\frac{dQ}{Q} = -\frac{1}{\sigma}\frac{dC_D}{C_D} = -\frac{1}{\sigma}\frac{dI_D}{I_D} = -\frac{1}{\sigma}\frac{dC_F}{C_F} = -\frac{1}{\sigma}\frac{dI_F}{I_F}. \qquad (46)
$$

Given the assumptions underlying equations (37) through (40), a change in Q will result in an equiproportional change in the utilisation of small-open-economy output in all four uses so that we can express (46) in the neighborhood of full employment simply as

$$
\frac{dQ}{Q_f} = -\frac{1}{\sigma}\frac{dY}{Y_f} \qquad (47)
$$

or to a linear approximation, letting Q_R be the ratio of Q to its full-employment level,

$$\frac{\Delta Y}{Y_f} = -\sigma \left(Q_R - 1 \right) \tag{48}$$

where $\frac{\Delta Y}{Y_f}$ is now interpreted as a change in income relative to its full-employment level resulting strictly from a change in Q. Identical equations result from each of the four equalities in (46). Equation (48) gives the rightward shift of the GG curve in response to a decline in the small open economy's real exchange rate. As purchasing-power-parity is approached, $\sigma \to \infty$. To incorporate the possibility that the equilibrium level of Q may change in response to short-run exogenous real shocks, the latter equation can be rewritten as

$$Q_R = -\frac{1}{\sigma} \frac{\Delta Y}{Y_f} + \Psi_Q + 1 \tag{49}$$

where Ψ_Q represents a temporary exogenous shift in the equilibrium level of Q.

Note now that the fraction of the capital stock utilised depends upon the sum of the deviation of output from its full-employment level due to short-run changes in investment and the deviation due to short-run changes in the real exchange rate, so that

$$K_U = \frac{K_U}{K} = \frac{\Delta I}{K} + \frac{\Delta Y}{Y_f} + 1. \tag{50}$$

This equation follows from the fact that $\frac{\Delta I}{K}$ is the change in the utilisation of K resulting from short-term investment changes and $\frac{\Delta Y}{Y_f}$ is proportional to the change in the utilisation of K resulting from short-term real exchange rate changes, so that the sum of these terms can be written as

$$\frac{K_U - K}{K} = \frac{K_U}{K} - 1$$

which equals $K_U - 1$ when the initial level of K is normalised at unity. Our third basic equation can now be obtained by substituting (49) into (50) to yield

$$III: \qquad K_U = \Delta I_D + \Delta I_F - \sigma \left(Q_R - \Psi_Q - 1 \right) + 1. \tag{51}$$

Finally, we impose the small open economy condition that

$$IV: \qquad r = \tilde{r} + \Theta - E \left[\frac{Q_t - Q_{t-1}}{Q_t} \right] \tag{52}$$

where $\tilde{r}$ is the interest rate in the large rest-of-world economy, Θ is the risk premium on claims on real capital employed in the small country and $E\left[\frac{Q_t - Q_{t-1}}{Q_t}\right]$ is the expected rate of change in the small country's real exchange rate. Barring changes in risk or in the expected future time path of the real exchange rate, changes in the real interest rate in the small open economy can arise only as a result of interest rate changes in the rest of the world.

Equations I, II and III, with the constraint IV imposed, form a three equation system in the variables K_U, ΔI and Q_R when the small open economy lets the nominal exchange rate float and K_U, ΔI and L when the nominal exchange rate is fixed, with L being exogenous when the exchange rate is flexible and Q_R being exogenous when the exchange rate is fixed. This applies, of course, only in the short run when the nominal price of domestic output is assumed fixed. As noted in the intuitive discussion in Section 2, short run equilibrium is determined by the intersection of the horizontal world interest rate line and the GG curve when the nominal exchange rate is fixed, and by the intersection of the interest rate line and the AA curve when the nominal exchange rate is flexible. In the former case the AA curve adjusts automatically through changes in L brought about by the commitment of the authorities to maintain the fixed exchange rate, while in the latter, the GG curve adjusts automatically through changes in Q.

When prices are completely flexible in the small country, K_U equals unity, ΔI and Ψ_Q are both zero and, from equation III, Q_R also equals unity. Given $\tilde{r}$, the equilibrium level of real liquidity, achieved through price level adjustment, is then determined by equation I. When $K_U = 1$ and L takes its full-employment steady-state equilibrium value, equation II becomes identical with (44), the equation determining the steady-state g_k. The small open economy will experience a steady-state net capital outflow if $g_c > g_k$. Little can be gained by numerically solving the model in the small open economy case, since the resulting AA and GG curves will be virtually identical with those in Figures 1a and 4a.

4 Price Rigidity and Overshooting

Thus far it has been assumed that the price of a country's output, or the domestic price level, does not change in the short-run but will fully adjust in the long-run to its new equilibrium level. Whether this failure to adjust in the short-run is due to lack of information of producers about current changes in demand, or to costs of continually making immediate price adjustments in response to frequent and often temporary changes in demand, is of little concern—all that is necessary for validity of our analysis is that prices do not change immediately but do change eventually. Moreover, since the speed at which agents learn about economic changes that have occurred will almost certainly vary from instance to instance, and the cost of making price changes will vary in accordance with the institutional setting and the particular industries involved, any model of dynamic adjustment paths will be dependent upon assumptions that are specific to the time and place. Trying to extend our basic general model to encompass the dynamic path of adjustment between the short and long runs is therefore not worthwhile.

4.1 The Basis for Overshooting

There is, however, a further type of price rigidity to which our analysis can and must be extended. These are the rigidities that lead to exchange rate overshooting—that is, a short-term response of the nominal exchange rate to a monetary shock that exceeds its ultimate long-term response. Under a flexible exchange rate the process of adjustment to excess liquidity involves an attempted purchase of assets abroad that leads to a devaluation of the real and nominal exchange rates and an increase in the level of output and income sufficient to induce the domestic representative agent to willingly hold this greater liquidity. It is inevitable that the process of adjustment of the current account balance and output in response to a devaluation will take time. In the very short run—say, a day or week—very little adjustment, if any, can occur.

The nature of these issues can be seen from a log linear representation of the AA equation I and the constraint IV

$$l_t = h_t + p_t + \epsilon\, y_t - \eta\, r_t - \eta\,(E\{p_{t+1}\} - p_t) \tag{53}$$

$$r_t = \tilde{r_t} + \Theta - (E\{q_{t+1}\} - q_t) \tag{54}$$

where y_t is the logarithm of domestic income, q_t is the logarithm of the real exchange rate, l_t is the logarithm of the nominal stock of liquidity and p_t is the logarithm of the domestic price level—that is, $l_t - p_t$ is the logarithm of

L—and ϵ and η are, respectively, the income elasticity and the absolute value of the negative interest semi-elasticity of demand for liquidity. Note that r_t and $\tilde{r_t}$ are the levels, not the logarithms, of small-country and rest-of-world interest rates. It should be noted that (53) presents the equation of the AA curve in its demand-for-money form, with l_t on the left side instead of r_t, and with h_t representing a logarithmic shock to the demand for liquidity. Assuming that the price level in the rest of the world is normalised at unity, the logarithm of the real exchange rate can be expressed

$$q_t \quad = \quad p_t + \pi_t \tag{55}$$

where π_t is the logarithm of the nominal exchange rate Π, defined as the price of the small country's currency in rest-of-world currency.

It can be easily seen that if price level adjustment is instantaneous, a positive monetary shock—that is, an one-time increase in l_t or a decline in h_t—will result in an immediate increase p_t and fall in π_t in the same proportion as the monetary shock. Since the shock is a one-off occurrence, the expected inflation rate and rate of change of the real exchange rate will be unaffected. The real interest rate will remain unchanged and price level flexibility will guarantee that y_t stays constant at its full employment level. As a result, from equations (53) and (55),

$$\Delta p_t = -\Delta\pi_t = \Delta(l_t - h_t)$$

and

$$\Delta q_t = 0.$$

Now suppose a length of run sufficiently short for changes in q_t to have no effect on y_t as well as for p_t and $E\{p_{t+1}\}$ to remain unchanged. Assume also that the risk premium on domestic assets does not change and that agents regard the real exchange rate as a random walk, implying that $E\{q_{t+1}\} = q_t$. This will imply an unchanged level of r_t in (54) and the left side of equation (53) will exceed the right side. Because of the attempt of the domestic representative agent to dispose of the excess liquidity by purchasing claims on real capital from abroad, π_t and q_t will fall, both in the same proportion. Since neither r_t or y_t are affected, there is no mechanism to bring the right and left sides of (53) together and the small country's currency will devalue without limit. There will be no short-run equilibrium!

4.2 Two Avenues to Equilibrium

It turns out that there are two ways in which a stable equilibrium can occur. The first involves the fact that domestic and foreign outputs can be divided into tradeable and non-tradeable components. In the previous section we assumed that each country's consumption and investment is divided between both countries' outputs, with the relative price of the two outputs denoted by the real exchange rate Q. We can visualise a part of each country's output, the non-tradeable component, as available only for home consumption and investment, and the remaining part, the tradeable component, as available for consumption and investment in both countries. The tradeable components can be thought of as having the same price, measured in either currency, in both countries while the output components restricted to consumption and investment in the country in which they are produced will, of course, have different prices. One need not equate traded and non-traded output components with traded and non-traded goods, as every good will have embedded in it both traded and non-traded components.[17]

We can therefore express the small country's price level as a weighted average of the domestic-currency prices of the traded and non-traded components of output.

$$p_t \quad = \quad \theta\, p_t^n - (1-\theta)\,\pi_t \tag{56}$$

where θ is the share of output that is non-traded, p_t^n is the domestic-currency price of the non-traded output component, and the price of the traded output component in rest-of-world currency units is normalised at unity.[18] It is obvious from this equation that a devaluation of the small country's currency will increase its price level even if the home-currency price of domestic non-traded component of output and price of the traded component in rest-of-world currency are fixed. It follows from (53) that the short-term equilibrium change of the logarithm of the nominal exchange rate will be

$$\Delta\pi_t = -\frac{1}{1-\theta}\,\Delta(l_t - h_t) = -\frac{1}{1-\theta}\,\Delta p_t. \tag{57}$$

[17] For example, the classic non-traded good, haircuts, will typically contain a traded component because the hair-stylist will use hair cream, scissors, chairs and other materials that may be traded across the international border. Of course, these imported items will themselves contain non-traded as well as traded components because domestic labour will be used in the process of importing, warehousing and distributing them.

[18] Since Π_t is the price of domestic currency in terms of foreign currency, $-\pi_t$ is the logarithm of the domestic currency price of foreign currency, which the rest-of-world price of the traded output component must be multiplied by to express it in units of domestic currency.

Since none of the other variables in (53) are affected, the increase in the price level will be proportional to the increase in the stock of liquidity, while the nominal exchange rate will decrease by some multiple of the increase in liquidity—the exchange rate overshoots its long-run equilibrium value, which will be below its initial value by an amount proportional to the increase in the money stock. The short-run change in the real exchange rate will be

$$\begin{aligned} \Delta q_t &= \Delta p_t + \Delta \pi_t \\ &= \left[1 - \frac{1}{1-\theta}\right] \Delta(l_t - h_t) \\ &= -\frac{\theta}{1-\theta}\, \Delta(l_t - h_t). \end{aligned} \tag{58}$$

which will be negative as long as the nominal exchange rate overshoots its final equilibrium value, which will happen whenever the non-traded component of output is positive—that is, $\theta > 0$.

It has recently been argued that even traded goods prices may not respond to exchange rate movements because of local-currency-pricing by firms that have monopsony power in international markets.[19] Letting κ represent the fraction of the small country's traded component of output that is *not* priced in local currency independently of the nominal exchange rate, the change in p_t in response to a change in π_t becomes, from (56),

$$\Delta p_t = -\kappa\,(1-\theta)\,\Delta \pi_t. \tag{59}$$

and equation (57) will become

$$\Delta \pi_t = \frac{-1}{\kappa\,(1-\theta)}\,\Delta(l_t - h_t). \tag{60}$$

If the price of the traded component is set entirely in the small country's currency and does not respond to movements in the exchange rate, $\kappa = 0$ and Δp_t will therefore also be zero. The nominal and real exchange rates will fall without limit in response to a positive monetary shock even if the traded component represents a substantial fraction of output.

[19] For recent literature on this issue, see Michael B. Devereux, "Real Exchange Rates and Macroeconomics: Evidence and Theory," *Canadian Journal of Economics*, Vol. 30, No. 4 (November), 1997, 773-808, Caroline Betts and Michael B. Devereux, "Exchange Rate Dynamics in a Model of Pricing-to-Market," *Journal of International Economics*, Vol. 50, No. 1 (February), 2000, 215-44 and Michael B. Devereux and Charles Engel, "Exchange Rate Volatility, Exchange Rate Pass-Through and Exchange Rate Disconnect," *Journal of Monetary Economics*, Vol. 49, No. 5 (July), 2002, 913-40.

We have thus-far assumed that the expected future rate of change in the real exchange rate is unaffected by this short-run overshooting movement in the nominal exchange rate. This would be reasonable if there is a lot of noise in the nominal and real exchange rates and agents act as though the real exchange rate is a random walk. Suppose, however, that agents know when the real exchange rate has fallen below its long-run equilibrium level and expect it to rise back to that level.[20] The term $(E\{q_{t+1}\}-q_t)$ in equation (54) will become positive, causing the domestic interest rate to fall. The quantity of liquidity demanded will thus increase in equation (53), offsetting some of the shock to the excess supply of money and moderating, and possibly even eliminating, the overshooting of the nominal exchange rate. This will be the case whenever the real exchange rate falls in response to a monetary shock as long as agents understand that the fall is temporary.[21]

Suppose that agents currently expect future capital gains, as a result of subsequent reversal of the decline in the real exchange rate, yielding a current-period return of ϱ times the real exchange rate decline. Equation (54) now yields

$$\begin{aligned} \Delta r_t &= -\Delta(E\{q_{t+1}\} - q_t) \\ &= \varrho\,(\Delta p_i + \Delta \pi_t) \\ &= \varrho\,[1 - \kappa\,(1-\theta)]\,\Delta \pi_i \end{aligned} \tag{61}$$

which, along with equations (53) and (59), gives us

$$\begin{aligned} \Delta(l_t - h_t) &= -\,\kappa\,(1-\theta)\,\Delta\pi_t - \eta\,\varrho\,[1-\kappa\,(1-\theta)]\,\Delta\pi_t \\ &= -[\kappa\,(1-\theta)(1-\eta\,\varrho) + \eta\,\varrho]\,\Delta\pi_t \end{aligned}$$

or

$$\Delta\pi_t \;=\; \frac{-1}{\kappa\,(1-\theta)(1-\eta\varrho) + \eta\,\varrho}\,\Delta(l_t - h_t). \tag{62}$$

A stable equilibrium will occur unless one of κ and $(1-\theta)$ and one of η and ϱ are zero. If either κ or $(1-\theta)$ are zero, the above expression becomes

$$\Delta\pi_t \;=\; -\frac{1}{\eta\,\varrho}\,\Delta(l_t - h_t) \tag{63}$$

[20] If this is the case, it can no longer be assumed that the prices of non-traded output components are fixed because agents are unaware that a shock of aggregate demand has occurred. The only basis for price level rigidity becomes menu and other costs of initiating price change. I would like to thank Allan Hynes for noting this point.

[21] This idea originated with Rudiger Dornbusch, “Expectations and Exchange Rate Dynamics,”, *Journal of Political Economy*, Vol. 84, No. 6 (December), 1976, 1161-76.

and the nominal exchange rate will overshoot its long-run equilibrium level whenever $\eta\,\varrho \leq 1$. If either η or ϱ are zero, the expression reduces to

$$\Delta\pi_t \;=\; -\frac{1}{\kappa\,(1-\theta)}\,\Delta(l_t - h_t) \tag{64}$$

and overshooting will occur unless $\kappa = 1$ and $\theta = 0$. Assuming that the demand for liquidity is negatively sloped with respect to the interest rate and some component of output is tradeable, an equilibrium will exist if not all domestic output is priced in local currency without regard to exchange rate movements or if agents expect the current change in the real exchange rate to be temporary.

4.3 Will Overshooting in Fact Occur?

For reasonable values of the parameters it would seem likely that monetary shocks will lead to very substantial temporary movements in nominal and real exchange rates. Based on evidence summarised by David Laidler, -0.2 would represent a reasonable upper bound on the short-run elasticity of demand for money.[22] If we set the interest cost of holding money at 5%, the resulting interest semi-elasticity of demand for money will be -4.0. Although real exchange rates are not random walks, the degree of mean reversion is extremely small in that 50% of the full adjustment to temporary shocks will take from 3 to 5 years.[23] If agents expect the reversion of the real exchange rate to its equilibrium value to be evenly spread out over future months, ϑ will be less than .02 when the unit of time is one month.[24] If we set the share of non-traded output components in total output equal to 0.6 and assume that the domestic price of the entire traded component adjusts to incorporate nominal exchange rate changes, we would conclude that a 1% excess supply of liquidity will cause the exchange rate to devalue by over 3%. And if we allow for the possibility that some fraction of the traded component of output is priced in domestic currency without regard to the exchange rate, the overshooting will be much greater.

Of course, once sufficient time has elapsed for domestic output to respond to declines in the real exchange rate the overshooting will dissipate. Overshooting will also be smaller if agents can determine which exchange

[22] David E. W. Laidler, *The Demand for Money*, Second Edition, New York: Dun-Donnelly, 1977, p. 133.

[23] See Kenneth Rogoff, 'The Purchasing Power Parity Puzzle," *The Journal of Economics Literature*, Vol. 34, No. 2, June) 1996, 647-668.

[24] $\vartheta = (1 - .5)^{1/n}$ where n is the number of months until 50% adjustment is achieved.

rate movements are due to short-term monetary shocks and which are not. In this respect, however, it must be kept in mind that no forecasting approach has been able to consistently outperform the simple prediction that tomorrow's real exchange rate will, on average, equal today's.[25]

5 Current Account and Balance of Payments Adjustment

Two issues that are modelled correctly in Sections 2 and 3 but remained in the background must now be addressed. The first deals with the relationship between the net international capital flow, the real exchange rate and current account adjustment. The second notes the absence of any causal relationship between changes in the current account and changes in any balance of payments surplus or deficit.

5.1 The Real Exchange Rate and the Current and Capital Accounts

It was noted that if consumption in the small open economy is growing faster in the steady state than the domestically employed capital stock, the domestic representative agent will be continually investing some fraction of domestic savings in capital employed in the rest of the world, and continually earning a fraction domestic income from that foreign-employed capital. Domestic income is equal to domestic output plus net earnings from domestically owned capital employed abroad minus earnings from domestically employed capital owned by foreigners.

$$\hat{Y}_t \quad = \quad Y_t + DSB_t \tag{65}$$

where $\hat{Y}_t$ and Y_t are respectively GNP and GDP, as conventionally defined, and DSB_t is the debt service balance, defined as earnings from domestically owned foreign capital minus earnings from foreign owned domestic capital. Along the steady-state growth path, consumption and income will be growing at the same constant rate and domestic output and the domestically employed capital stock will also be growing at the same constant rate, although their growth rate may differ from the growth rate of consumption

[25]This was first noticed by Richard Meese and Kenneth Rogoff in "Empirical Exchange Rate Models of the Seventies: Do They Fit Out-of Sample?" *Journal of International Economics*, Vol. 14, No. 1-2 (February), 1983, 3-24.

and income in an open economy. If income is growing faster than output in the steady-state, the difference between them—that is, the debt service balance—must be growing constantly through time. This in turn implies that in every period a portion of domestic savings must be flowing to investment in foreign employed capital—that is, domestic savings must exceed domestic investment. From the fact that domestic output, net of depreciation, is the sum of domestic consumption, domestic net investment, and net sales of output to foreigners,

$$Y_t = C_t + I_t + B_{Tt}, \tag{66}$$

where B_{Tt} is the balance of trade, equation (65) can be expressed[26]

$$\hat{Y}_t = C_t + I_t + B_{Tt} + DSB_t. \tag{67}$$

Subtracting $C_t + I_t$ from both sides of this equation, we obtain

$$\hat{Y}_t - C_t - I_t = B_{Tt} + DSB_t \tag{68}$$

or

$$NCO = S_t - I_t = B_{Tt} + DSB_t = CAB \tag{69}$$

where $S_t = \hat{Y}_t - C_t$ is savings, NCO is the net capital outflow and CAB is the current account balance. All these magnitudes are measured in real terms.

The relationship between domestic saving and investment, the real exchange rate and the current account balance in the long run can be analysed with reference to Figure 10. The real exchange rate is on the vertical axis and the real current account balance is on the horizontal one. The vertical SI line imposes the condition that real exchange rate changes do not lead directly to changes in real savings and investment—they simply involve changes in the relative price of domestic output in terms of foreign output. Such real exchange rate changes primarily reflect changes in the price of the

[26] All these magnitudes include transactions on both private and public account with the result that taxes and government expenditures do not appear separately as in traditional Keynesian models. Also, we could equivalently write output as

$$Y_t = C'_t + I'_t + E'_t$$

where C'_t, I'_t and E'_t are the outputs of consumption, net investment and exported goods. To obtain (66) we simply add and subtract imports and define C_t and I_t and B_{Tt} as consumption, net investment and net exports of both domestic and imported output.

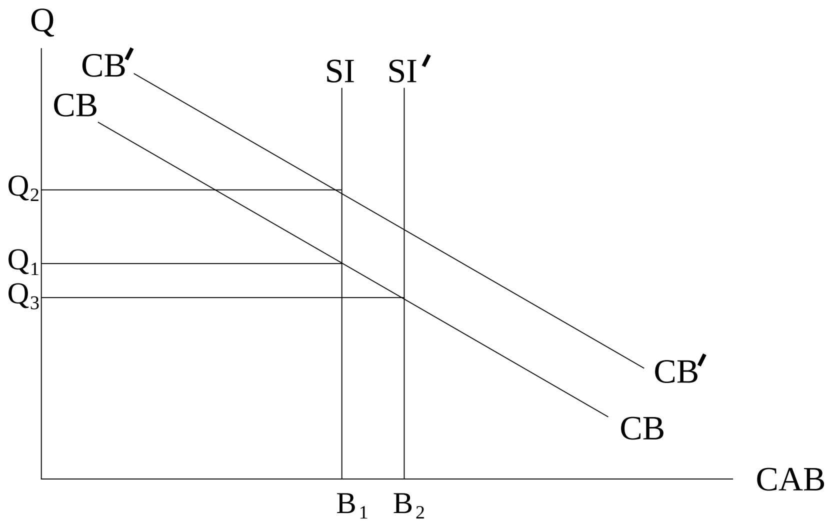

Figure 10: The real exchange rate and current account balance in a small open economy in the long run.

non-traded component of domestic output relative to the price of the non-traded component of foreign output, where prices are measured in one country's currency. Although traded components of output will have the same prices all over the world, the relative prices of different traded components can change through time as the world economy grows, and the particular traded components may not all be produced in all countries. These relative prices may also be reflected in real exchange rate changes, in which case terms of trade changes and real exchange rate changes may be positively related. One therefore cannot rule out the possibility that a decline in the real exchange rate may lead to a reduction in domestic real income via an effect on the terms of trade and thereby affect savings. In the long-run, this would appear as a slight change in the steady-state growth rate of income and consumption. At the same time, however, this reduction in the terms of trade would reduce slightly the profitability of investment in the domestic economy, since output of traded components is less valuable than before. This would be reflected in a slight decline in the productivity of the aggregate domestic capital stock and hence a slight reduction in its growth rate. The effect of these forces on the slope of SI would be ambiguous in that the savings and investment will be affected in the same direction. The assumption that the SI line is vertical in the long-run steady state would therefore seem to be reasonable as an approximation.

The negatively sloping CB curve states simply that a fall in the real exchange rate will lead to an increase in net purchases of consumption and investment goods abroad. The real exchange rate will adjust to bring the equilibrium current account balance into line with the net capital flow. In the long run, the current account balance will thus be determined primarily by the domestic representative agent's choices regarding real savings and investment—that is, it represents net intertemporal lending.[27]

Contrary to what often is claimed in the popular press, the imposition of a tariff on imports or a subsidy on exports that has no impact on saving and investment will not increase the current account balance in the long-run—the real exchange rate will simply adjust to maintain its equality with the unchanged real net capital flow. It is also the case that an increase in rest-of-world demand for a country's exports will not lead to an increase in the current account balance unless domestic savings and investment are also affected. As indicated by the rightward shift of the CB curve to CB', the effect will be a rise in the real exchange rate with no change in the current account balance.

Another common fallacy is the argument that an observed increase in, or so-called 'improvement' of, the current account balance implies that the country is better off because it now has a better market for its goods abroad. As can be seen in Figure 10, an increase in the current account surplus results from a rightward shift of the SI line. Assuming that wealth, and therefore the steady-state growth path of income and consumption, has not changed, domestic investment must have declined relative to domestic saving—that is, the domestic economy has become a poorer place in which to invest. Indeed, unless the demand for the country's exports increases along with the decline in domestic investment, the 'improvement' of the current account balance will have resulted from the fall in the real exchange rate required to create an increase in the current account surplus equal to the increase in the net capital outflow.

A tariff on imports or subsidy of exports will nevertheless have a positive effect on the current account balance in the short run when the utilisation of the current human and physical capital stock can change. This can be seen from Figure 11, where the current account balance is on the horizontal axis and the vertical axis now gives the level of income and employment. The positive slope of the NL curve indicates that as income temporarily

[27] For a rigorous presentation of this argument, see Maurice Obstfeld and Kenneth Rogoff, *Foundations of International Macroeconomics*, MIT Press, 1996, Chapter 1.

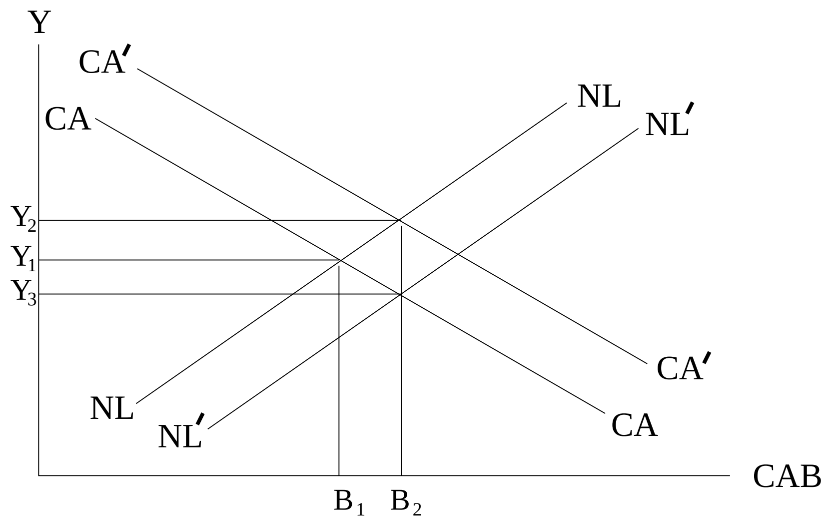

Figure 11: The real exchange rate, real income and the current account balance in a small open economy in the short run.

rises relative to its full-employment level Y_f, saving increases because the representative agent wants to intertemporally smooth consumption. The desired net capital outflow thereby increases. The negative slope of the *CA* curve indicates that as income increases relative to its full-employment level, holding the real exchange rate constant, imports will increase relative to exports, reducing the current account balance. Given a constant real exchange rate, the levels of domestic employment, output and income must adjust to maintain equality between the current account balance and the net capital inflow. If the nominal exchange rate is flexible, both the real exchange rate and output and income can adjust simultaneously in the short-run, making the causal forces bringing about particular observed current account and real exchange rate adjustments difficult to determine.

5.2 Balance of Payments Disequilibria and the Current Account

The standard Fleming-Mundell result regarding the short-run effects of real and monetary shocks on output and income in a small open economy has been fully verified in the analysis of the previous sections and can be reviewed with reference to Figure 12. When the nominal exchange rate is flexible, equilibrium output and income in the small open economy is established by monetary factors at the intersection of the AA curve and the rest-of-world determined interest rate line rZ. The real exchange rate will then adjust until the GG curve crosses rZ at that same intersection point. Unless they somehow affect asset equilibrium, real shocks will simply lead to equilibrium real exchange rate adjustments without affecting output and employment. When the nominal exchange rate is fixed, equilibrium output and income is established by real factors at the intersection of the GG curve and the rZ line. To the extent that the AA curve does not pass through that intersection there will be an excess demand or supply of liquidity resulting in attempts to reestablish portfolio equilibrium by buying or selling the monetary asset in return for claims on real capital. To maintain the fixed exchange rate, the authorities are forced to provide the stock of liquidity the public wishes to hold at the fixed interest rate and equilibrium level of output and income. The AA curve will thereby adjust until it passes through the GG-rZ intersection.

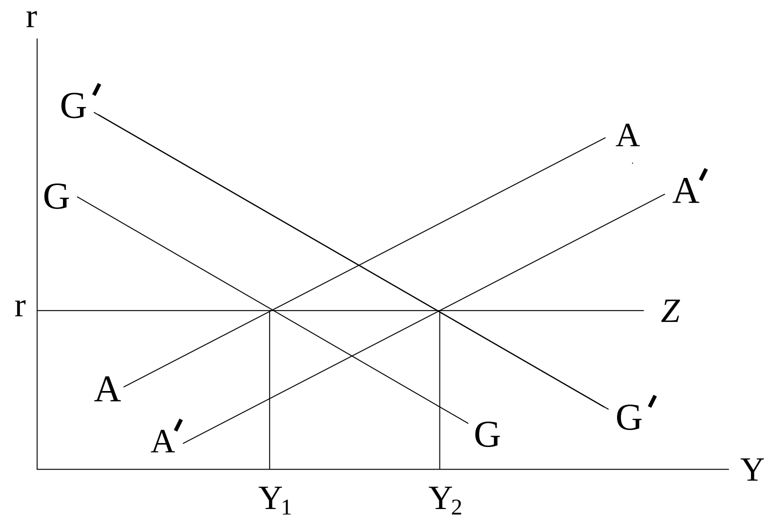

Figure 12: Response of a small open economy to short-term monetary and real shocks.

These adjustments of the AA curve under fixed exchange rates are directly related to the process by which balance of payments equilibrium is maintained. The stock of liquidity in the economy can be altered in two ways by the actions of the authorities—by purchasing or selling foreign exchange reserves in return for the domestic monetary asset, or by purchasing real capital from domestic residents in exchange for the monetary asset and holding ownership of this capital on public account. Simply printing nominal units of the monetary asset and handing them out would be equivalent to the latter alternative. The foreign exchange reserves are, of course, claims on foreign employed capital held on public account. Assuming for simplicity of argument that the price of domestic output, being fixed in the short run, is normalised at unity,

$$L_t = R_t + D_t \tag{70}$$

where R_t is the stock of foreign exchange reserves and D_t is the quantity of the monetary asset that has been created without the purchase of foreign exchange reserves. The balance of payments surplus is the period-to-period change in the stock of foreign exchange reserves, which can be obtained directly from (70) and written as

$$R_t - R_{t-1} = (L_t - L_{t-1}) - (D_t - D_{t-1}). \tag{71}$$

If asset markets are in equilibrium, $(L_t - L_{t-1})$ must be equal to the change in the quantity of liquidity demanded between periods $t-1$ and t. This can be obtained from equation (53), and equals

$$L_t - L_{t-1} = (H_t - H_{t-1}) + \; \Xi \; (Y_t - Y_{t-1}) \tag{72}$$

where the real interest rate is constant according to equation (54) under the assumption that the world-market risk premium for holding domestic capital and the expected future rate of change in the real exchange rate are both constant. Shifts in the demand for liquidity are represented by $(H_t - H_{t-1})$ and the parameter Ξ gives the change in the quantity of liquidity demanded in response to a change in real income.[28] Substituting (72) into (71), we obtain

$$R_t - R_{t-1} = (H_t - H_{t-1}) + \; \Xi \; (Y_t - Y_{t-1}) - (D_t - D_{t-1}). \tag{73}$$

The shift in the GG curve in Figure 12 is represented by $(Y_t - Y_{t-1})$ and the remaining terms represent shifts in the AA curve. A change in Y will lead to a change in R in the same direction to maintain L constant unless the authorities compensate with an equivalent opposite change in D. A change in the demand for liquidity H will require the authorities to change D by an identical amount to maintain the AA curve in an initial equilibrium position if the stock of foreign exchange reserves is to be unaffected. And

[28]Note that h_t in equation (53) is the logarithm of H_t.

an exogenous increase in D by the authorities will result in an equivalent reduction in R with no change in the stock of liquidity—the authorities have no monetary control once they fix the nominal exchange rate.

The important thing to notice here is that there is no relationship between the current account balance and the balance of payments. The current account balance is determined by the real forces outlined in Figures 10 and 11. Those same real forces establish the equilibrium levels of income and the real exchange rate in the short- and long-runs. A balance of payments disequilibrium is consistent with any level of the real current account balance and the associated level of real capital outflow—it arises simply because the authorities are expanding D at a slower or faster rate than the demand for liquidity is growing.

It is traditional to define balance of payments equilibrium as a situation where autonomous receipts from the sale of goods and capital abroad equal the autonomous payments for goods and capital abroad or, what is the same thing, a situation where net induced payments are zero. Induced receipts and payments are defined as transactions conducted by the authorities to maintain a pre-determined level of the nominal exchange rate, or to influence the level of the exchange rate under a floating rate system. In the above analysis, induced transactions are represented by changes in the level of the stock of foreign exchange reserves held by the authorities. While these induced transactions are strictly monetary, one can also think of induced transactions designed to alter the real and nominal exchange rates by subsidising exports or taxing imports—as noted, however, unless these restrictions affect savings or investment differentially, they will have no effect on the current account balance.

The balance of payments surplus is traditionally defined as

$$R_t - R_{t-1} = CAB_t - ANCO_t \tag{74}$$

where CAB_t and $ANCO_t$ are, respectively, the current account balance and the autonomous net capital outflow. The actual observed level of the current account balance is always assumed equal to the autonomous level. As a matter of arithmetic, the total capital outflow, NCO_t, must equal the autonomous net capital outflow plus the induced capital outflow, which is simply the purchase of foreign exchange reserves by the authorities. That is,

$$NCO_t = ANCO_t + (R_t - R_{t-1}) = CAB_t \tag{75}$$

which implies that the balance of payments surplus must be equal to the difference between the total and autonomous net capital outflows,

$$R_t - R_{t-1} = NCO_t - ANCO_t. \tag{76}$$

Observed balance of payments disequilibria are exclusively an asset phenomenon. Because the authorities want to maintain a particular level of the exchange rate, they are forced to make up any difference between the existing stock of non-monetary assets and the amount the private sector wishes to hold by accumulating or decumulating foreign exchange reserves. Any difference between the existing stock of non-monetary assets and the desired stock—that is the excess supply of non-monetary assets—is always equal to the excess demand for the monetary asset. An increase in the current account balance, holding the autonomous net capital outflow constant, will be matched an increase in the balance of payments surplus because it represents an equivalent increase in the total net capital outflow.

As noted in the earlier sections, whenever the nominal exchange rate is fixed, a balance of payment disequilibrium can always be corrected simply by changing the rate of growth of the supply of the monetary asset through manipulation of our variable D. No short-term changes in income and employment and other real variables will result. And the size of the domestic stock of foreign exchange reserves is irrelevant as long as it is sufficient to cover normal day-to-day variations in the demand for liquidity—any surplus or deficiency of the level of the stock of reserves can be immediately corrected by an appropriate change in the level of D.

6 Conclusion

The focus of this paper has been to develop a model of the short-term effects of real and monetary shocks on the real interest rate, real output and employment, and the nominal and real exchange rates in open economies. The model is embedded in an intertemporal AK model of economic growth that is sufficiently general to incorporate, in the background, the usual range of inefficiencies of resource allocation. The resulting short-run analytical framework is similar in appearance to the standard IS-LM model and yields very similar results with respect to the effects of monetary shocks. The results with respect to the effects of real shocks, however, are much more limited—no general conclusions can be reached, for example, about the effects of fiscal policy once we embed the analysis in an intertemporal framework and allow for the possibility that public-sector production of goods and services may frequently substitute for their private-sector production.[29] Within the framework developed here, real shocks can arise in three ways. There can be an increase in the actual and perpetual future flow of output and income from the existing capital stock. Or there can be an increase in the expected future income stream per period from the capital stock in the absence of an actual observed change in current output and income. Thirdly, there can be an increase in the flow of income from the capital stock in the current period followed by an actual and expected return to the original income level in subsequent periods. An increase in the actual and expected income flow from the capital stock in the current and several future periods with a subsequent return to the initial income level can be viewed as roughly equivalent in effect to an increase in the perpetual income flow from the capital stock having the same present value, with the difference between the current and permanent levels of income in each period being channelled into additions to the stock of capital.

For the small-open economy, the standard Fleming-Mundell propositions about the effects of real and monetary shocks under fixed and flexible exchange rates hold.[30] When the exchange rate is fixed, short-term variations in output and employment are generated by real shocks with the stock of liquidity adjusting automatically as a result of the actions of the authorities in maintaining the exchange rate parity. Monetary shocks are accommodated by endogenous adjustments of the stock of liquidity while real shocks lead to changes in employment and income. In the case of a flexible exchange rate, short-term adjustments in output and employment are generated by shocks to either the demand or supply of liquidity with the real exchange rate adjusting endogenously to maintain real sector equilibrium. Monetary policy is effective in controlling output while real shocks, including those that might be generated by fiscal policy, have no effect.

As our small open economy gets bigger the results under fixed exchange rates remain unchanged as long as foreign exchange reserves are not held in the form of liquid foreign assets. The authorities must always provide the level of liquidity the public demands at the equilibrium determined by

[29] For corroborating literature, see Olivier Jean Blanchard and Stanley Fischer *Lectures on Macroeconomics*, The MIT Press, 1989, Chapter 2, and Robert Barro, "Are Government Bonds Net Wealth?" *Journal of Political Economy*, Vol. 81, No. 6 (December), 1974, 1095-1117. This limitation also applies to the more sophisticated analyses of fiscal policy within the *IS-LM* framework—see for example, Martin Bailey, *National Income and the Price Level*, McGraw-Hill, 1962, 71-80.

[30] Subject, of course, to the previously noted limitations with respect to fiscal-policy analysis.

domestic real forces and the world interest rate. Since the domestic stock of liquidity must always adjust to match that in the rest of the world at the world real interest rate, the rest-of-world monetary authorities determine that interest rate, regardless of the relative size of the domestic economy. Obviously, in a multi-country world any key-currency country to whose currency other countries fix their nominal exchange rate will run monetary policy and interest rates in those countries.[31]

When the nominal exchange rate is flexible, increasing the size of our small open economy will give it increasing influence on the world real interest rate—the greater the fraction of world liquidity the country holds, the greater the influence. Positive monetary shocks then operate through two avenues—they cause the domestic real exchange rate to decline, shifting world demand to domestic goods from those produced abroad, and they increase the world stock of liquidity, lowering the world real interest rate and thereby resulting in an expansion of investment and output in all countries. If rest-of-world authorities endogenously adjust their stocks of liquidity to avoid possible overshooting effects on exchange rates of the expansion of domestic liquidity, the effects of domestic monetary expansion on world interest rates and world income and employment will be greater.

Domestic real shocks will also have increasing effects on the world interest rate under flexible exchange rates when the country increases in size, resulting in possible world-wide effects on real output and income, although endogenous real exchange rate adjustments will prevent differential effects across countries.

Our most important conclusion from the point of view of modern policy analysis relates to the channels through which short-run monetary policy operates. In the small open economy, monetary policy operates on domestic output and employment through its effects on the exchange rate. There is little reason to expect that the underlying real interest rate relevant for investment decisions can be much affected. Why would the risk premium on domestic real capital fall as a result of a one- or two percent current change in the stock of liquidity? This does not rule out the possibility that in a small economy in which there is a large array of intermediate assets there might be an effect of short-term monetary expansion on the interest rates on some of these assets. But this need not imply a change in the underlying interest rate on real capital. An overshooting fall in the exchange rate in response to

[31] Recall that if the peripheral countries hold foreign exchange reserves in the monetary asset of the key-currency country, changes in the stocks of those reserves will affect the key-currency country's stock of liquidity and thereby have a real interest rate effect.

a monetary expansion could, of course, lead to the expectation that the real exchange rate will rise in the future as the overshooting dissipates, causing the real interest rate to fall in response to the resulting expected capital gains. The problem with this argument is that it is virtually impossible to distinguish the actually observed patterns of real exchange rate movements from random walks, implying that real exchange rates are equally likely to change in either direction from the current period to the next. As the economy gets bigger, of course, domestic monetary expansion will affect world interest rates—an unanticipated short-term monetary expansion in the U.S., for example, will undoubtedly cause world and U.S. real interest rates to fall, especially if other countries also expand liquidity to prevent appreciation of their exchange rates with respect to the U.S. dollar.

It is sometimes argued that the authorities of even a small country can reduce domestic interest rates by monetary expansion under a fixed exchange rate because of the portfolio effects of the resulting increase of domestic asset holdings on private account and reduction of foreign assets on public account associated with the decline in the stock of foreign exchange reserves. This increase in the demand for domestic relative to foreign assets, on combined public and private account, it is argued, will cause domestic interest rates to fall relative to foreign rates. Given that foreign exchange reserve holdings are typically a small fraction of the overall stock of liquidity which, in turn, is perhaps one-third the size of income, and given that the broadly-measured domestic capital stock will be at least ten times the size of income, the overall portfolio shift resulting from the typical change in the stock of foreign exchange reserves has to be tiny as a fraction of total wealth. It is therefore difficult to believe that such portfolio-mix changes could lead to a significant change in the interest rate relevant for real investment.

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