Stochastic Monetary Interdependence, Currency-Regime Choice and the Operation of Monetary Policy

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1 Introduction

This paper develops a theory that explains important stylized facts now well known to international economists and examines the implications of that theory for countries' decisions as to whether to adopt fixed or flexible exchange rates and, in the latter case, how to conduct and measure the effects of monetary policy. The stylized facts are: first, countries' real and nominal exchange rates tend to move together under flexible exchange rate systems with the ratios of domestic to rest-of-world price levels showing much less variability;[1] second, business cycles and medium-term price level movements tend to be international in scope;[2] third, forward exchange rates track the corresponding spot rates very well but show much less variability than spot rates. And forward discounts perform poorly as predictors of subsequent changes in spot rates.[3] The theory also explains why real exchange rates have traditionally shown less low-frequency variability under fixed as opposed to flexible exchange rate regimes.

These phenomena are ascribed to the maximizing behavior of central banks, each pursuing similar objectives, in a world where information about the future course of economic activity and the timing of the influence of central bank policy upon it is extremely poor. Central banks, given the limited information available to them, are induced to set paths of monetary growth that will neutralize the effects of international portfolio shocks on domestic exchange rates while allowing exchange rate movements that result from changes in technology, oil shocks, the terms of trade, and other real forces to go through unopposed. The result is a similarity of credit conditions across countries leading to world-wide variations of output and inflation rates. This monetary coordination in response to exogenous local and world-wide stochastic shocks suggests an appropriate name of the theory: *Stochastic Monetary Interdependence*.

[1] This fact was noted early on by Michael Mussa, "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," in Karl Brunner and Allan H. Meltzer, eds., *Carnegie-Rochester Conference Series on Public Policy*, Vol. 11, (*Policies for Employment, Prices, and Exchange Rates*), North Holland, 1979, pp. 9–57.

[2] See Arthur Burns and W.C. Mitchell, *Measuring Business Cycles*, New York: National Bureau of Economic Research, 1941, Victor Zarnowitz, "Recent Work on Business Cycles in Historical Perspective: A Review of Theories and Evidence," *Journal of Economic Literature*, Vol. 23, June 1985, 523–80, David K. Backus, Patrick J. Kehoe and Finn E. Kydland, "International Real Business Cycles," Working Paper, Leonard N. Stern School of Business, New York University, February 1991, and Marianne Baxter and Allan C. Stockman, "Business Cycles and the Exchange Rate Regime: Some International Evidence," *Journal of Monetary Economics*, Vol. 23, May 1989, 377–400.

[3] See again the paper by Michael Mussa cited above.

In providing a coherent explanation of how world monetary policy works, both monetary shocks and speculative bubbles are ruled out as factors explaining recent major (though not minor) real exchange rate movements. The results indicate why purchasing power parity has not been a useful empirical model of exchange rate behavior and explain the tendency of forward discounts to grossly under-predict, and often wrongly predict, future movement in spot rates.[4] Finally, a new perspective is presented on the old question of whether fixed or flexible exchange rate systems are best by establishing the circumstances under which it will pay a country to adopt one regime rather than the other. This brings us a bit closer to a theory of optimal currency areas.

The results developed here are in sharp contrast with the literature on the international coordination of economic policies pioneered by Oudiz and Sachs[5] and Canzoneri and Henderson.[6] That literature explores the logical consequences of maximizing behavior of individual countries' authorities when they know the parameters governing the time-paths of their economies and concludes that substantive attempts at coordination are rare; this study explores the consequences when the maximizing authorities know the signs but not the magnitudes of the parameters and cannot forecast dynamic paths, and suggests that coordination is inadvertent but widespread. This difference in the assumed information sets of the authorities also distinguishes this present work from the old targets and indicators literature.[7]

The model and the results that follow from it have important applications to contemporary issues in international macroeconomics. First, they

[4]See, Kenneth Rogoff, "The Purchasing Power Parity Puzzle," *The Journal of Economic Literature*, 34, 2 (June), 1996, 647–668. See also, Bennett T. McCallum, "A Reconsideration of the Uncovered Interest Parity Relationship," *Journal of Monetary Economics*, 33, 1994, 105–132.

[5]Gilles Oudiz and Jeffrey Sachs,, "Macroeconomic Policy Coordination Among the Industrial Economies," *Brookings Papers on Economic Activity*, 1, 1984, 1–77.

[6]Matthiew Canzoneri and Dale W. Henderson, *Monetary Policy in Interdependent Economies: A Game Theoretic Approach*, MIT Press, 1991.

[7]See William C. Brainard, "Uncertainty and the Effectiveness of Monetary Policy," *American Economic Review, Papers and Proceedings*, 57, May 1967, 411–25, William Poole, "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," *Quarterly Journal of Economics*, 84, 1976. 631–40, Stephen J. Turnovsky, "The Relative Stability of Alternative Exchange Rate Systems in the Presence of Random Disturbances," *Journal of Money, Credit and Banking*, 8, February, 1976, pp. 29–50, Russel S. Boyer, "Optimal Foreign Exchange Market Intervention," *Journal of Political Economy*, Vol. 86, No. 6 (December), 1978, 1045–55, and Warren Weber, "Output Variability Under Monetary Policy and Exchange Rate Rules," *Journal of Political Economy*, Vol. 89, No. 4 (August), 1981, 733–751.

make possible important insights into the problem of speculative attacks on currencies and on the efficacy of exchange rate target zones.[8] Questions are raised that have not been adequately dealt with in the technical literature. And the resulting answers suggest that the practice of specifying target zones to would appear to be of limited value. Second, light is shed on problems that will be experienced in attempts, like those currently underway in Europe, to establish currency unions. And the conditions under which such currency unions are likely to be beneficial can be more clearly established. On these issues, the present analysis bears out the arguments and predictions of the contemporary literature.[9] Finally, the implications of the model for exchange rate and interest rate determination are of considerable importance for evaluating and extending modern work on the mechanism by which money shocks are transmitted to economic activity. The substantial literature on observed dynamic empirical responses to the U.S. Federal Reserve System's policy actions must reconcile with the theory developed here if the latter is to constitute a believable approach to understanding world monetary policy.[10] It is shown that a consistent interpretation of this evidence in terms of our theory poses few difficulties, although loose ends remain in interpreting the observed responses of exchanges rates to monetary shocks. The approach here can probably also be extended without difficulty to incorporate recent work on the credit channel of monetary transmission although that mechanism would have to be viewed from a world-wide rather than an individual country perspective.[11] On other details of the transmission mechanism, however, the results here suggest that a number of current perspectives need to be revised, particularly those involving the relationship between interest rates and exchange rates and the relevance of interest rate targeting for the operation of monetary policy in small countries.[12]

[8]For an introduction to this literature see Paul Krugman, "Exchange Rates in a Currency Band: A Sketch of the New Approach," and Robert P. Flood and Peter M. Garber, "The Linkage Between Speculative Attack and Target Zone Models of Exchange Rates: Some Extended Results," in Paul Krugman and Marcus Miller, eds., *Exchange Rate Targets and Currency Bands*, Cambridge University Press, 1992.

[9]For a window into this literature, see Barry Eichengreen, *European Monetary Unification*, The MIT Press, 1998.

[10]This empirical response literature is surveyed in Lawrence J. Christiano, Martin Eichenbaum, and Charles Evans, "Monetary Policy Shocks: What Have We Learned and to What End?" Unpublished Manuscript, Northwestern University, 1997.

[11]See Ben Bernanke and Mark Gertler, "Inside the Black Box: The Credit Channel of Monetary Transmission," *Journal of Economic Perspectives*, Vol. 9, No. 4, Fall 1995, pp. 27–48.

[12]See John B. Taylor, "The Monetary Transmission Mechanism: An Empirical Framework," *The Journal of Economic Perspectives*, Vol. 9, No. 4, Fall 1995, pp. 11–26.

2 The Model

The analysis focuses on the problem of a small country that faces domestic technology and demand for money shocks and operates in an international economy in which there are also technology, demand for money, and supply of money shocks. The basic model is a two-country one, consisting of the small country and a big country which can be treated, as the situation requires, either as a rest-of-world aggregate or as a major component of that aggregate. When the rest-world-aggregate consists entirely of small countries that do not cooperate explicitly in the setting of monetary policy, the solution of the individual country's policy problem defines world monetary policy. When the rest of the world consists of the big country and a collection of small countries, world monetary policy involves an interaction between the big country and many small countries identical to the one modeled.

When full employment is continually maintained the foreign (big) economy produces an output consisting of Y^*_{ft} units of a single world consumption characteristic and the domestic (small) economy produces $Q_{ft}Y_{ft}$ units, where Y_{ft} is domestic output and Q_{ft} is the number of units of the world consumption characteristic yielded by each unit of domestic output.[13] Q_{ft} thus represents the full-employment real exchange rate defined as the relative price of domestic output in terms of foreign output when the two economies are fully employed.

Outputs are determined by short-run aggregate supply curves of the form

$$\begin{aligned} y_t &= y_{ft} + \theta\,\gamma\,(p_t - E_{t-1}p_t) + \lambda\,(y_{t-1} - y_{f(t-1)}) && (1)\\ y^*_t &= y^*_{ft} + \theta^*\gamma^*\,(p^*_t - E_{t-1}p^*_t) + \lambda^*\,(y^*_{t-1} - y^*_{f(t-1)}) && (2)\end{aligned}$$

where y_t is the logarithm of current output, y_{ft} is the logarithm of full-employment output, p_t is the logarithm of the actual price level and $E_{t-1}p_t$ the expected log price level based on information in the previous period. A superscript $*$ indicates that the parameter or variable applies to the rest of the world. Neither private agents nor governments observe current-period price levels.

When these equations are interpreted in a rational expectations framework following Lucas, the parameters θ and θ^* vary inversely with the variance of the current price level about its mean conditioned on past history

[13] One unit of foreign output thus equals one unit of the world consumption characteristic. As will be evident later, we assume that one unit of foreign output can be transformed costlessly into one unit of foreign-employed capital and one unit of domestic output can be transformed into one unit of domestically employed capital, although in both cases adjustment costs will be involved in putting the capital into productive use.

and the parameters γ and γ^* are the limiting values of the respective aggregate supply curves as the general price levels approach stability.[14] Since agents use all available information in setting wages, the expected price levels are those that will produce full employment if information about money, technology and other exogenous variables turns out to be correct. Output deviates from its full-employment level as the result of price level surprises. The fact that the previous period's price level is observed might suggest that price-setting errors would be corrected each period, so that deviations of output from its full-employment level would show no persistence. But current output will be affected by past price-level surprises because of irreversible past investment decisions based on them. Also, wage settings may be locked into contracts written in previous periods so that the unemployment rate will vary persistently even though previous periods' price levels are observed by agents. The inclusion of the lagged deviation of output from its full-employment level introduces some rudimentary dynamics to create appropriate persistence in the time-path of unemployment. The parameters θ and θ^* will not be constant across different monetary regimes because agents' information about the variance of unexpected price level shocks will affect their behavior.

Although the approach here follows Lucas, the results will be the same if the response coefficients of output to current unanticipated price level shocks ($\theta\gamma$ and $\theta^*\gamma^*$) are constant across regimes as we will assume that central banks do not know the magnitudes of these parameters in any event. Thus, equations (1) and (2) can be alternatively treated as giving standard short-run supply responses to price level shocks. It will make no difference to the ultimate conclusions whether the output and employment fluctuations result from errors in wage setting that lead to "involuntary unemployment" or from the misperception that real wages have temporarily changed when they have not, leading to "voluntary" substitution between current and future leisure.

Agents are assumed to be free to exchange ownership of existing stocks of capital, and finance new capital formation, across international borders. World asset or portfolio equilibrium is thus determined by the demand functions for money together with the Euler condition determining the relationship between domestic and foreign interest rates. The demand functions for money are standard with everything but interest rates expressed in loga-

[14] Robert E. Lucas Jr., "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review*, 63, June 1973, pp. 326–334. Lucas's analysis is elaborated in Appendix B.

rithms.

$$p_t = m_t - \psi_t + \eta\, r_t + \eta\,(E_{t-1}p_{t+1} - E_{t-1}p_t) - \epsilon\, y_t \tag{3}$$
$$p^*_t = m^*_t - \psi^*_t + \eta^*\, r^*_t + \eta^*\,(E_{t-1}p^*_{t+1} - E_{t-1}p^*_t) - \epsilon^*\, y^*_t \tag{4}$$

where p_t is the price level, m_t is the nominal money stock, r_t is the real interest rate, ψ_t is a demand-for-money shift variable and E_{t-1} is, as before, the expectations operator based on the information available in the period $t-1$. Each country's residents are assumed to hold only home money. The expressions

$$i_t = r_t + (E_{t-1}p_{t+1} - E_{t-1}p_t)$$

and

$$i^*_t = r^*_t + (E_{t-1}p^*_{t+1} - E_{t-1}p^*_t)$$

are the domestic and foreign nominal interest rates. Unlike real interest rates, nominal interest rates are observable within-period by private agents and governments.

Each country's securities are denominated in their own output goods and the domestic real interest rate differs from the foreign real rate by a risk differential ρ_t minus the expected rate of change in the real exchange rate, represented by the change in the logarithm of the real exchange rate q_t. Hence,

$$r_t = r^*_t + \rho_t - (E_{t-1}q_{t+1} - E_{t-1}q_t). \tag{5}$$

Since each country's residents must hold their existing stocks of wealth as either monetary or non-monetary assets, a zero excess demand for money implies a zero excess demand for the aggregate of non-monetary assets. Equations (3) and (4) thus imply that domestic and foreign residents hold their desired mixes of monetary and non-monetary assets. Equation (5) ensures that the existing mix of domestic and foreign non-monetary assets is willingly held by world residents. A situation where domestic and foreign residents together want to hold a greater ratio of domestic to foreign securities in their portfolio than the ratio of domestic to foreign securities in actual existence, for example, will result in a rise in the price of domestic securities relative to foreign securities and a fall in the domestic interest rate relative to the foreign interest rate.

The real exchange rate is defined in logarithms as

$$q_t = p_t - p^*_t - \pi_t \tag{6}$$

where π_t is the logarithm of the nominal exchange rate (domestic currency price of foreign currency). Agents and governments observe the nominal exchange rate but not the real exchange rate in the current period. We can express the deviation of the real exchange rate from its full-employment level as

$$q_t - q_{ft} \quad = \quad -\sigma\left[(y_t - y_{ft}) - (y_t^* - y_{ft}^*)\right] \tag{7}$$

where $\sigma > 0$. The real exchange rate is at its full-employment level when output is at its full-employment level. The full-employment real exchange rate gives the relative valuation of the two countries' outputs determined by technology and tastes. When domestic output and employment rises relative to rest-of-world output and employment at given technology and tastes, an excess supply of domestic output goods on the world market occurs and the marginal valuation of domestic output in producing the world consumption characteristic declines, reducing the real exchange rate relative to its full-employment level.

Finally, the deviation of the rest-of-world real interest rate from its full employment level is negatively related to the deviation of rest-of-world output from its full-employment level according to

$$r_t^* - r_{ft}^* \quad = \quad -\phi^*(y_t^* - y_{ft}^*). \tag{8}$$

The intuition here is that a temporary expansion of output and income resulting from an increase in employment will flow to transitory savings so that agents can smooth their consumption through time. This increased demand for future relative to current goods will cause the real interest rate to be bid down. Keep in mind that the domestic economy is very small in relation to the rest of the world so that changes in domestic employment and income and domestic consumption smoothing have no effect on the rest-of-world interest rate.

Equations (1) through (8) solve for the eight variables y_t, y_t^*, p_t, p_t^*, r_t, r_t^*, q_t and π_t, given the stocks of and demands for nominal money holdings in the two countries, the full-employment levels of output, interest rates and the real exchange rate, and the expected levels of prices in periods t and $t+1$ as viewed from period $t-1$. Of the eight equations, the Lucas supply curves, equations of monetary equilibrium, the equality of domestic and foreign interest rates and the definition of the real exchange rate are standard. The latter two equations, however, appear unconventional. One steeped in the Keynesian tradition might be looking for the standard *IS* goods market equilibrium equations. We could follow this tradition more closely by expressing the goods market equilibrium conditions in the two countries as

$$y_t^* - y_{ft}^* \quad = \quad -\frac{1}{\phi^*}\,(r_t^* - r_{ft}^*). \tag{9}$$

and

$$y_t - y_{ft} \quad = \quad -\frac{1}{\phi}\,(r_t - r_{ft}) - \vartheta\,(q_t - q_{ft}) + \xi\,(y_t^* - y_{ft}^*) \tag{10}$$

where $-1/\phi\,(<0\,)$ and $-1/\phi^*\,(<0\,)$ are the interest semi-elasticities of response of output to real interest rates along the domestic and foreign "*IS*" curves, $-\vartheta\,(<0\,)$ is the elasticity of the horizontal shift of the domestic "*IS*" curve in response to changes in the real exchange rate, and $\xi\,(>0\,)$ is the elasticity of the horizontal shift of the domestic "*IS*" curve in response to an increase in foreign output. We put quotation marks around *IS* here because the above equations do not define traditional *IS* curves. Here, interest rates and outputs enter the equation as deviations from their full-employment levels. This is necessary to ensure that the "*IS*" curves are defined holding technology constant. Technological change in typical textbook *IS–LM* models, with r on the vertical axis and y on the horizontal one, will shift both the *IS* curve and the full-employment output level and may result in either a rise or a fall in the full-employment real interest rate. The formulation here separates the effects of output movements into those arising from changes in full-employment output and those arising from changes in the deviation of output from its full-employment level.

First note that (9) and (8) are the same equation. Then substitute (5) into (10), letting ρ_t and $(E_{t-1}q_{t+1} - E_{t-1}q_t)$ equal zero for expositional convenience, to eliminate r_t and r_{ft} and then (8) into (10) to eliminate, $r_t^* - r_{ft}^*$. This yields

$$y_t - y_{ft} \quad = \quad \left(\xi + \frac{\phi^*}{\phi}\right)(y_t^* - y_{ft}^*) - \vartheta\,(q_t - q_{ft}) \tag{11}$$

When a change in the world real interest rate relative to its full-employment level has proportionally the same effects on domestic and foreign outputs relative to their full-employment levels (11) reduces to (7) with $\sigma = 1/\vartheta$. Expansion in the domestic economy has a trivial effect on foreign output because of the relative sizes of the two economies. But expansion in the foreign economy leads to a significant increase in domestic exports and output. Imagine the foreign economy as an aggregate of small economies identical to the domestic one, each with a direct output response to world interest rate changes equal to $1/\phi$. When all the little economies comprising the rest-of world aggregate expand in response to a change in the world real interest rate, each has a trivial trade-balance effect on the demand for the others' outputs. When we aggregate, however, these trivial effects add up to a proportional increase in rest-of-world output relative to its full-employment level larger than

$$\frac{1}{\phi}\,\Delta(r_t^* - r_{ft}^*).$$

Under the assumption that each little component of the foreign economy is identical to the domestic economy, the increase in the logarithm of aggregate rest-of world output will equal

$$\Delta(y_t^* - y_{ft}^*) = \frac{1}{\phi^*}\,\Delta(r_t^* - r_{ft}^*) = \frac{1}{\phi}\,\Delta(r_t^* - r_{ft}^*) + \xi\,\Delta(y_t^* - y_{ft}^*)$$

which implies that

$$\frac{1}{\phi^*}\,\Delta(r_t^* - r_{ft}^*) = \frac{1}{\phi}\,\Delta(r_t^* - r_{ft}^*) + \frac{\xi}{\phi^*}\,\Delta(r_t^* - r_{ft}^*)$$

$$1 = \frac{\phi^*}{\phi} + \xi.$$

This permits us to write (11) as

$$y_t - y_{ft} \;\;=\;\; -\vartheta\,(q_t - q_{ft}) \,+\, (y_t^* - y_{ft}^*), \tag{12}$$

which is identical to (7) with $\vartheta = 1/\sigma$. So we can obtain the equilibrium levels of the variables using either equations (1) through (8) or equations (1) through (6) plus (9) and (12).

3 Solution for the Comparative Statics Effects of Exogenous Real and Monetary Shocks[15]

First we consolidate the eight equations above into a four-equation system. Substituting the Lucas supply curves (1) and (2) into the respective demand for money functions (3) and (4) and utilizing the equality of domestic and rest-of-world interest rates (5) we obtain

$$\begin{aligned}
p_t &= m_t - \psi_t + \eta\, r_t^* + \eta\rho_t - \eta\,(E_{t-1}q_{t+1} - E_{t-1}q_t) \\
&\qquad + \eta\,(E_{t-1}p_{t+1} - E_{t-1}p_t) - \epsilon\, y_{ft} \\
&\qquad\qquad - \epsilon\,\theta\,\gamma\,(p_t - E_{t-1}p_t) - \epsilon\,\lambda\,(y_{t-1} - y_{f(t-1)}) \qquad (13) \\
p_t^* &= m_t^* - \psi_t^* + \eta^*\, r_t^* + \eta^*\,(E_{t-1}p_{t+1}^* - E_{t-1}p_t^*) - \epsilon^* y_{ft}^* \\
&\qquad - \epsilon^*\,\theta^*\gamma^*(p_t^* - E_{t-1}p_t^*) - \epsilon^*\,\lambda^*\,(y_{t-1}^* - y_{f(t-1)}^*). \qquad (14)
\end{aligned}$$

Rearranging the definition of the real exchange rate (6) and substituting in (7) (or (12)), (1) and (2) we obtain

$$\begin{aligned}
\pi_t &= p_t - p_t^* - q_{ft} + \sigma\,[\theta\,\gamma\,(p_t - E_{t-1}p_t) - \theta^*\gamma^*(p_t^* - E_{t-1}p_t^*)] \\
&\qquad + \sigma\,[\lambda(y_{t-1} - y_{f(t-1)}) - \lambda^*(y_{t-1}^* - y_{f(t-1)}^*)]. \qquad (15)
\end{aligned}$$

Finally, substitution of the big country's Lucas supply curve (2) into its "*IS*" curve (8) yields

$$r_t^* = r_{ft}^* - \phi^*\theta^*\gamma^*(p_t^* - E_{t-1}p_t^*) - \phi^*\lambda^*(y_{t-1}^* - y_{f(t-1)}^*). \qquad (16)$$

A system of four equations determining the expected levels of p_t, p_t^*, π_t, and r_t^* can now be obtained by passing the expectations operator through equations (13) through (16). Here, the expected risk premium on domestic assets next period is equal to the actual risk premium in the current period.

$$\begin{aligned}
E_{t-1}p_t &= E_{t-1}m_t - E_{t-1}\psi_t + \eta\, E_{t-1}r_t^* + \eta\rho_{t-1} - \eta\,(E_{t-1}q_{t+1} - E_{t-1}q_t) \\
&\qquad + \eta\,(E_{t-1}p_{t+1} - E_{t-1}p_t) - \epsilon\, E_{t-1}y_{ft} - \epsilon\,\lambda\,(y_{t-1} - y_{f(t-1)}) \qquad (17) \\
E_{t-1}p_t^* &= E_{t-1}m_t^* - E_{t-1}\psi_t^* + \eta^*\, E_{t-1}r_t^* + \eta^*\,(E_{t-1}p_{t+1}^* - E_{t-1}p_t^*) \\
&\qquad - \epsilon^* E_{t-1}y_{ft}^* - \epsilon^*\,\lambda^*\,(y_{t-1}^* - y_{f(t-1)}^*) \qquad (18) \\
E_{t-1}\pi_t &= E_{t-1}p_t - E_{t-1}p_t^* - E_{t-1}q_{ft} \\
&\qquad + \sigma\,[\lambda(y_{t-1} - y_{f(t-1)}) - \lambda^*(y_{t-1}^* - y_{f(t-1)}^*)] \qquad (19) \\
E_{t-1}r_t^* &= E_{t-1}r_{ft}^* - \phi^*\lambda^*(y_{t-1}^* - y_{f(t-1)}^*). \qquad (20)
\end{aligned}$$

[15] Readers can skip this section and proceed directly to the diagrammatic analysis of section 4 but they should return and work through this section before beginning section 5.

To simplify notation let us define the deviation of the variables from their expected levels using the operator D. This yields expressions of the form $Dp_t = p_t - E_{t-1}p_t$ and so forth. For those variables expressed in logarithms, this is equivalent to expressing the deviations as percentages of expected levels. For r_t ρ_t and r^*_t the deviations are expressed in percentage points. Subtracting (17), (18), (19) and (20) from (13), (14), (15) and (16) respectively, we obtain the following equations determining the equilibrium unanticipated shocks to the system:[16]

$$\begin{aligned}
Dp_t &= \frac{1}{1+\epsilon\,\theta\,\gamma}Dm_t - \frac{1}{1+\epsilon\,\theta\,\gamma}D\psi_t + \frac{\eta}{1+\epsilon\,\theta\,\gamma}D\rho_t \\
&\qquad + \frac{\eta}{1+\epsilon\,\theta\,\gamma}Dr^*_t - \frac{\epsilon}{1+\epsilon\,\theta\,\gamma}Dy_{ft} && (21) \\
Dp^*_t &= \frac{1}{1+\epsilon^*\theta^*\gamma^*}Dm^*_t - \frac{1}{1+\epsilon^*\theta^*\gamma^*}D\psi^*_t \\
&\qquad + \frac{\eta^*}{1+\epsilon^*\theta^*\gamma^*}Dr^*_t - \frac{\epsilon^*}{1+\epsilon^*\theta^*\gamma^*}Dy^*_{ft} && (22) \\
D\pi_t &= -\,Dq_{ft} + (1+\sigma\,\theta\,\gamma)Dp_t - (1+\sigma\,\theta^*\gamma^*)Dp^*_t && (23) \\
Dr^*_t &= -\,\phi^*\theta^*\gamma^*Dp^*_t + Dr^*_{ft} && (24)
\end{aligned}$$

The interpretation of these equations is straight forward. In (21) and (22) unanticipated increases in the nominal money stock or decreases in the demand for money unexpectedly raise the country's price level. An unanticipated rise in the world interest rate reduces the demand for money, unexpectedly raising both price levels. A rise in the risk premium on domestic assets increases the domestic relative to the foreign real interest rate, reducing the demand for domestic money holdings and unexpectedly increasing the domestic price level. Unanticipated positive shocks to full-employment output increase the demand for money, unexpectedly lowering the country's price level. Unanticipated shocks to the full-employment real exchange rate are transmitted directly to the nominal exchange rate in (23) and shocks to the domestic and foreign price levels have two effects—they affect the nominal exchange rate directly at each level of the real exchange rate and they create deviations of the real exchange rate from its full-employment level by changing the relative domestic and foreign output levels at given technology

[16]The terms $(1+\epsilon\,\theta\,\gamma)$ and $(1+\epsilon^*\theta^*\gamma^*)$ in the denominators of the first two equations result from the fact that Dp_t and Dp^*_t appear on the right as well as left sides of the equations obtained by subtracting (17) from (13) and (18) from (14). A rise in the current price level is thus moderated by its positive effect on output and employment and the demand for money.

and tastes. In (24), unanticipated shocks to the rest-of-the-world real interest rate arise either from a technology shock to the full-employment real interest rate or from unanticipated shocks to the price level and employment.

Note that the only rest-of-world variable that appears as a determinant of the unanticipated shock to the domestic price level in (21) is the real interest rate Dr^*_t. This demonstrates the well-known result that a flexible exchange rate insulates the domestic economy from all foreign shocks except shocks to the world interest rate. Fixing the exchange rate by setting $D\pi_t = 0$ in equation (23) destroys that insulation by imposing a rigid relationship between the unanticipated shocks to the domestic and foreign price levels.

We can now proceed to derive the unanticipated shocks to the domestic and foreign price levels that will occur under fixed as compared to flexible exchange rates.

3.1 Rest-of-World Equilibrium

Since the domestic economy is small in relation to the rest of the world changes in domestic variables have but a trivial effect on rest-of-world variables. Accordingly, equilibrium in the rest of the world is independent of whether the domestic authorities adopt a fixed or flexible exchange rate.

The equilibrium unanticipated shock to the rest-of-world real interest rate is obtained by substituting (22) into (24) and collecting the terms.

$$\begin{aligned} Dr^*_t \;\; = \;\; & -\frac{\phi^*\theta^*\gamma^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,[Dm^*_t - D\psi^*_t] + \frac{\epsilon^*\phi^*\theta^*\gamma^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,Dy^*_{ft} \\ & + \frac{1+\epsilon^*\theta^*\gamma^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,Dr^*_{ft}. \end{aligned} \tag{25}$$

And the equilibrium unanticipated shock to the rest-of-world price level is obtained by substituting (24) into (22) and collecting the terms.

$$\begin{aligned} Dp^*_t \;\; = \;\; & \frac{1}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,[Dm^*_t - D\psi^*_t] - \frac{\epsilon^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,Dy^*_{ft} \\ & + \frac{\eta^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,Dr^*_{ft}. \end{aligned} \tag{26}$$

The world interest rate is negatively affected by a positive shock to the supply of money relative to the demand for it and positively affected by positive shocks to world full-employment output and the full-employment world real interest rate. The rest-of-world price level is positively affected by unanticipated excess money supply shocks and unanticipated shocks to the full-employment real interest rate and negatively affected by unanticipated shocks to full-employment output. The shock to rest-of-world output and employment can be obtained by substituting (26) into (2).

$$
\begin{aligned}
Dy_t^* \;=\; & \frac{\theta^*\gamma^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,[Dm_t^* - D\psi_t^*] - \frac{\theta^*\gamma^*\epsilon^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,Dy_{ft}^* \\
& + \frac{\theta^*\gamma^*\eta^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\,Dr_{ft}^* + \lambda^*\,Dy_{t-1}^*. \qquad (27)
\end{aligned}
$$

Shocks to output and employment are directly related to the unanticipated price level shocks—to eliminate fluctuations in employment the authorities need only eliminate the unanticipated shocks to the price-level.

The big-country authorities are interested not only in the unanticipated shocks to the price level and output and employment, but in movements in the expected price level as well. If the expected price level is rising rapidly through time there is substantial on-going inflation. Even though that inflation is fully anticipated, it is generally undesirable because it causes the public to hold an inefficiently small stock of real money holdings and imposes costs of having to more frequently adjust prices, taxes and other nominal contracts to compensate for changes in the value of the currency.[17] The equilibrium expected price level in the rest of the world can be obtained by substituting (20) into (18). This yields

$$
\begin{aligned}
E_{t-1}p_t^* \;=\; & E_{t-1}m_t^* - E_{t-1}\psi_t^* + \eta^*\,E_{t-1}r_{ft}^* + \eta^*\,(E_{t-1}p_{t+1}^* - E_{t-1}p_t^*) \\
& - \epsilon^* E_{t-1}y_{ft}^* - (\eta^*\,\phi^* + \epsilon^*)\,\lambda^*\,(y_{t-1}^* - y_{f(t-1)}^*) \qquad (28)
\end{aligned}
$$

To calculate the expected rate of inflation in the rest of the world, we advance (28) one period to yield

$$
\begin{aligned}
E_{t-1}p_{t+1}^* \;=\; & E_{t-1}m_{t+1}^* - E_{t-1}\psi_{t+1}^* + \eta^*\,E_{t-1}r_{f(t+1)}^* \\
& + \eta^*\,(E_{t-1}p_{t+2}^* - E_{t-1}p_{t+1}^*) - \epsilon^* E_{t-1}y_{f(t+1)}^* \\
& - (\eta^*\,\phi^* + \epsilon^*)\,\lambda^*\,(y_t^* - y_{f(t)}^*) \qquad (29)
\end{aligned}
$$

and then subtract (28) from it. This yields

$$
\begin{aligned}
E_{t-1}p_{t+1}^* - E_{t-1}p_t^* \;=\; & [E_{t-1}m_{t+1}^* - E_{t-1}m_t^*] - [E_{t-1}\psi_{t+1}^* - E_{t-1}\psi_{t+1}^*] \\
& + \eta^*\,[E_{t-1}r_{f(t+1)}^* - E_{t-1}r_{f(t)}^*] - \epsilon^*[E_{t-1}y_{f(t+1)}^* - E_{t-1}y_{f(t)}^*] \\
& + \eta^*\,[(E_{t-1}p_{t+2}^* - E_{t-1}p_{t+1}^*) - (E_{t-1}p_{t+1}^* - E_{t-1}p_t^*)] \\
& - (\eta^*\,\phi^* + \epsilon^*)\,\lambda^*\,[(y_{t+1}^* - y_{f(t+1)}^*) - (y_t^* - y_{f(t)}^*)]. \qquad (30)
\end{aligned}
$$

[17] An exception would arise when the tax system is so primitive that a tax on money is the most efficient way for the government to obtain revenue.

In the special case where the expected future inflation rate is constant, actual future output equals its full-employment level and the full-employment real interest rate does not change through time, this reduces to

$$\begin{aligned} E_{t-1}p^*_{t+1} \; - \; E_{t-1}p^*_t \;\; &= \;\; [E_{t-1}m^*_{t+1} \; - \; E_{t-1}m^*_t] \; - \; [E_{t-1}\psi^*_{t+1} \; - \; E_{t-1}\psi^*_{t+1}] \\ & \qquad - \epsilon^*[E_{t-1}y^*_{f(t+1)} \; - \; E_{t-1}y^*_{f(t)}]. \end{aligned} \tag{31}$$

The expected inflation rate in the rest of the world thus depends positively on the expected rate of growth of the rest-of-world nominal money supply, negatively on the expected rate of growth of the rest-of-world demand for money, and negatively on the expected rate of growth of full-employment output in the rest of the world.

Turning now to domestic equilibrium, which depends on whether the country is on a fixed or a flexible exchange rate, we begin with the flexible exchange rate case.

3.2 Domestic Equilibrium under Flexible Exchange Rates

The unanticipated shock to the domestic price level is obtained by substituting the equilibrium shock to the world interest rate (24) into (21), yielding

$$\begin{aligned} Dp_t \;\; &= \;\; \frac{1}{1+\epsilon\,\theta\,\gamma}\,[Dm_t - D\psi_t] \; - \; \frac{\epsilon}{1+\epsilon\,\theta\,\gamma}\,Dy_{ft} \; + \; \frac{\eta}{1+\epsilon\,\theta\,\gamma}\,D\rho_t \\ & \quad - \frac{\eta\,\phi^*\theta^*\gamma^*}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))}\,[Dm^*_t - D\psi^*_t] \\ & \qquad + \frac{\eta\,\epsilon^*\phi^*\theta^*\gamma^*}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))}\,Dy^*_{ft} \\ & \qquad + \frac{\eta\,(1+\epsilon^*\theta^*\gamma^*)}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))}\,Dr^*_{ft}. \end{aligned} \tag{32}$$

Not surprisingly, in the special case where the unanticipated monetary and technology shocks are identical in the domestic and rest-of-world economies— i.e., where $Dm_t - D\psi_t = Dm^*_t - D\psi^*_t$ and $Dy_{ft} = Dy^*_{ft}$—and the economies are structurally identical except for scale with $D\rho_t = 0$, the shocks to the domestic and rest-of-world price levels are also the identical. This can be seen by noting that

$$\begin{aligned} & \frac{1}{1+\epsilon\,\theta\,\gamma} - \frac{\eta\,\phi^*\theta^*\gamma^*}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))} \\ = \;\; & \frac{1}{1+\epsilon\,\theta\,\gamma}\left[1 - \frac{\eta\,\phi^*\theta^*\gamma^*}{1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*)}\right] \end{aligned}$$

$$= \frac{1}{1+\epsilon\,\theta\,\gamma}\left[1 - \frac{\theta^*\gamma^*(\epsilon^* + \eta^*\phi^* + \eta\,\phi^*)}{1+\theta^*\gamma^*(\epsilon^* + \eta^*\phi^*)}\right].$$

When all starred coefficients are equal to their unstarred counterparts, this reduces to

$$\frac{1}{1+\theta^*\gamma^*(\epsilon^* + \eta^*\phi^*)},$$

which is the coefficient of $[Dm_t^* - D\phi_t^*]$ in (26) and ϵ^* times the coefficient of Dy_{ft}^* in (26). The identical effects of identical shocks can also be seen intuitively from the set of equations (21) through (24). The shock to the world interest rate given by (24) is the same for both countries. It is therefore obvious that if the unanticipated excess money supply and full-employment real income shocks are the same in the domestic economy as in the rest of the world and the parameters are the same at home and abroad the unanticipated price level shocks will also be the same. When this is true (23) reduces to

$$D\pi_t = -Dq_{ft}.$$

The unanticipated shocks to the nominal exchange rate will entirely reflect the shocks to the full-employment real exchange rate. The equilibrium shocks to domestic output and employment will be

$$\begin{aligned} Dy_t \quad = \quad & \frac{\theta\,\gamma}{1+\epsilon\,\theta\,\gamma}\,[Dm_t - D\psi_t] \,-\, \frac{\theta\,\gamma\,\epsilon}{1+\epsilon\,\theta\,\gamma}\,Dy_{ft} \,+\, \frac{\theta\,\gamma\,\eta}{1+\epsilon\,\theta\,\gamma}\,D\rho_t \\ & - \frac{\theta\,\gamma\,\eta\,\phi^*\theta^*\gamma^*}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))}\,[Dm_t^* - D\psi_t^*] \\ & + \frac{\theta\,\gamma\,\eta\,\epsilon^*\phi^*\theta^*\gamma^*}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))}\,Dy_{ft}^* \\ & + \frac{\theta\,\gamma\,\eta\,(1+\epsilon^*\theta^*\gamma^*)}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*+\eta^*\phi^*))}\,Dr_{ft}^* + \lambda Dy_{ft}. \qquad (33) \end{aligned}$$

The equilibrium expected level of domestic prices can be obtained by substituting (20) into (17). This yields

$$\begin{aligned} E_{t-1}p_t \quad = \quad & E_{t-1}m_t \,-\, E_{t-1}\psi_t \,+\, \eta\,E_{t-1}r_{ft}^* \,+\, \eta\,(E_{t-1}p_{t+1} - E_{t-1}p_t) \\ & + \eta\,\rho_{t-1} \,-\, \epsilon E_{t-1}y_{ft} \,-\, (\eta\,\phi \,+\, \epsilon)\,\lambda\,(y_{t-1} - y_{f(t-1)}). \qquad (34) \end{aligned}$$

If the expected future domestic inflation rate is constant, actual future output equals its full-employment level and the world full-employment real interest rate and the risk premium on domestic assets are constant through time, the expression for the expected rate of domestic inflation becomes

$$\begin{aligned} E_{t-1}p_{t+1} \,-\, E_{t-1}p_t \;&=\; [E_{t-1}m_{t+1} \,-\, E_{t-1}m_t] \,-\, [E_{t-1}\psi_{t+1} \,-\, E_{t-1}\psi_{t+1}] \\ &\quad - \epsilon\,[E_{t-1}y_{f(t+1)} \,-\, E_{t-1}y_{f(t)}]. \end{aligned} \tag{35}$$

The expected rate of domestic inflation then depends positively on the expected rate of growth of domestic nominal money supply, negatively on the expected rate of growth of the domestic demand for money, and negatively on the expected rate of growth of domestic full-employment output. It is completely independent of the expected rate of inflation in the rest of the world.

If we ignore deviations of output from full employment over the long run, the expected level of the nominal exchange rate becomes, from (19),

$$E_{t-1}\pi_t \;=\; E_{t-1}p_t \,-\, E_{t-1}p^*_t \,-\, E_{t-1}q_{ft} \tag{36}$$

Subtracting (36) from itself advanced one period yields the expected rate of change in the nominal exchange rate, which equals the excess of the domestic over rest-of-world inflation rates plus the expected rate of devaluation of the full-employment real exchange rate. That is,

$$\begin{aligned} E_{t-1}\pi_{t+1} - E_{t-1}\pi_t \;&=\; [E_{t-1}p_{t+1} - E_{t-1}p_t] \\ &\quad - [E_{t-1}p^*_{t+1} - E_{t-1}p^*_t] - [E_{t-1}q_{f(t+1)} - E_{t-1}q_{ft}]. \end{aligned} \tag{37}$$

3.3 Domestic Equilibrium under Fixed Exchange Rates

Under flexible exchange rates, the set of equations (21) through (24) solved for $Dp_t, Dp^*_t, D\pi_t$ and Dr^*_t. Under a fixed exchange rate, $D\pi_t = 0$ and the domestic nominal money stock Dm_t becomes endogenous—the domestic authorities are forced to adjust the money supply through non-sterilized foreign exchange market intervention to maintain the exchange rate at its pegged level. So the system now solves for Dp_t, Dp^*_t, Dm_t and Dr^*_t.

Fixing the exchange rate in equation (23) sets up a dependency of the domestic price level on the price level abroad and the real exchange rate:

$$Dp_t \;=\; \frac{1+\sigma^*\theta^*\gamma^*}{1+\sigma\,\theta\,\gamma}\,Dp^*_t + \frac{1}{1+\sigma\,\theta\,\gamma}\,Dq_{ft} \tag{38}$$

When the parameters are the same in the domestic and rest-of-world economies this reduces to

$$Dp_t \;=\; Dp^*_t \,+\, \frac{1}{1+\sigma\,\theta\,\gamma}\,Dq_{ft}. \tag{39}$$

If there are no shocks to the real exchange rate, the domestic price level will move in unison with the foreign price level and the unanticipated shock to the domestic price level will be given by (26). The shock to domestic output and employment becomes

$$\begin{aligned} Dy_t &= \theta\,\gamma\,Dp_t^* + \frac{\theta\,\gamma}{1+\sigma\,\theta\,\gamma}\,Dq_{ft} \\ &= Dy_t^* + \frac{\theta\,\gamma}{1+\sigma\,\theta\,\gamma}\,Dq_{ft}. \end{aligned} \tag{40}$$

Domestic unemployment will also move in unison with unemployment abroad.

The expected domestic price level is obtained from (19). When we ignore deviations of output from full employment in the long run, it becomes

$$E_{t-1}p_t = E_{t-1}p_t^* + E_{t-1}q_{ft}. \tag{41}$$

The expected rate of domestic inflation will equal the expected rate of foreign inflation minus the expected rate of change in the real exchange rate,

$$\begin{aligned} [E_{t-1}p_{t+1} - E_{t-1}p_t] &= [E_{t-1}p_{t+1}^* - E_{t-1}p_t^*] \\ &\quad + [E_{t-1}q_{f(t+1)} - E_{t-1}q_{ft}]. \end{aligned} \tag{42}$$

4 A Diagrammatic Exposition: Symmetric and Asymmetric Shocks

A better intuitive grasp of the model and a deeper understanding of its implications can be obtained through the use of some simple diagrams. The left panels of Figures 1, 2 and 3 present the equilibrium of the small country and the right panels the equilibrium of the rest of the world. Real outputs in logarithms are on the horizontal axes and the real interest rate in percent per annum is on the vertical axes. The scales of the horizontal axes are set so that a given percentage increment to domestic and rest-of-world output represents the same distance along the respective axes. The curves gg and g^*g^* give the combinations of outputs and the real interest rate for which the demands for aggregate output equals the quantities produced—that is, for which the goods markets are in equilibrium. Movements along and shifts of these curves are defined in equations (7) and (8) (or (12)). The slopes of both gg and g^*g^* equals ϕ^* in equation (8). A devaluation of the domestic currency in real terms will shift gg to the right, as will an exogenous fall in domestic savings, increase in investment in the domestic economy or increase in desired exports relative to desired imports. The curve shifts to the right because the equilibrium level of output associated with each level of the real interest rate increases. The domestic economy is too small in relation to the rest of the world for changes in the real exchange rate or domestic exports and imports to have an effect on g^*g^*—the latter is affected only by rest-of-world savings and investment decisions. World asset equilibrium is given by the curves mm and m^*m^* in combination with the relationship between the domestic and foreign real interest rates given by equation (5). The mm and m^*m^* curves, which represent equations (3) and (4), give the combinations of real outputs and the real interest rate for which the demands for money balances in the respective countries equal the existing supplies. An increase in the supply of money or fall in the demand for money shifts mm or m^*m^*, as the case may be, to the right—at any given real interest rate, output will have to increase to get the public to hold the existing money stock. The horizontal lines running across the figures at r^*_f give the full-employment real interest rates and the vertical lines at y_f and y^*_f give the full-employment output levels in the two economies.

As will be seen below the shocks to the two economies can be regarded as real if there is a change in the full-employment world interest rate or the domestic full-employment real exchange rate and monetary if the full-employment levels of these variables remain unchanged. Real shocks can be

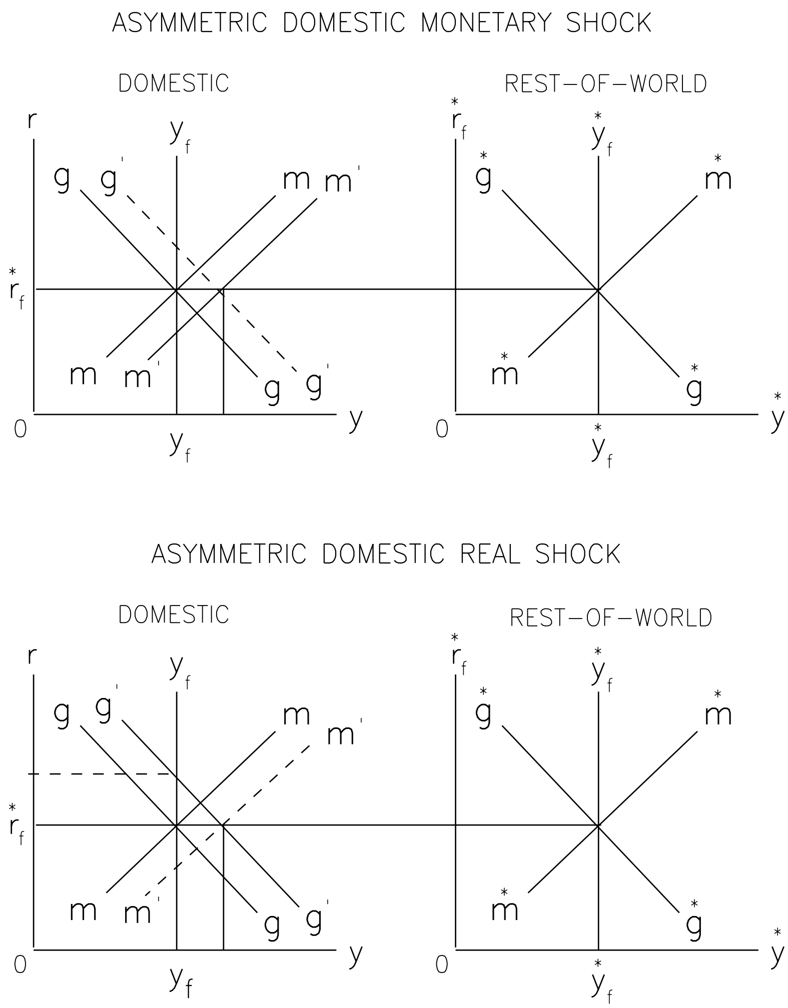


Figure 1: Asymmetric domestic real and monetary shocks.

regarded as symmetric if the domestic full-employment real exchange rate remains unchanged and asymmetric if it does not. Monetary shocks are viewed as symmetric or asymmetric according to whether the the nominal exchange rate would or would not be affected under a flexible exchange rate regime.

4.1 Asymmetric Domestic Monetary Shocks

An unanticipated expansion of the domestic money supply or contraction of the domestic demand for money shifts the mm curve to the right in the top left panel of Figure 1. Domestic residents will try to reestablish portfolio equilibrium by purchasing assets from abroad with their excess money holdings. If the domestic authorities otherwise hold the money supply constant and allow the exchange rate to float, this will create an excess supply of domestic currency on the foreign exchange market causing the domestic currency to devalue and the real exchange rate to fall. This will shift the gg curve to $g'g'$ and cause domestic output to rise above its full-employment level. In the long-run domestic wages and prices will rise, reducing real money holdings to the point where the mm curve will return to its original intersection with $y_f y_f$. At this point the price level will have risen, and the domestic currency will have devalued, in proportion to the initial excess supply of money. The real exchange rate, along with the gg curve, will have returned to its initial position.

When the authorities maintain the nominal exchange rate constant at a fixed level, the domestic portfolio response to the excess supply of money will lead to a one-shot balance of payments deficit and a loss of foreign exchange reserves. The sale of official reserves for domestic currency reduces the domestic money supply to equal the demand for money—the mm curve thus shifts back to (or more correctly, remains in) its original position and the monetary shock has no effect on either output and employment or prices. Under a fixed exchange rate the domestic money supply is endogenous. A domestic monetary shock in the face of unchanged real and monetary conditions abroad—i.e., a domestic asymmetric monetary shock—is rendered harmless as far as output, employment, and prices are concerned by fixing the nominal exchange rate. The domestic authorities cannot conduct monetary policy when they choose a fixed exchange rate regime—the only useful function of monetary actions is to maintain the stock of foreign exchange reserves at an appropriate level. Under a fixed exchange rate regime, the effects of asymmetric domestic monetary shocks on the domestic economy are automatically neutralized.

4.2 Asymmetric Domestic Real Shocks

The bottom two panels of Figure 1 show the effect of an asymmetric domestic real shock—that is a shock to the gg curve under conditions where the corresponding curve in the rest of the world remains unchanged. The rightward shift of gg to $g'g'$ is equivalent to a rise in the full-employment real exchange rate. If the nominal money supply is held constant, the upward pressure on domestic output will increase domestic residents' demand for money, causing them to reestablish portfolio equilibrium by selling assets abroad. This will lead to an appreciation of the domestic currency and a rise in the real exchange rate, shifting gg back to the left to maintain its intersection with the world interest rate line at the full-employment level of output. The effects on the domestic economy of asymmetric real shocks are automatically neutralized by a flexible exchange rate when the domestic authorities hold the quantity of money constant.

If the authorities maintain a fixed nominal exchange rate, the sale of assets abroad will be successful and the authorities will supply the necessary increase in the domestic money supply, thereby shifting mm to $m'm'$. Output and employment will increase in the short-run. In the long-run when wages and prices fully adjust, the resulting rise in the price level will increase the real exchange rate, shifting gg back to its original position, and also reduce real money holdings, shifting mm back to its original position.

Ultimately, the real exchange rate rises to its new full-employment level—either through an increase in the nominal exchange rate holding prices constant, or through an increase in the price level with the nominal exchange rate held constant. By holding the nominal money supply constant and letting the exchange rate float the authorities can neutralize the effects on output, employment and prices of asymmetric real shocks.

Note that a shock to the full-employment level of domestic output (leaving full-employment output in the rest of the world constant) is an asymmetric domestic monetary, not real, shock. As long as the full-employment real exchange rate remains constant, a full-employment output shock will have the same effect as a shock to the supply or demand for money. In this event everything in the top left panel of Figure 1 except the mm curve will shift—a decline in y_f, for example, will be accompanied by equivalent leftward shifts of gg and $y_f y_f$ with mm remaining unchanged. The gg curve shifts to the left along with $y_f y_f$ because the equilibrium full-employment real exchange rate is assumed to be unaffected. Since the money supply and the demand function for money remain unaffected, the level of output (and income) sufficient to make domestic residents willing to hold that money supply will not change, while the full-employment level of output will fall. The analysis proceeds in the same way as the analysis of a rightward shift of mm—the fall in full-employment output reduces the desired quantity of money relative to the existing supply.

4.3 Asymmetric Rest-of-World Monetary Shocks

Now suppose that there is a shock to the demand or supply of money in the rest of the world unaccompanied by any change in the domestic demand and supply of money—an asymmetric rest-of-world monetary shock. The effects are analyzed in the top panels of Figure 2. A positive monetary shock in the rest of the world shifts m^*m^* to the right. Rest-of-world residents bid down the world interest rate until the resulting lower cost of holding money makes them willing to hold the excess quantity. This shifts the horizontal real interest rate line downward. The lower world interest rate will increase the profitability of domestic investment creating upward pressure on domestic output and prices. This will increase the quantity of money demanded causing domestic residents to sell assets abroad to maintain portfolio equilibrium. When the exchange rate is fixed the domestic authorities will have to increase official foreign exchange reserve holdings to maintain the desired level of the exchange rate and in the process the domestic money stock will increase shifting mm endogenously to the right to $m'm'$. If the domestic and rest-of-world economies are identical in structure, differing only in scale, domestic output will rise in the same proportion as output abroad, just as if both governments had exogenously expanded their money supplies in the same proportion.

Alternatively, suppose that the domestic authorities react to the money shock abroad by maintaining the domestic money supply constant and letting the exchange rate float. The fall in the world interest rate will create an increase in the domestic demand for money causing domestic residents to try to sell assets abroad to reestablish portfolio equilibrium. This will create an excess demand for the domestic currency on the foreign exchange market resulting in both a nominal and real appreciation of the domestic currency, shifting the gg curve to the left until it intersects mm at the new world interest rate. Domestic output and employment contract—the expansion abroad causes a contraction at home.

In the long run wages and prices in the rest of the world will rise to drive m^*m^* back to its original position, thereby shifting the r^* line back up to its original position. In the case where the domestic authorities have been maintaining the domestic money supply constant, domestic residents'

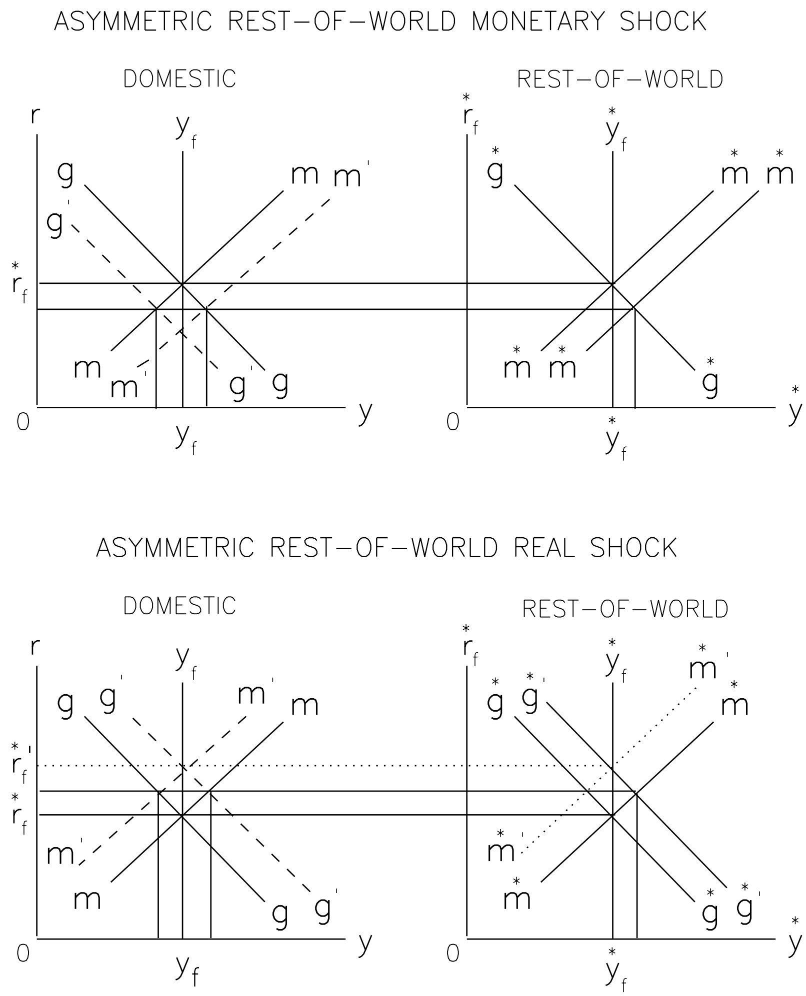


Figure 2: Asymmetric rest-of-world real and monetary shocks.

demand for money will shift back to its original level and they will attempt to purchase assets abroad, putting downward pressure on the nominal and real exchange rates. The real exchange rate will depreciate back to its full-employment level. The nominal exchange rate will not return to its original level, however, because the rise in the rest-of-world price level, at a given level of domestic prices, will require an appreciated nominal exchange rate if the real exchange rate is to return to its initial pre-shock level. So ultimately the domestic currency will appreciate by an amount equivalent to the decline in the real value of foreign currency and the domestic price level (and the real value of the domestic currency) will remain at its original level.

When the domestic authorities maintain the nominal exchange rate constant the long-run effect will be an increase in the domestic price level in the same proportion as the increase in the price level abroad. The rise in output in the short-run will put upward pressure on domestic wages and prices. As the nominal exchange rate is kept fixed, the domestic price level increase will be proportional to the price level increase in the rest of the world (when the economies are identical except for scale) and the real exchange rate, which had been unaffected by the foreign shock in the short run) will remain unchanged in the long-run. The domestic mm curve will return to its original position as the domestic price level rises. As in the short-run, the result will be the same as would have occurred had the domestic authorities exogenously increased their money supply to match the increase in the money supply abroad.

So an asymmetric rest-of-world monetary shock will have equivalent effects on the domestic and foreign price and employment levels in the short and long runs when the domestic authorities fix the exchange rate (given that the economies are identical except for scale). It will have a short-run effect on domestic output and employment in the opposite direction to the output and employment effect in the rest of the world when the domestic authorities hold the money stock constant and let the exchange rate float. The long-run result in this latter case will be an unchanged domestic price level accompanied by an nominal appreciation of the domestic currency equal to the rise in the price level in the rest of the world.

4.4 Asymmetric Rest-of-World Real Shocks

Now consider an asymmetric real shock in the rest of the world. This will take the form of an upward shift of g^*g^* with gg remaining unchanged. The result will be an increase in output and employment in the rest of the world in the short-run, followed in the long-run by an increase in wages and in prices sufficient to shift m^*m^* up to $m^{*\prime}m^{*\prime}$. The world interest rate will rise in the short-run and then further in the long-run, ultimately reaching $r_f^{*\prime}$.

If the domestic authorities hold the money supply constant and let the exchange rate float there will be a decline in desired money holdings as the world interest rate rises, causing domestic residents to rebalance their portfolios by purchasing assets abroad. The domestic currency will devalue and the real exchange rate will fall, shifting gg upward to $g'g'$. Output and employment will increase in the same proportion as output and employment in the rest of the world (assuming that the economies are identical except for scale).

When the domestic authorities fix the exchange rate the increase in the world interest rate will lead to a contraction of domestic investment and a reduction in output and employment—the opposite of what is occurring abroad. As domestic income falls, domestic residents will demand less money and rebalance their portfolios by purchasing assets abroad. The domestic authorities will sell sufficient foreign exchange reserves to make this possible, reducing the domestic money supply in the process and shifting mm to the left to $m'm'$.

In the long-run as the rest-of-world's price level fully adjusts, the world interest rate will rise further to $r_f^{*\prime}$. When the exchange rate is flexible and the domestic nominal money supply is constant the domestic price level will rise in response to the excess levels of domestic output and employment. In addition, the further rise in the world interest rate, by reducing the demand for money, will result in a further rise in the domestic price level, ultimately increasing it in proportion to the rise in the price level abroad. Since in the final equilibrium gg must shift upward to cross through the vertical y_f line at its intersection with the world real interest rate line, the equilibrium real exchange rate must be lower in the long run as a result of the asymmetric rest-of-world real shock.

When the domestic residents hold the nominal exchange rate fixed, the rise in the price level in the rest of the world would, barring changes in the real exchange rate, cause a proportional rise in the domestic price level. But there will obviously be downward pressure on the domestic price level as a result of the short-fall of output and employment from full-employment levels in the short-run. The gg curve must shift to the right to cross though $y_f y_f$ at the new world real interest rate line $r_f^{*\prime}$. This requires a decline in the real exchange rate which, when the nominal exchange rate is fixed, can only happen as a result of some combination of a rise in the rest-of-world price level and decline in the domestic price level. While it is clear that the domestic price level must decline relative to the price level in the rest of the world, it is not possible to determine whether domestic prices will fall relative to its level before the foreign real shock occurred. This will depend on the magnitude of σ in equation (7), the elasticity of the horizontal shift of gg in response to a decline in the real exchange rate. Any positive value of this parameter is consistent with the two economies being identical except for scale.

An important thing to notice about asymmetric rest-of-world real shocks, and asymmetric real shocks in general, is that they necessarily involve a change in the full-employment real exchange rate. In the example above, the real exchange rate has to depreciate in response to the shock, either through a nominal depreciation of the domestic currency or through a decline in the domestic price level relative to the price level abroad. The shift of gg to $g'g'$ resulting from the devaluation of the real exchange rate is permanent. Real shocks are asymmetric or symmetric according to whether or not the full-employment real exchange rate changes.

4.5 Symmetric Monetary Shocks

Finally, we turn in Figure 3 to symmetric monetary and real shocks. A symmetric monetary shock occurs in the top panels—the domestic and rest-of-world mm curves shift in the same proportion to $m'm'$ and $m^{*\prime}m^{*\prime}$ respectively. The world interest rate falls and output and employment increase in the same proportion in both economies when they are identical except for scale. The real and nominal exchange rates will remain unaffected regardless of whether the authorities follow a fixed exchange rate regime or one of flexible exchange rates with constancy of the money stock (subsequent to any initiating exogenous change). In the long run the price levels will rise in both economies to shift the two mm curves back to their original positions.

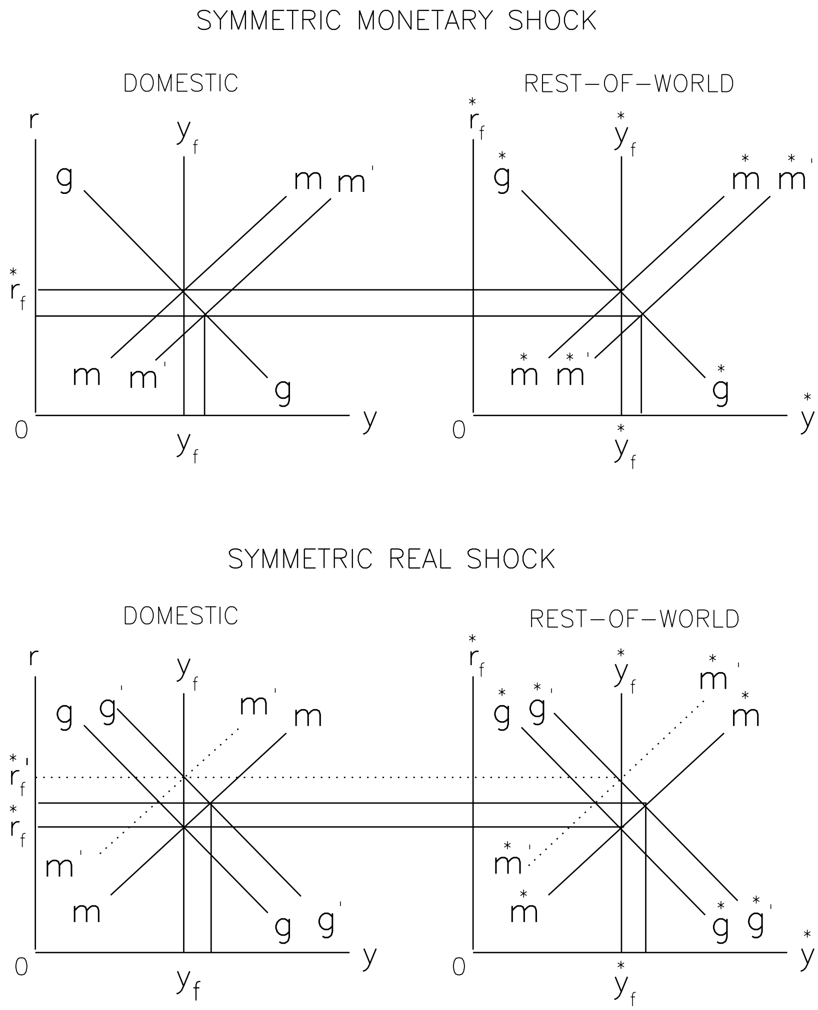


Figure 3: Symmetric real and monetary shocks.

4.6 Symmetric Real Shocks

A symmetric real shock necessarily involves no change in the full-employment real exchange rate. This means that gg and g^*g^* both shift upward in the same proportion so that they cross the respective countries' vertical full-employment output lines at the world real interest rate. In our model this is equivalent to a shock to the world full-employment real interest rate with no change in the full-employment real exchange rate. Symmetric real shocks could occur as a result of a proportional change in the marginal productivity of capital in both countries, an equivalent shift in their residents' respective rates of time preference, or a change in the world human or physical capital stock appropriately distributed between the countries.

As in the case of a symmetric monetary shock, the effects on output and employment in the two economies are the same (assuming that the economies are identical except for scale) whether a fixed or flexible exchange rate regime is adopted. This occurs, of course, because the equilibrium real and nominal exchange rates never change. In the case where the domestic authorities hold the money shock constant and let the exchange rate float the excess demand for goods in the domestic economy increases output and employment to the same degree as output and employment increase abroad. Then in the long run the price levels in the two economies rise at the same rate, driving the output levels back to full-employment at the same rate. When the domestic authorities fix the exchange rate output and employment again expand to the same degree in both economies. This puts equivalent pressure on their price levels and the path to the final equilibrium is the same as it would have been under a flexible exchange rate regime with no change in the money supply.

4.7 Some General Issues

The general conclusion that emerges is that the domestic authorities can neutralize asymmetric domestic monetary shocks and render the effects of asymmetric rest of world monetary shocks symmetric by fixing the exchange rate. And they can neutralize asymmetric domestic real shocks and render the effects of asymmetric foreign real shocks symmetric by holding the domestic money supply constant and letting the exchange rate float.

When the domestic authorities float the exchange rate and hold the money stock constant, positive asymmetric domestic monetary shocks have a positive effect on domestic output and employment while positive rest-of-world asymmetric monetary shocks have negative effects on the domestic economy—domestic output and employment thus expand when there is an excess positive domestic relative to foreign asymmetric shock. When they fix the exchange rate the authorities render the effects of positive asymmetric domestic real shocks on domestic output and employment positive and the domestic effects of positive asymmetric foreign real shocks negative—domestic output and employment expand when there is an excess positive domestic relative to foreign asymmetric real shock.

When the combination of asymmetric domestic and foreign shocks causes the full-employment real exchange rate to rise their net effect on domestic relative to foreign output and employment will be positive under a fixed exchange rate regime—when the full-employment real exchange rate falls the net effect will be negative.

It should be evident that the domestic authorities can only neutralize the effects on the economy of ongoing asymmetric money and real shocks if they can observe the magnitudes of the shocks and know the exact change in the nominal money supply (and, consequently, the exchange rate) that will be required to maintain domestic output at the full-employment level. In the case of known domestic asymmetric monetary shocks that occur alone, this involves offsetting the shock by an appropriate change in the nominal money supply—or, alternatively, pegging the nominal exchange rate at its (necessarily known) equilibrium level. In the case of known asymmetric domestic real stocks that occur alone, it involves letting the exchange rate float and holding the domestic money supply constant. Exact knowledge of the shocks, and of the structure of the domestic economy, is required when domestic asymmetric real and monetary shocks occur at the same time.

In the case of foreign asymmetric shocks the situation is more complicated because, in addition to knowing the nature and magnitudes of the shocks, the authorities generally have to know the structure of both the domestic and foreign economies. In the case of a positive foreign monetary shock, for example, the authorities have to let the currency appreciate and contract the domestic money supply in precise proportions. In the case of a positive foreign real shock they have to reduce the domestic money supply sufficiently to accommodate the decline in the demand for money caused by the rise in the world interest rate and then allow the currency to devalue by the full amount of the decline in the full-employment real exchange rate. This can not be done without knowing the structures of both economies. Knowledge of the structure of the rest-of-world economy is required to establish the effect of the shock on the world interest rate so that the domestic authorities can respond appropriately.

To offset the effects of symmetric shocks the domestic authorities also have to know the magnitudes of the shocks and the underlying structures of the two economies. If the foreign authorities have information about the foreign shocks and the structure of the foreign economy, and successfully offset their effects on the foreign economy, all shocks facing the domestic authorities essentially become domestic asymmetric shocks. This makes the domestic authorities' problem easier to the degree that they now only have to worry about the magnitudes of the domestic shocks. But if the foreign authorities do an imperfect job of offsetting the effects of the shocks to their economy, the domestic authorities then have to know the actions of the foreign authorities together with the structures of, and magnitudes of the shocks to, both economies.

Relaxation the assumption that the domestic economy is small in relation to the rest of the world introduces interactions between the policies implemented in the two economies which both countries' authorities must also understand to get things right. The complexity further increases when there are more than two economies.

5 Dynamics

The evolution of the underlying real and monetary shocks through time and the responses of the domestic and foreign economies these shocks and to policy initiatives will depend critically on dynamics of the model that have yet to be specified. Two issues are involved. The first is the evolution of tastes and technology through time, which determine the stochastic properties of the full-employment levels of income, the real exchange rate, the real interest rate and the demand for money. The second is the dynamics of the response of domestic and rest-of-world output and employment and the real exchange rate to unanticipated real and monetary shocks.

Consider technology and taste shocks first. The full-employment output levels Y_{ft} and Y^*_{ft} are assumed to vary stochastically around logarithmic trend values determined by ongoing technological change. And the logarithm of the full-employment real exchange rate, q_{ft}, is assumed to be a random walk determined by both technology and tastes.[18] Since agents then cannot distinguish the observed real exchange rate from a random walk,

$$E_{t-1}q_{t+1} - E_{t-1}q_t = 0,$$

so shocks to the expected rate of change of the real exchange rate, and any effects they might have on domestic output and employment, are eliminated.

The big country's full-employment riskless real interest rate on securities denominated in the world consumption characteristic, r^*_{ft}, is also determined by technology and is assumed to vary stochastically around a constant mean. The risk premium on domestic assets is assumed to be constant at zero. This assumption will later turn out to be innocuous once the implications of the other stochastic shocks are taken into account. It is also assumed that the domestic and rest-of-world demand functions for money vary stochastically about constant trends. All stochastic shocks, taken as proportions of their trend levels, or of the previous period's level in the case of q_{ft}, have zero means and constant variances. The analysis that follows assumes that neither private agents nor governments observe the current period shocks to the full-employment real exchange rate, output levels, the demand functions for money and real interest rate, although they eventually learn about the

[18] This assumption is a rough but convenient approximation to the true behaviour of the full-employment real exchange rate which is now generally regarded as a stationary but highly persistent process. (See Kenneth Rogoff, "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, Vol.34, (June 1996), pp. 647–668.) The argument put forward here depends only upon the exchange rate having a high degree of persistence and not upon whether or not it is technically a random walk.

variances of the stochastic shocks and the logarithmic trends of the domestic and foreign output levels and demand functions for money.

One feature of the dynamic adjustment to these unobserved stochastic shocks is already contained in our formulation of the Lucas-type aggregate supply curves, (1) and (2), which are repeated here for convenience.

$$y_t = y_{ft} + \theta\,\gamma\,(p_t - E_{t-1}p_t) + \lambda\,(y_{t-1} - y_{f(t-1)}) \tag{1}$$
$$y^*_t = y^*_{ft} + \theta^*\gamma^*\,(p^*_t - E_{t-1}p^*_t) + \lambda^*\,(y^*_{t-1} - y^*_{f(t-1)}) \tag{2}$$

The response of output to unanticipated price shocks is determined by the parameters θ and θ^* which will depend in a Lucas world on the fractions of observed price shocks that agents attribute to movements in the general price level—as these fractions approach unity, θ and θ^* go to zero and as they approach zero the two parameters approach unity. Obviously, these parameters will depend on the information available to agents at each point in time and will therefore not be constant either through time or across regimes. Their magnitudes will also be unknown to the authorities.

The non-constancy of θ and θ^* together with the fact that the authorities do not observe any of the stochastic shocks to the system will make it impossible for central banks to offset these shocks within-period or to determine the proportions of previous period's output changes that represented deviations from the (never accurately observed) full-employment output levels.

Persistence of deviations of output from its full-employment level is incorporated into (1) and (2) by means of non-zero values of λ and λ^*. These parameters will depend on agents' information regarding the permanence of shocks to r^*_t and the response of the economy to past shocks to it. The world real interest rate is not observed directly at any time because it differs from observed nominal rest-of-world interest rates by the unobserved foreign expected inflation rate. Since the authorities do not possess good information about the magnitudes of λ and λ^* they cannot forecast the timing of the future effects on their economies of current monetary policy initiatives. So even when it is known that the economy is in recession, the time path of unanticipated (by private agents) nominal money supply movements that will achieve full-employment without aggravating the next inflationary boom cannot be determined.

The above conditions are sufficient to render attempts at discretionary monetary policy nugatory even if we make the unreasonable assumption that the authorities have precise knowledge of the parameters other than θ, θ^*, λ and λ^*. And there are further lagged responses that should be added before our model can be treated as one that the authorities could use for current analysis and attempts at forecasting. The full adjustment of the real exchange rate to deviations of domestic and foreign output from their full-employment levels in equation (7), reproduced below,

$$q_t - q_{ft} \quad = \quad -\,\sigma\,[(y_t - y_{ft}) - (y_t^* - y_{ft}^*)] \tag{7}$$

is likely to take a number of periods, so the equation should really contain a number of lags of the right-hand-side variable as well as its current level. The same applies to the equation determining the world real interest rate, (8),

$$r_t^* - r_{ft}^* \quad = \quad -\,\phi^*(y_t^* - y_{ft}^*). \tag{8}$$

Given that savers and investors maximize inter-temporally subject to information that gets revised each period, the deviation of the real interest rate from its full-employment level in any period is likely to be a response to the history of deviations of output from full employment so that lagged values of $(y_t^* - y_{ft}^*)$ should really appear in (8).

Our subsequent analysis of the conduct of monetary policy and exchange rate regime choice can proceed most easily using equations (7) as (8) originally defined. The authorities do not have sufficient information to conduct discretionary monetary policy even without more realistic definitions of these equations, and the introduction of lagged right-hand-side variables to these equations will only reenforce that result. Furthermore, given that the dynamic properties of the economic system are unknown to the authorities their best strategy will always be to choose exchange rate regimes and monetary rules to minimize the impact of unknown current-period shocks on the grounds that the subsequent consequences of those shocks will most likely then also be minimized.

6 Conducting Monetary Policy

While the diagrammatic analysis in section 4 provided useful insights, a comparative analysis of alternative policy regimes can best be conducted by working with the underlying mathematical model. Assume that in each country there is an underlying desired or core inflation rate based on the willingness of the public, as revealed in the political process, to use an inflation tax to finance some part of government expenditure. The function of the country's central bank is, then, to minimize deviations of the actual inflation rate from the core inflation rate. There are three dimensions to this minimization problem. First, unanticipated period-to-period deviations of the inflation rate from core, represented by unanticipated shocks to the current-period price level, lead to associated deviations of output and employment from their full-employment levels. The central bank will therefore want to minimize the variance of the unanticipated shocks to output and employment by minimizing the variance of the unanticipated price level shocks. Second, persistent deviations of the inflation rate from core, even when fully anticipated, are costly because they signify the wrong level of monetary finance of government expenditure, imposing unwarranted costs of continually adjusting prices and taxes and other nominal contracts. Third, variations of the long-term inflation rate impose costs because the institutions through which contracting takes place have to constantly adjust to accommodate changes in expected inflation. Examples would be the necessity of adjusting public old-age and disability pensions, fee schedules under medical care schemes, and the various tax rates through which the government distributes the burden of public finance across different income and interest groups.

The domestic and foreign central banks can minimize the costs of unanticipated shocks to prices and employment by minimizing

$$\textit{I:}\quad Var\{Dy_t^*\}\ \ [Var\{Dp_t^*\}] \quad \text{and} \quad Var\{Dy_t\}\ \ [Var\{Dp_t\}].$$

They can minimize the expected deviation of the long-term inflation rate from core by minimizing

$$\textit{II:}\quad E_{t-1}\{(E_{t+k-1}p_{t+k}^* - E_{t-1}p_t^* - k^*\,\alpha^*)\} \quad \text{and}$$
$$E_{t-1}\{(E_{t+k-1}p_{t+k} - E_{t-1}p_t - k\,\alpha\}.$$

where α and α^* are the domestic and foreign core inflation rates and the relevant time horizon is k periods. Finally, they can minimize the costs of variability of their long-term inflation rates by minimizing

III: $Var\{E_{t+k-1}p^*_{t+k} - E_{t-1}p^*_t - k^*\,\alpha^*\}$ and $Var\{E_{t+k-1}p_{t+k} - E_{t-1}p_t - k\,\alpha\}$.

Note the distinction between *II* and *III*. Objective *II* can be satisfied by minimizing the predicted average deviation of the inflation rate from the core rate over the succeeding k periods. To satisfy objective *III*, the central bank must minimize the variance of its error in predicting the k-period inflation rate.

Formally, these objectives can be incorporated into the loss functions

$$L = w_1\,V_{Dy} + w_2\,E_{Tk} + w_3\,V_{Tk} \tag{43}$$

and

$$L^* = w^*_1\,V^*_{Dy^*} + w^*_2\,E^*_{Tk} + w^*_3\,V^*_{Tk} \tag{44}$$

where L and L^* are the losses, V_{Dy} and $V^*_{Dy^*}$ are the variances of the current period shocks to output and employment, E_{Tk} and E^*_{Tk} are the expected absolute differences between the trend rates of inflation and the underlying core inflation rates over a k-period horizon and V_{Tk} and V^*_{Tk} are the variances of the differences of the k-period trend inflation rates from the core rates. The selection of the time horizons k and k^* is arbitrary but it will turn out that the conditions for minimizing L and L^* are the same regardless of the k and k^* selected. The weights w_i and w^*_i sum to unity for each country.

Perfect monetary policies would involve manipulation of m^*_t and m_t to offset within period the effects of all shocks to the full-employment output, interest rate and real exchange rate levels and all shocks to the demand functions for money to maintain time paths of the price levels identical to the paths implied by the countries' core inflation rates. Such policies are impossible to implement, of course, when the current shocks to the full-employment levels of the relevant exogenous variables and the current shocks to the demand for money are unobservable. Moreover, since the authorities do not know the magnitudes of the economies' structural parameters governing dynamic adjustment they cannot forecast the time-path of the response of the price level and output to any corrective monetary action they might take. Attempts to use discretionary policy will be fruitless.

When the big country's monetary authority has no information about the current shocks to full-employment output and interest rate levels and the demand for money but has some information about trends in full-employment output and demand for money growth, its optimal policy will be to maintain a constant rate of money growth sufficient to produce an expected average rate of increase in the price level equal to the core inflation rate. Although objective I above must be abandoned, the other two objectives can be accomplished to the extent that the underlying growth rates of output and the demand for money are correctly perceived. Unanticipated variations in the trend rates of full-employment output and money growth will, of course, lead to persistent deviations of the longer-term inflation rate from the core rate through time.

In fact, policy makers generally believe that they can obtain some information about output and demand for money shocks by observing credit conditions and nominal interest rates within period. Acting on this belief, the big country's authorities would introduce variations in m_t^* to offset some of these shocks. The potential success of such actions is a controversial issue that we will not pursue here except to note that a policy rule that would use variations in the money stock to minimize variations in the world real interest rate will be ineffective because the authorities cannot observe real interest rates within period or, in an inflationary environment, with any accuracy across periods.

The small country's central bank faces an even more difficult task in attempting to implement discretionary policy because it must evaluate shocks to the big country's economy as well as to its own. In compensation, however, its authorities have the additional option of fixing the exchange rate. The first question that must be considered, before attempting to wrestle with discretionary policy issues, is whether it would be better for the small country to fix the exchange rate or let it float and adopt a constant rate of domestic money growth.

6.1 Fixed Exchange Rates vs. Flexible Rates with Constant Money Growth

We begin by making three assumptions that will later be relaxed: that the big country follows constant money growth designed to eliminate deviations of the long-term inflation rate from the core rate; that the full-employment real exchange rate is constant and known to be so; and that the domestic and rest-of-world core inflation rates are the same. We also adopt the working assumption that the domestic and foreign economies are identical except for scale. Under these assumptions, E_{Tk}, V_{Tk}, E_{Tk}^* and V_{Tk}^*—the losses in terms of objectives II and III—will be minimized whichever regime the small country chooses. The best choice hinges on satisfying objective I. Constancy of the real exchange rate implies from (39) and (41) that under a fixed exchange rate $Dy_t = Dy_t^*$, $Dp_t = Dp_t^*$ and $E_{t-1}p_t = E_{t-1}p_t^*$ when the economies are identical except for scale—the time-path of the domestic price level will be identical with the time-path of the price level in the rest of the world. A fixed exchange rate regime will be preferred to a flexible exchange rate with constant growth if the variance of the unanticipated shocks to domestic output and employment, and hence the domestic price level, is greater under flexible exchange rates with constant money growth than the variance of the unanticipated output, employment and price level shocks abroad. Comparing (32) and (26), this condition reduces to

$$Var\{Dp_t\} \quad > \quad Var\{Dp_t^*\}$$

or

$$\begin{aligned} Var\{\, A\ [Dm_t - D\psi_t] \ + \ B\ [Dm_t^* - D\psi_t^*] \ + \ \epsilon\, A\ Dy_{ft} \\ +\, \epsilon\, B\ Dy_{ft}^* \ + \ \eta\, C\ Dr_{ft}^*\} \ > \\ Var\{\, C\ [Dm_t^* - D\psi_t^*] \ + \epsilon^*\, C\ Dy_{ft}^* \ + \ \eta^*\, C\ Dr_{ft}^*\}. \end{aligned} \tag{45}$$

where

$$\begin{aligned} A &= \frac{1}{1+\epsilon\,\theta\,\gamma} \\ B &= -\frac{\eta\,\phi^*\theta^*\gamma^*}{(1+\epsilon\,\theta\,\gamma)(1+\theta^*\gamma^*(\epsilon^*-\eta^*\phi^*))} \\ C &= \frac{1}{1+\theta^*\gamma^*(\epsilon^*-\eta^*\phi^*)}. \end{aligned}$$

Passing the variance operator through (45) and setting all foreign parameters equal to their domestic counterparts, we obtain

$$\begin{aligned} A^2\ Var\{Dm_t - D\psi_t\} \ + \ B^2\ Var\{Dm_t^* - D\psi_t^*\} \ + \ \epsilon^2\, A^2\ Var\{Dy_{ft}\} \\ +\, \epsilon^2\, B^2\ Var\{Dy_{ft}^*\} \ > \ C^2\ Var\{Dm_t^* - D\psi_t^*\} \ + \ \epsilon^2\, C^2\ Var\{Dy_{ft}^*\}. \end{aligned}$$

which reduces to

$$\begin{aligned} A^2\ Var\{Dm_t - D\psi_t\} \ - \ [C^2 - B^2]\ Var\{Dm_t^* - D\psi_t^*\} \\ +\, \epsilon^2\, A^2\ Var\{Dy_{ft}\} \ - \ \epsilon^2\, [C^2 - B^2]\ Var\{Dy_{ft}^*\} \ > \ 0. \end{aligned} \tag{46}$$

It has already been shown in the discussion following equation (32) that $A + B = C$. Since $B < 0$ and hence $A > C$, this implies that $A^2 > C^2 > C^2 - B^2$. Thus, when the variances of the money and full-employment output shocks are the same in the domestic and foreign economies—i.e., $Var\{Dm_t -$

$D\psi_t\} = Var\{Dm_t^* - D\psi_t^*\}$ and $Var\{Dy_{ft}\} = Var\{Dy_{ft}^*\}$—the variance of the domestic price level shock will be smaller under a fixed exchange rate than under flexible exchange rates with constant money growth. This conclusion becomes stronger when we take account of the effect of the greater variance of the domestic price level on θ relative to θ^*. As Lucas argued, greater variability of the unanticipated shocks to the price level will make agents less inclined to view observed within-period price shocks in their own industries as relative price shocks, with the result that employment will fluctuate less in response to unanticipated shocks to the price level. From the definitions of A and C above, it is clear the existence of an excess of A over C at equal values of θ and θ^* will cause a decline in θ relative to θ^*, making that excess of A over C greater.[19]

It turns out that the variances of the small-country money and full-employment output shocks are likely to be many times larger than the variances of the corresponding big-country shocks. Think of the world as consisting of n equal-sized areas each with a variable S_t^i (not in logarithms) subject to random shocks. Let one of these areas be the small country and denote the magnitude of its variable by S_t^1 and let the remaining $n-1$ areas be the big country whose aggregate level of the variable is denoted by $S_t^R = \sum_{i=2}^n S_t^i$. The deviation of the big-country's aggregate variable from its initial value taken as a proportion of that initial value will equal

$$\begin{aligned} DS_t^R = \sum_{i=2}^n \left[\frac{(S_t^i - S_{t-1}^i)}{S_{t-1}^R}\right] &= \sum_{i=2}^n \frac{S_{t-1}^i}{S_{t-1}^R}\left[\frac{(S_t^i - S_{t-1}^i)}{S_{t-1}^i}\right] \\ \simeq \sum_{i=2}^n \frac{1}{n-1}\left[\frac{(S_t^i - S_{t-1}^i)}{S_{t-1}^i}\right] &= \frac{1}{n-1}\sum_{i=2}^n DS_t^i \end{aligned} \tag{47}$$

When the shocks in the different areas are independent and have the same variance, the variance of this aggregate shock will equal

$$\begin{aligned} Var(DS_t^R) &= \frac{1}{(n-1)^2}\sum_{i=2}^n Var(DS_t^i) \\ = \frac{(n-1)}{(n-1)^2} Var(DS_t^i) &= \frac{1}{n-1} Var(DS_t^i). \end{aligned} \tag{48}$$

The variance of the shock in the small country will be $(n-1)$ times the variance of the shock in the big country. By pegging its currency to the

[19] Robert E. Lucas Jr., "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review*, 63, June 1973, pp. 326–334, and "Econometric Policy Evaluation: A Critique", *Journal of Monetary Economics*, 2, 1976, Carnegie-Rochester Conference Series, Vol. 1, 1976, pp. 19–46. See also Appendix B.

large-country's currency the small country in effect pools its income and demand for money shocks with the big-country shocks—the average percentage shock over a large area will always be smaller than the percentage shocks of the individual small areas that comprise it. We call this the *pooling advantage* of fixed exchange rates. A classic example is the Canada/U.S. case. During the period 1963–70 when the Canadian dollar was pegged to the U.S. dollar the variance of the residual from an AR36 forecast of the logarithm of monthly Canadian $M1$ was ten times the corresponding residual from a similar forecast of the logarithm of U.S. $M1$.[20] During this fixed exchange rate period the Canadian money supply can be assumed to have been adjusted endogenously in response to demand for money shocks to maintain the fixed exchange rate. By tying the Canadian dollar to the U.S. dollar, the Canadians effectively pool their income and demand for money shocks with the U.S. shocks so that unanticipated domestic price changes are dependent on U.S. shocks rather than their own.

So when purchasing power parity holds a fixed exchange rate regime will always beat flexible exchange rates with constant money growth as long as the country is small in relation to the rest of the world and not too different from it in structure. This also assumes, of course, that the small country is willing to accept the big country's core inflation rate. If the core inflation rates differ, objective II will be violated and the small country's authorities will have to decide whether to sacrifice objective II in order to pursue objective I. The extent of the difference in the core inflation rates together with w_1 and w_2, the relative weights of objectives I and II in the central bank's loss function, will be the determining factors.

Relaxation of the assumption that purchasing power parity holds modifies these results in two ways. First, the real exchange rate variations lead to additional price level and employment variations in (39) making the fixed exchange rate regime less favorable in relation to flexible exchange rates with constant money growth. Since the coefficient of the variable Dq_t is less than unity, the variance of the current shocks to the full-employment real exchange rate would have to be greater than that of the excess variances of the small-country over large country income and demand for money shocks to tip the balance in favour of flexible exchange rate regime with constant money growth. In the Canada/U.S. case noted earlier, the first difference of the logarithm of the real exchange rate, which equals the ratio of the Cana-

[20]Seasonally unadjusted data were used and seasonal dummies were included in the regressions. Running AR24 forecasts were also used, in which case the variance of the Canadian forecast error was about five times the variance of the U.S. forecast error.

dian to the U.S. price level, has about twice the variance of the forecast error of logarithm of the U.S. money shock. This compares with a variance of the Canadian money shock ten times larger than the variance of the U.S. money shock. The variance of the observed real exchange rate shocks, however, will certainly be smaller than the variance of the full-employment real exchange rate shocks because the nominal exchange rate is constant and the real exchange rate shocks, being unanticipated, will only partially affect prices and will therefore be partly absorbed by unanticipated changes in output. The residuals from an AR36 regression of Canadian industrial production have only twice the variance of the residuals from a similar forecast of U.S. industrial production[21] so it would seem rather unlikely that the variance of the shocks to the full-employment real exchange rate would have exceeded the difference in the variances of the money shocks. In any case, were this condition to have consistently failed for countries in general it would be reasonable to expect that some country at some point in history would have discovered that it could benefit by adopting flexible exchange rates with constant money growth rather than fixed exchange rates. Flexible exchange rates with constant money growth have never been observed, central bank rhetoric of the 1970's notwithstanding.

A second modification of the results is required due to the fact that real exchange rates are typically indistinguishable from random walks over periods as long as two or three decades. Real exchange rate shocks that are far too small to compensate for the difference between the variances of the small-country and big-country income and monetary shocks will cause the real exchange rate to wander far and wide. This means that the price level will be a random walk under a fixed exchange rate system—since $E_{t-1}q_{ft}$ becomes a random walk in (41), objective III will be unattainable. Objective II will still be obtainable in the sense that the authorities can control the drift term in the resulting random-walk process determining the price level. Whether this will lead the small country to prefer flexible exchange rates with constant money growth to fixed exchange rates will depend upon the size of the variance of the period-to-period innovation of the full-employment real exchange rate and the weight w_3 attached to objective III in the authorities' loss function.

Flexible exchange rates thus have an *insulation advantage* over fixed exchange rates in that they insulate the domestic price level from both short- and long-run variations in the real exchange rate—if it lets the exchange rate float, the small country's central bank can maintain an appropriate trend

[21]Seasonally adjusted data was used in these calculations.

rate of inflation, and thereby achieve objectives II and III regardless of movements in the full-employment real exchange rate and regardless of the core inflation rate in the big economy. This comes, of course, at the sacrifice of objective I.

If the rest-of-world authorities can abandon constant money growth and offset some of the price level shocks by clever within-period money stock adjustments they can make fixed exchange rates more desirable for the domestic economy. This assumes, quite reasonably, that the domestic authorities cannot engage in even more clever within-period departures from constant money growth under flexible exchange rates—the domestic authorities have the same access to technical expertise as their counterparts abroad and face a more difficult within-period stabilization task because they need to observe and counteract both the domestic and rest-of-world exogenous money and full-employment output shocks. The possibility that political instability in the rest of the world could result in worse performance than could be obtained with constant money growth must also be noted. Flexible exchange rates with constant money growth could insulate the domestic economy against this instability, although cases where this has actually happened have never been observed.

So the small country's authorities have to balance the insulation advantage of flexible exchange rates against the pooling advantage of fixed exchange rates in minimizing their loss function. We thus need to know the loss function and the magnitudes and variances of the underlying exogenous shocks to determine which regime will be chosen. But the fact that flexible exchange rate regimes are frequently adopted but no country has ever adopted constant money growth for any significant period suggests that there must be additional options available.

6.2 The Best of Both Worlds: Smoothing International Portfolio Shocks Under Flexible Exchange Rates

Under a fixed exchange rate the adjustments of the small country's nominal money supply are endogenous—the necessary consequence of fixing the exchange rate. These money supply changes finance portfolio adjustments necessitated by shocks to the domestic quantity of money demanded. When domestic residents wish to hold additional nominal money balances they reestablish portfolio equilibrium by selling assets to foreign residents in return for money. The domestic authorities must supply that money to keep the home currency from appreciating. This excess demand for money can arise from a shock to the domestic demand function for money or domestic full-employment output or from a movement of the world interest rate consequent on money and full-employment income shocks in the rest of the world. These same portfolio pressures occur under flexible exchange rates but their effect is to cause an adjustment of the nominal exchange rate and, in response, an adjustment of the domestic price level. Portfolio pressures on the exchange rate result when credit conditions become tighter or easier in the domestic economy than in the rest of the world—that is, when there are differential excess money supply shocks in the small and large countries. This is evident in the model from the fact that when the money and full-employment income shocks are the same in the small and large countries and the two countries are structurally the same except for scale, (21) and (22) become identical.

6.2.1 Portfolio Smoothing Using Current-Period Information

Now imagine the possibility that the small country authorities, from within-period observation of credit and foreign exchange market conditions, can sense when these portfolio pressures on the exchange rate are occurring and offset them with either unsterilized foreign exchange market intervention or open market operations. Such interventions would fit the conventional descriptions of "orderly markets" or "even keel" policies. They do not require that the authorities have better information about domestic economic conditions than do private agents since information about portfolio pressures on the exchange rate is useful only to the authorities, given that they alone control the stock of base money. These policies also do not require any special forecasting ability on the part of the central bank or knowledge of current-period demand function for money and full-employment output shocks—when recognizable portfolio pressure on the exchange rate, from whatever source, appears in the current period the bank simply provides the necessary finance to maintain portfolio equilibrium without the exchange rate being affected. Meanwhile, the nominal exchange rate is free to move without restriction in response to underlying shocks to the full-employment real exchange rate and differences between the domestic and foreign core inflation rates. Perfectly done, this would achieve simultaneously the pooling advantages of fixed exchange rates along with the insulation advantages of flexible rates. The small economy will follow the same cyclical pattern as its big neighbor, albeit with perhaps a different trend inflation rate.

In fact, the small country's inflation rate will be driven in this case by private sector expectations based on the underlying core inflation rate and recent inflation experience. Money holders will require increases in their holdings sufficient to finance their inflation expectations—excess or deficient holdings will lead to portfolio pressure on the exchange rate. The time path of nominal money growth in the small country would thus be determined endogenously and the credibility of the central bank's commitment to the core inflation rate would be extremely important.

It is of course visionary to suggest that the small country's central bank could perform the above portfolio smoothing operation without error. To make a regime of flexible exchange rates with portfolio smoothing viable, these errors must be insufficient to merit the adoption of either constant money growth on the one hand, or a fixed exchange rate on the other. When these smoothing errors are substantial and the underlying stochastic shocks to the full-employment real exchange rate are small, a fixed exchange rate may be the better solution, given that only small persistent deviations from the core inflation rate will likely occur under such a regime. The smoothing errors have to be greater than the domestic income and demand for money shocks to make the portfolio smoothing policy inferior to flexible exchange rates with constant money growth.

6.2.2 Managed Floating

There is in fact an upper bound to the smoothing errors that need be tolerated. This can be seen by approaching the smoothing process from an exchange rate management rather than a money supply management perspective. The objective is to allow the nominal exchange rate to adjust to reflect full-employment real exchange rate movements and differences between the domestic and foreign core inflation rates but to maintain it constant in response to portfolio shocks.

Suppose that the small country's authorities have no within-period knowledge of the stochastic shocks to the demand function for money and the full-employment levels of output and the real exchange rate and cannot observe from within-period foreign exchange and capital market conditions when portfolio pressures on the exchange rate are occurring. In the absence of such information, they can nevertheless approximately identify movements of the full-employment real exchange rate with a one-period lag and use this information to manipulate the nominal exchange rate to remove their long-term effects on the domestic price level. The object is to use unsterilized foreign exchange market intervention (combined with adjustments of the domestic source component of base money sufficient to maintain appropriate holdings of foreign exchange reserves) to keep the nominal exchange rate at a target level equal to the estimated current-period full-employment real exchange rate adjusted to take account the differences between the domestic and foreign core inflation rates.[22] The target nominal exchange rate would thus equal (with everything in logarithms)

$$\bar{\pi}_t \quad = \quad \bar{p}_t - p_t^* - \hat{q}_{ft} \tag{49}$$

where $\bar{\pi}_t$ is the target level of the nominal exchange rate, $\bar{p}_t$ is the target domestic price level given the foreign price level and the difference between the domestic and foreign core inflation rates, and $\hat{q}_{ft}$ is the current period forecast of the full-employment real exchange rate. If the domestic and foreign core inflation rates are the same, $\bar{p}_t = p_t^*$. Otherwise they will differ by the excess of the domestic core inflation rate over the foreign core inflation rate multiplied by the number of periods, denoted by n, that have passed since the two price levels were equal.[23] As a result,

$$\bar{p}_t - p_t^* \quad = \quad n\,(\alpha - \alpha^*) \tag{50}$$

or

$$\bar{p}_t \quad = \quad p_t^* + n\,(\alpha - \alpha^*). \tag{51}$$

Equation (49) then becomes

$$\bar{\pi}_t \quad = \quad n\,(\alpha - \alpha^*) - \hat{q}_{ft}. \tag{52}$$

Given that the full-employment real exchange rate is a random walk, the best forecast of its current-period level is its level in the previous period's level. And it would be natural to use the observed real exchange rate in the previous period as a forecast of that period's full-employment real exchange rate. We thus obtain

$$\bar{\pi}_t \quad = \quad n\,(\alpha - \alpha^*) - q_{t-1}. \tag{53}$$

If the authorities maintain the current-period nominal exchange rate on its target the resulting current price level follows directly from the definition of the real exchange rate. That is

$$p_t \quad = \quad q_t + \bar{\pi}_t + p_t^*, \tag{54}$$

[22]Unsterilized foreign exchange market intervention would be used for day-to-day stabilization and open market operations in domestic bonds would be used to force domestic portfolio adjustments that will bring about desirable one-off adjustments in the stock of reserves in the course of maintaining the desired level of the nominal exchange rate.

[23]Presumably at some point in the past when a decision was made as to what the domestic core inflation rate would be as compared to the foreign core inflation rate the units in which the price levels are measured were chosen to make the domestic and foreign price levels equal.

which upon substitution of (53) yields

$$p_t \quad = \quad q_t - q_{t-1} + n\,(\alpha - \alpha^*) + p_t^*. \tag{55}$$

Upon subtraction of (51) we obtain

$$\begin{aligned} p_t - \bar{p}_t &= q_t - q_{t-1} + n\,(\alpha - \alpha^*) + p_t^* - n\,(\alpha - \alpha^*) - p_t^* \\ &= q_t - q_{t-1}. \end{aligned} \tag{56}$$

The domestic price level will differ from its target level only by the change in the real exchange rate from the previous period—it will therefore follow a trend that will differ from that of the foreign price level by the difference in the core inflation rates. This trend will be unaffected by the stochastic trend of the full-employment real exchange rate. The nominal and real exchange rates and the target nominal exchange rate will be random walks.

Equation (55) can be rewritten as

$$p_t - p_t^* \quad = \quad q_t - q_{t-1} + n\,(\alpha - \alpha^*) \tag{57}$$

and expanded using the relationships

$$\begin{aligned} p_t &= E_{t-1}p_t + Dp_t \\ p_t^* &= E_{t-1}p_t^* + Dp_t^* \\ q_t &= q_{ft} + Dq_t \\ q_{t-1} &= q_{f(t-1)} + Dq_{t-1} \end{aligned}$$

to obtain

$$\begin{aligned} Dp_t + E_{t-1}p_t - Dp_t^* - E_{t-1}p_t^* \quad = \quad & q_{ft} + Dq_t - q_{f(t-1)} \\ & -Dq_{t-1} + n\,(\alpha - \alpha^*). \end{aligned} \tag{58}$$

Substituting (7) to eliminate Dq_t and Dq_{t-1} we obtain

$$\begin{aligned} Dp_t + E_{t-1}p_t - Dp_t^* - E_{t-1}p_t^* \quad = \quad & q_{ft} - q_{f(t-1)} - \sigma\,[(y_t - y_{ft}) - (y_t^* - y_{ft}^*)] \\ & +\sigma\,[(y_{t-1} - y_{f(t-1)}) - (y_{t-1}^* - y_{f(t-1)}^*)] + n\,(\alpha - \alpha^*). \end{aligned} \tag{59}$$

Equations (1) and (2) can now be substituted to eliminate $(y_t - y_{ft})$ and $(y_t^* - y_{ft}^*)$.

$$
\begin{aligned}
&Dp_t + E_{t-1}p_t - Dp_t^* - E_{t-1}p_t^* \\
&\quad = \quad q_{ft} - q_{f(t-1)} - \sigma\,\theta\,\gamma\,(p_t - E_{t-1}p_t) - \sigma\lambda\,(y_{t-1} - y_{f(t-1)}) \\
&\qquad + \sigma\,\theta^*\gamma^*\,(p_t^* - E_{t-1}p_t^*) + \sigma\lambda^*\,(y_{t-1}^* - y_{f(t-1)}^*) + \sigma\,(y_{t-1} - y_{f(t-1)}) \\
&\qquad\qquad - \sigma\,(y_{t-1}^* - y_{f(t-1)}^*) + n\,(\alpha - \alpha^*) \\
&\quad = \quad q_{ft} - q_{f(t-1)} - \sigma\,\theta\,\gamma\,Dp_t - \sigma\lambda\,Dy_{t-1} + \sigma\theta^*\gamma^*\,Dp_t^* + \sigma\lambda^*\,Dy_{t-1}^* \\
&\qquad\qquad + \sigma\,Dy_{t-1} - \sigma\,Dy_{t-1}^* + n\,(\alpha - \alpha^*). \qquad (60)
\end{aligned}
$$

This simplifies to

$$
\begin{aligned}
Dp_t(1 + \sigma\,\theta\,\gamma) - Dp_t^*(1 + \sigma\theta^*\gamma^*) \quad &= \quad Dq_{ft} + \sigma\,(1 - \lambda)Dy_{t-1} \\
&- \sigma\,(1 - \lambda^*)Dy_{t-1}^* + n\,(\alpha - \alpha^*) - E_{t-1}p_t + E_{t-1}p_t^*. \qquad (61)
\end{aligned}
$$

Because the difference in the expected domestic and foreign price levels equals the difference in the trend levels implied by the difference in the core inflation rates,

$$
E_{t-1}p_t - E_{t-1}p_t^* \quad = \quad n\,(\alpha - \alpha^*),
$$

these terms can be eliminated from (61). When the parameters are the same in the two economies we thus obtain

$$
Dp_t \quad = \quad Dp_t^* + \frac{1}{1 + \sigma\,\theta\,\gamma} Dq_{ft} + \frac{\sigma\,(1 - \lambda)}{(1 + \sigma\,\theta\,\gamma)}[Dy_{t-1} - Dy_{t-1}^*]. \qquad (62)
$$

This is the same as the unanticipated domestic price level shock under a fixed exchange rate, given by (39), except for the addition of the term

$$
\frac{\sigma\,(1 - \lambda)}{(1 + \sigma\,\theta\,\gamma)}[Dy_{t-1} - Dy_{t-1}^*].
$$

This term compensates for the fact that under managed floating the nominal exchange rate target is based on the previous period's actual observed real exchange rate rather than full-employment real exchange rate. The previous period's actual and full-employment real exchange rates will differ by the differential effects of the deviations of the domestic and foreign outputs from full-employment in that period.

For a small country, the managed floating regime just described will almost certainly be superior to a flexible exchange rate regime with constant money growth. The condition is essentially the same as that for superiority of fixed exchange rates over flexible exchange rates with constant money growth and no trend in the real exchange rate—the stochastic trends in the real exchange rate are neutralized by managing the float. Managed floating is also superior to a fixed exchange rate regime to the extent that it insulates the domestic price level both from longer-term swings in the real exchange rate (i.e., the cumulative effects of the period to period shocks) and any differences between the domestic and foreign core inflation rates. Nevertheless, if the real exchange rate exhibits very little stochastic trend (i.e., the period-to-period shocks to it are small) and the domestic and foreign core inflation rates are not too different, a fixed exchange rate may be superior to managed floating in that, being automatic, it requires few resources to manage and precludes the possibility of management mistakes.

Any actual managed floating regime constructed on the above principles will differ in practical details from the above description. First, countries will not have precisely defined core inflation rates—modern central banks typically set a range in which they hope to maintain the inflation rate in the future. Also, while prices are clearly observable with a lag of at most a month or two, and observation is more accurate the further one looks back into the past, there are many competing price indexes that the authorities could contemplate stabilizing. One would expect price indexes that quickly reflect movements in real exchange rates occurring at given nominal rates to be of particular interest to the authorities as they construct a target level of domestic prices in relation to the price level abroad. This target price level will probably not be precisely defined. Manipulation of the nominal exchange rate using unsterilized intervention will nevertheless be a feasible method of keeping some appropriate measure of the domestic price level in rough alignment with the price level abroad after taking into account acceptable differences between the domestic and foreign long-term inflation rates. If the central bank has been maintaining the nominal exchange rate within a given narrow range for some time and observes that domestic prices have become, say, a couple of percentage points too high relative to the foreign price level, it merely forces a two-percent appreciation of the nominal exchange rate over the next several months. If domestic prices remain too high by the same percentage after that time has elapsed, there are grounds for concluding that the full-employment real exchange rate is rising, making a further appreciation of the nominal exchange rate desirable. If domestic prices rise further relative to the price level abroad as the nominal appreciation is being effected it would be appropriate to conclude that the real exchange rate is rising sharply and the external value of the domestic currency should be pushed up more rapidly. On the other hand, if domestic prices are falling relative to the price level abroad as desired, it will appear that the effects of past real exchange rate appreciation on the domestic relative to the foreign price level are being corrected and that the upward pressure the central bank is putting on the domestic currency should not extend beyond what was originally planned. Evidence of major downward pressure on domestic relative to the foreign prices would signal that the nominal exchange rate should be depreciated to accommodate a depreciating full-employment real exchange rate. Although these policy actions are taken without reference to the nominal money supply they are essentially an indirect method of monetary control when the authorities cannot observe shocks to the demand for money and hence cannot directly choose the desired level of base money.

Two features of this managed floating policy should be emphasized. First, the concern is with movements in domestic prices relative to foreign prices, not with movements in domestic prices alone. This is because pressure on the nominal exchange rate is being used to neutralize the effects on domestic relative to foreign prices of changes in the full-employment real exchange rate. The foreign price level, and the foreign monetary policy that ultimately determines it, are being used as an anchor. No attempt is being made to implement an independent domestic counter-cyclical monetary policy. Indeed, implementing such a policy by trying to force countercyclical changes in domestic relative to foreign prices would involve the authorities in forecasting and discretionary policy initiatives that have already been ruled out by our information assumptions.

The second noteworthy feature of the above approach to managed floating is that the authorities do not inform the private sector of their planned actions. The central bank is using nominal exchange rate adjustments to determine and engineer the appropriate change in the domestic nominal money supply. Informing private agents of its nominal exchange rate target might cause the rate to move to target more quickly but would not change the ultimate adjustment of the money supply required to keep it there.

Direct management of the exchange rate has an advantage over a policy of adjusting the money supply to smooth perceived portfolio shocks in that it gives the central bank clear control over the long-term domestic inflation rate relative to the inflation rate abroad. If private agents come to expect an increase in the future inflation rate their portfolio decisions will reflect that view. If the central bank commits itself to financing these portfolio adjustments to prevent them from affecting the exchange rate, it will unwittingly finance these private inflation expectations and the domestic inflation rate will become whatever private agents expect it to be. When the central bank manages the exchange rate, on the other hand, allowing it to adjust only to compensate for previously observed movements in the full-employment real exchange rate or deviations of the domestic inflation rate from its estimate of the core inflation rate, its resulting management of the money supply is independent of any changes in private inflationary expectations.

A small country's authorities can, of course, use a combination of managed floating and the portfolio smoothing approach outlined in the previous subsection. Together with the underlying core-inflation rate targets, such an approach could be termed *disciplined discretion*.[24] And both strategies can be implemented with respect to a basket of currencies, not just the currency of a particular large trading partner. In a world of many countries this will lead to pooling of income and demand for money shocks over a wider currency area than possible when countries peg to single key currencies. One could indeed imagine a world consisting entirely of small countries, each of which implements some combination of managed floating and portfolio smoothing policies with reference to the basket of all other world currencies. In this case the big country in the above analysis would consist of an aggregate of small countries identical to the one analyzed. The rate of world money growth in such a currency system will reflect the average of the core inflation rates of the respective countries augmented by unforeseen aggregate shocks to the world quantity of money demanded.

6.2.3 The Case for Fixed Exchange Rates

Whether the authorities attempt to observe and smooth portfolio shocks to the exchange rate or make the money supply adjustments necessary to directly manage the nominal exchange rate in a way that will accommodate underlying movements in the full-employment real exchange rate, the process will not be without error. These errors will appear as unanticipated shocks to the domestic price level and hence to domestic output and employment. By adopting a fixed exchange rate, these errors can be avoided. But the price is a failure to insulate the domestic price level from random-walk movements in the real exchange rate and an unacceptable core inflation rate in the country whose currency is being pegged to. If the shocks to the full-employment real exchange rate are small it will wander very slowly and the domestic price level will move only gradually, albeit persistently, from a path dictated by the core inflation rate of the country being pegged to. In this event, a fixed exchange rate may involve less costs that those resulting from errors in managing the float and may therefore be the best choice.

[24]This term is used in a somewhat different but not necessarily inconsistent fashion by Thomas Lauback and Adam S. Posen in "Disciplined Discretion: Monetary Targeting in Germany and Switzerland", *Essays in International Finance*, No. 206, Princeton University, December 1997.

6.2.4 Capital Controls and Currency Unions

An appropriate strategy for the small open economy would thus be a fixed exchange rate when shocks to the full-employment real exchange rate are small, with abandonment of the fixed exchange rate for managed floating or portfolio smoothing when the full-employment real exchange rate shocks are sufficient to produce price level movements under a fixed exchange rate that are more costly than the smoothing errors that will result under managed floating or portfolio smoothing. A flexible exchange rate with constant money growth would appear never to be appropriate when the country is very small in relation to the rest of the world—it could never beat the crudest regime of managed floating, which will neutralize demand for money shocks while insulating the economy from stochastic trends in, but not current-period shocks to, the real exchange rate.

Circumstances may arise, however, in which the rest of the world is unstable due to war or politically induced policy mismanagement. Neither a managed flexible exchange rate nor one accompanied by constant money growth will insulate the domestic economy from world real interest rate instability coming from abroad. Restrictions on international capital movements necessary to isolate the domestic capital market and permit competent closed-economy (or big-country) style monetary policy might be the best option when this happens. By isolating itself, the country makes the unanticipated shocks to the domestic price level less dependent on what is happening in the rest of the world and thereby easier to forecast and offset. But the exogenous shocks to the demand function for money and full-employment output will probably still be many times larger than would be the aggregate of these shocks in the rest of the world were the latter a stable policy environment. The argument here suggests a possible interpretation of nations' policies in the Great Depression together with their collective efforts to reduce balance of payments focused trade restrictions and maintain stable currency values after World War II.

At the other extreme, when there is no reason to expect that the full-employment real exchange rate will ever move significantly and the large country has a stable monetary policy and acceptable core inflation rate the costs of operating a foreign exchange market can be avoided by simply adopting the big country's currency. This, of course, forecloses the possibility of allowing nominal exchange rate movements to insulate the domestic price level from any future full-employment real exchange rate changes that may occur. It also binds the country irrevocably to the monetary policy and core inflation rate of the larger currency area. One would anticipate that future stability of the full-employment real exchange rate could only be expected over the very long run when there is labour mobility between the joining countries. The real exchange rate is an index of the relative prices of domestically produced traded and non-traded goods in terms of foreign produced traded and non-traded goods. Traded goods prices will be the same in both countries, though different traded goods may be produced in each. Non-traded goods prices can only differ if labour is unable to move to equalize wage rates, and labour mobility will also ensure that wage rates are the same in both countries regardless of relative traded goods prices. Cultural differences may be important in determining the degree of labour mobility—mere abolition of immigration restrictions may be insufficient. This provides an indication of the issues the nations of Europe must face in deciding to opt for a single currency.

7 Testable Hypotheses

The above theoretical analysis leads to a number of testable hypotheses. Some of these can be tested and provisionally accepted on the basis of readily available data. Others will require much further work. All are suggestive of useful directions in which future research can proceed.

First, the analysis above would lead us to expect that countries will adopt fixed exchange rates when the shocks to their full-employment real exchange rates are small and the latter exhibit stable and shallow trends. They will abandon fixed rates for flexible rates with managed floating when major real exchange rate movements occur. Evidence confirming this is presented in Figure 4 which plots the real exchange rates of Germany, France, Italy, United States, United Kingdom and Canada during the gold-standard period 1880–1913, the inter-war period 1924–38, the Bretton-Woods period 1950–71 and the managed-floating period 1973–1990.[25] During the gold standard and Bretton-Woods periods exchange rates were fixed (except for Canada in the latter period) and during the other two periods the fixed exchange rate system broke down. For every country but Canada, low-frequency real exchange rate variability was much greater during the flexible exchange rate periods than when exchange rates were fixed. Canada's real exchange rate varied as much in the Bretton-Woods period as it did in the flexible rate periods and, as the theory predicts, that country broke ranks and maintained a flexible exchange rate during much of the Bretton-Woods period. Canada also had substantial trend real exchange rate movements during the years of prairie settlement before the First World War but the average annual change between 1897 and 1908 was still only about 3 percent. The U.S. real exchange rate had a downward trend and the real exchange rates of Italy, Germany and the U.K. upward trends for the entire period after the Second World War but, again, the trend rates of change were less than 3 percent per year and the deviations around trend were far greater after 1973 than before.

A skeptic might argue that greater variability of real exchange rates in flexible exchange rate periods is the result of erratic monetary forces rather than natural real exchange rate movements, citing monetary instability as the reason for abandoning fixed rates. But this criticism implausibly assumes an absence of price level adjustments for long periods over which low-frequency real exchange rate adjustments were occurring.[26] To argue

[25] Similar historical evidence is presented in J. E. Floyd, *World Monetary Equilibrium*, Philip Allen and University of Pennsylvania Press, 1985, 69-81.

[26] It might also be argued that the major reason for adopting the gold standard was to

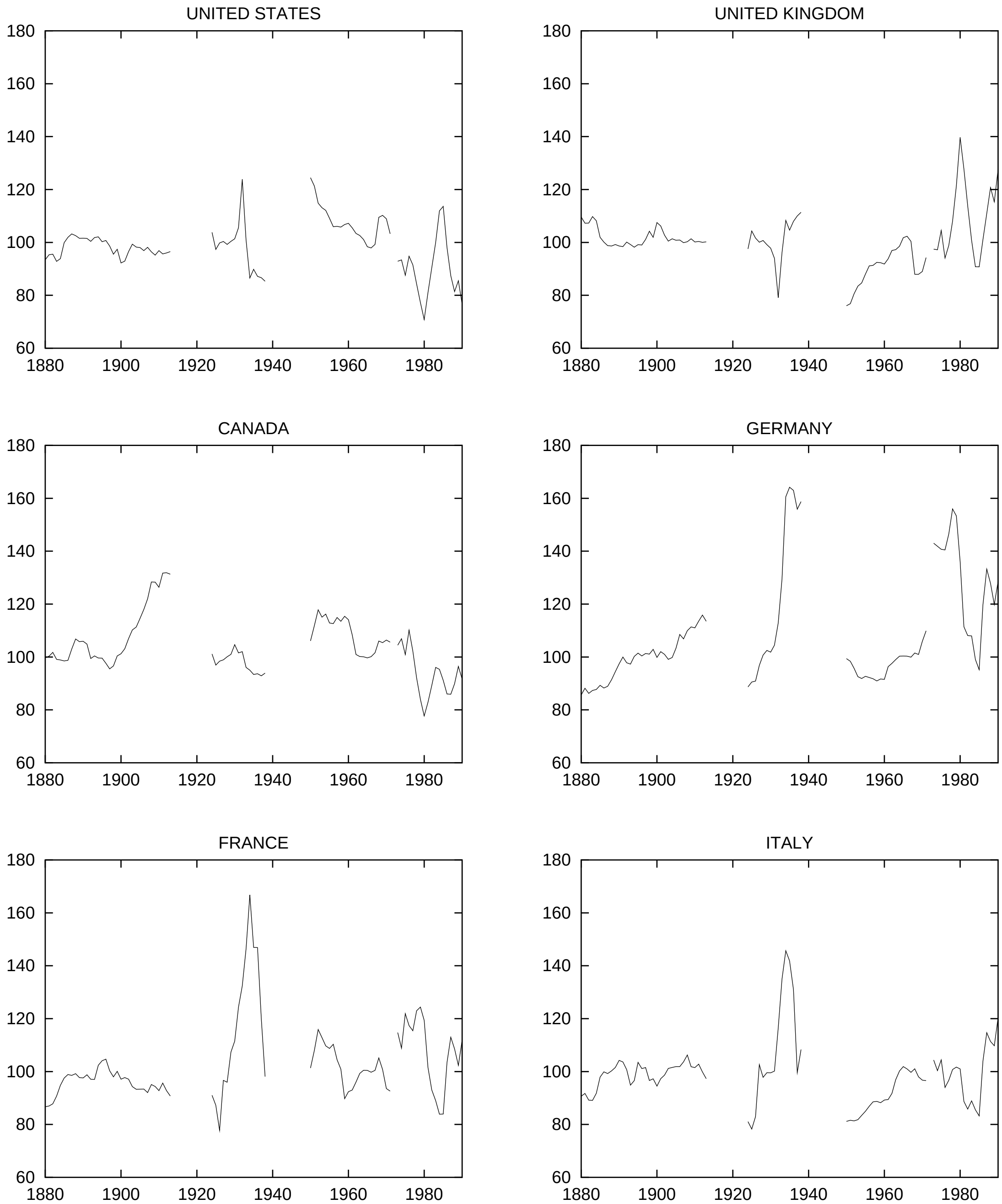


Figure 4: Real exchange rates of selected countries: 1880–1913 (1890–99 = 100), 1924–1938 (1927–29 = 100), and 1950–1971, 1973–1990 (1963–66 = 100). Source: See Appendix A.

Table 1: Inflation Rates of Selected Countries
Percent Per Year, 1980 and 1984

	1980	1984	Difference
United States	9	2	-7
United Kingdom	15	3	-12
Germany	6	0	-6
France	12	3	-9
Italy	17	5	-12
Sweden	14	4	-10

Sources: *Cansim* for Canada, *Citibase* for the U.S. and IMF, *International Financial Statistics* and OECD, *Main Economic Indicators* for the remaining countries.

that this greater variability was due to erratic monetary policy, one would have to postulate persistent rigidity of countries' *relative* price levels during a time when world inflation rates were changing substantially. To illustrate, consider the infamous real and nominal appreciation of the U.S. dollar during the period 1980–84. Over a four year period, the currency appreciated by more than 50 percent. Without sticky prices, a 50 percent appreciation of the U.S. real exchange rate not based on exogenous real factors would have been eliminated by some combination of a 50 percent fall in the price level in the U.S. or a 50 percent rise in the price level in the rest of the world. Yet all countries shown in Table 1 experienced substantial reductions in their inflation rates between 1980 and 1984. For a price stickiness argument to be convincing, U.S. prices would have had to be sticky downward and foreign prices sticky upward, but the fall in the foreign inflation rates implies downward rather than upward pressure on foreign prices.

A second obvious and testable implication of our theory is that countries operating in an integrated world capital market will have approximately the same business cycles and the deviations of their inflation rates from their (perhaps different) core inflation rates will be correlated. That is, their monetary policies will be stochastically interdependent. There is well-substantiated evidence that the business cycle is international in scope. Backus, Kehoe and Kydland calculate the contemporaneous correlations of the logarithms of outputs of a number of countries with the logarithm of U.S. output after detrending the output variables using the Hodrick-Prescott technique.[27] The cross-correlations for a selected group of countries are as follows:

---

keep monetary policy out of the control of the government, so that the fixed exchange rate regimes in that period had nothing to do with pooling demand for money shocks. Further analysis of the real exchange rate movements of countries that were slow to adopt the gold standard in preference to fiduciary money—for example, Austria-Hungary, Russia and Argentina—might prove fruitful.

[27] David K. Backus, Patrick J. Kehoe and Finn E. Kydland, "International Real Business Cycles," Working Paper, Leonard N. Stern School of Business, New York University, February 1991. The Hodrick-Prescott filter removes low-frequency variations from the data, acting as a flexible detrending procedure. For discussions of the technique, see Robert Hodrick and Edward C. Prescott, "Post-war U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, Vol. 29, No. 1, February 1997, pp. 1–16, Robert King and Sergio Rebelo, "Low Frequency Filtering and Real Business Cycles," Rochester Center for Research Working Paper No. 205, October 1989, and Finn E. Kydland and Edward C. Prescott, "Real Facts and a Monetary Myth," *Quarterly Review of the Minneapolis Federal Reserve Bank*, Spring 1990, 3–18.

Australia	.25	Austria	.31
Canada	.77	Finland	.02
France	.22	Germany	.42
Italy	.39	Japan	.25
Switzerland	.27	United Kingdom	.48
Europe	.70		

Contemporaneous correlations are not the best indication of the international scope of business cycle activity because they do not take account of differences in timing across countries and regions. In a spectral analysis of the GNP series of the U.S., U.K., Germany and Japan, Harris Dellas found the following pairwise coherence coefficients:[28]

U.S.-Germany	0.9 at 2.5 years
U.S.-U.K.	0.7 at 9 quarters
U.S.-Japan	0.55 at 4 years
U.K.-Germany	0.6 at 2.5 to 4 years
U.K.-Japan	0.5 at 5 years
Germany-Japan	0.7 at 3.5 years

Stefan Gerlach also finds that the output movements of a group of countries—in particular, Belgium, Canada, France, Germany, Italy, Netherlands, Norway, and Sweden—are correlated in the business cycle frequency band.[29]

The inflation rates of the major industrial countries during the past three decades are shown in Figure 5. They all show the same general pattern—low inflation during the 1960s, higher rates during the 1970s with peaks around 1973 and 1979, and declines during the 1980s toward the levels experienced in the 1960s. The simple correlations between the U.S. inflation rate and those of the other countries are given below:

United Kingdom	.74	Japan	.45
Germany	.58	Canada	.83
France	.79	Austria	.51
Italy	.72	Australia	.63

[28] Harris Dellas, "A Real Model of the World Business Cycle," *Journal of International Money and Finance*, Vol. 5, 1986, 381–394. The coherence coefficient indicates the proportion of the variance of one economic series that is accounted for by variation in another series at some frequency.

[29] Stefan Gerlach, "World Business Cycles Under Fixed and Flexible Exchange Rates," *Journal of Money, Credit and Banking*, November 1988, 620–630.

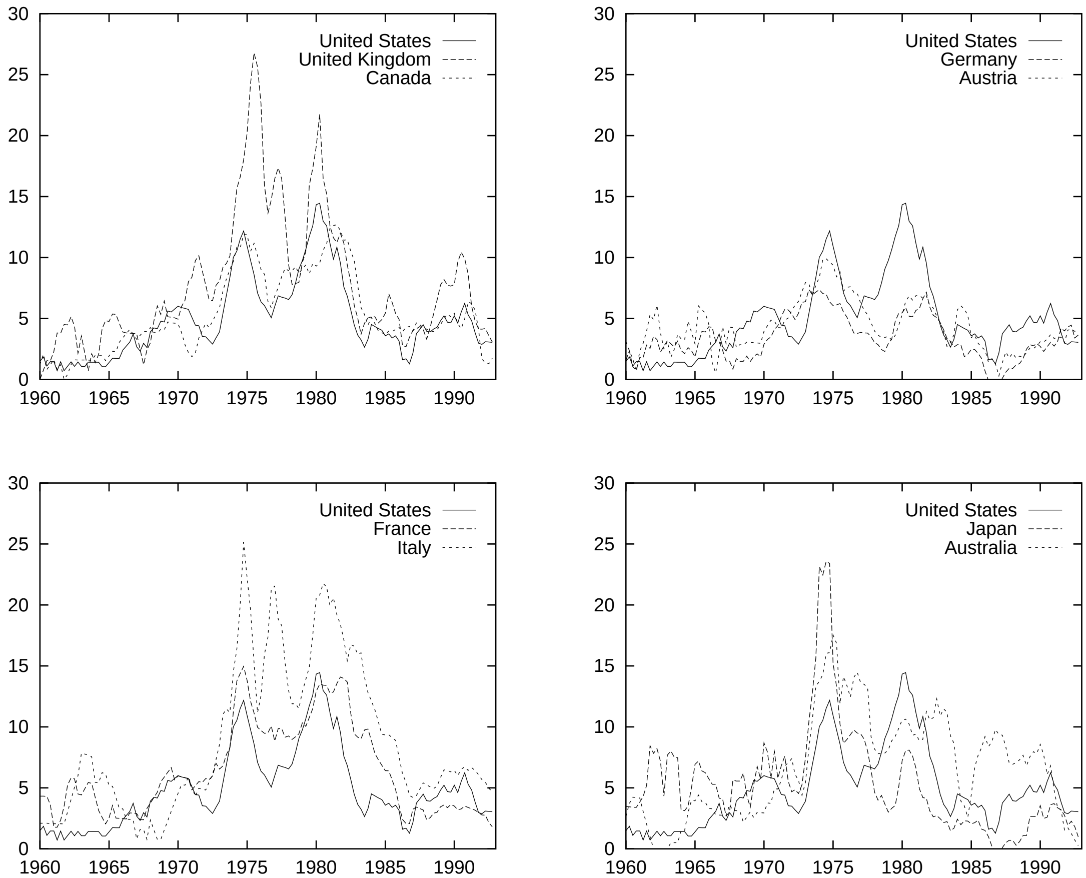


Figure 5: Inflation rates of selected countries as compared to the United States, 1960–1993, percent per year. Source: *International Financial Statistics*.

Third, our theory would lead us to expect that real and nominal exchange rates will be correlated with each other and exhibit much more variability than, and little correlation with, the ratios of nominal price levels of the corresponding countries. The real and nominal exchange rates and domestic/foreign price level ratios of nine industrial countries are plotted in Figure 6. For each country, the ratio of the domestic price level to the price level in the rest of the world is indicated by the solid line, and the real and nominal exchange rates by the broken lines. The rest-of-world aggregate consists of the remaining eight countries.[30] The correspondence of the real and nominal exchange rate movements and the high degree of both high- and low-frequency exchange rate as compared to price level variability is immediately evident. This stylized fact as well as others of interest here was noted in Mussa's excellent wide-ranging summary of the character of real exchange rate movements.[31]

Fourth, we would expect from the analysis above that major changes in world-wide inflation and deflation will normally occur in response to world-wide shocks affecting the demand for money—for example, the oil shocks of 1973 and 1978—or shocks affecting a major country or group of countries—such as the financial de-regulation that increased the U.S. demand for money in the early 1980s. It is tantalizing to speculate about whether the world-wide reduction in inflation rates in the 1980s was the result of a strong anti-inflation stance taken by the U.S. Federal Reserve System under Volker, as is conventionally believed, or the consequence of the positive demand for money effects of U.S. financial market deregulation. In both cases other countries would have adjusted their money supplies to eliminate the effects on their exchange rates of the portfolio shocks resulting from monetary tightness in the U.S. This hypothesis is more conjectural than the previous ones—the evidence remains to be explored.

Finally, the near random-walk nature of full-employment real exchange rate movements explains quite nicely the failure of the deviations of forward from spot exchange rates to predict future movements in spot rates. When the real exchange rate is not expected to change, forward rates will

[30]The price levels are consumer price indexes with the rest-of-world price level being an index of all other countries' CPI's using 1980 output shares as weights. The real exchange rate is the ratio of the domestic CPI to a 1980 output-weighted average of the CPI's of all other countries after all CPI's have been multiplied by the U.S. dollar prices of the individual countries' currencies.

[31]Michael Mussa, "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," in Karl Brunner and Allan H. Meltzer, eds., *Carnegie-Rochester Conference Series on Public Policy*, Vol. 11, (*Policies for Employment, Prices, and Exchange Rates*), North Holland, 1979, pp. 9–57.

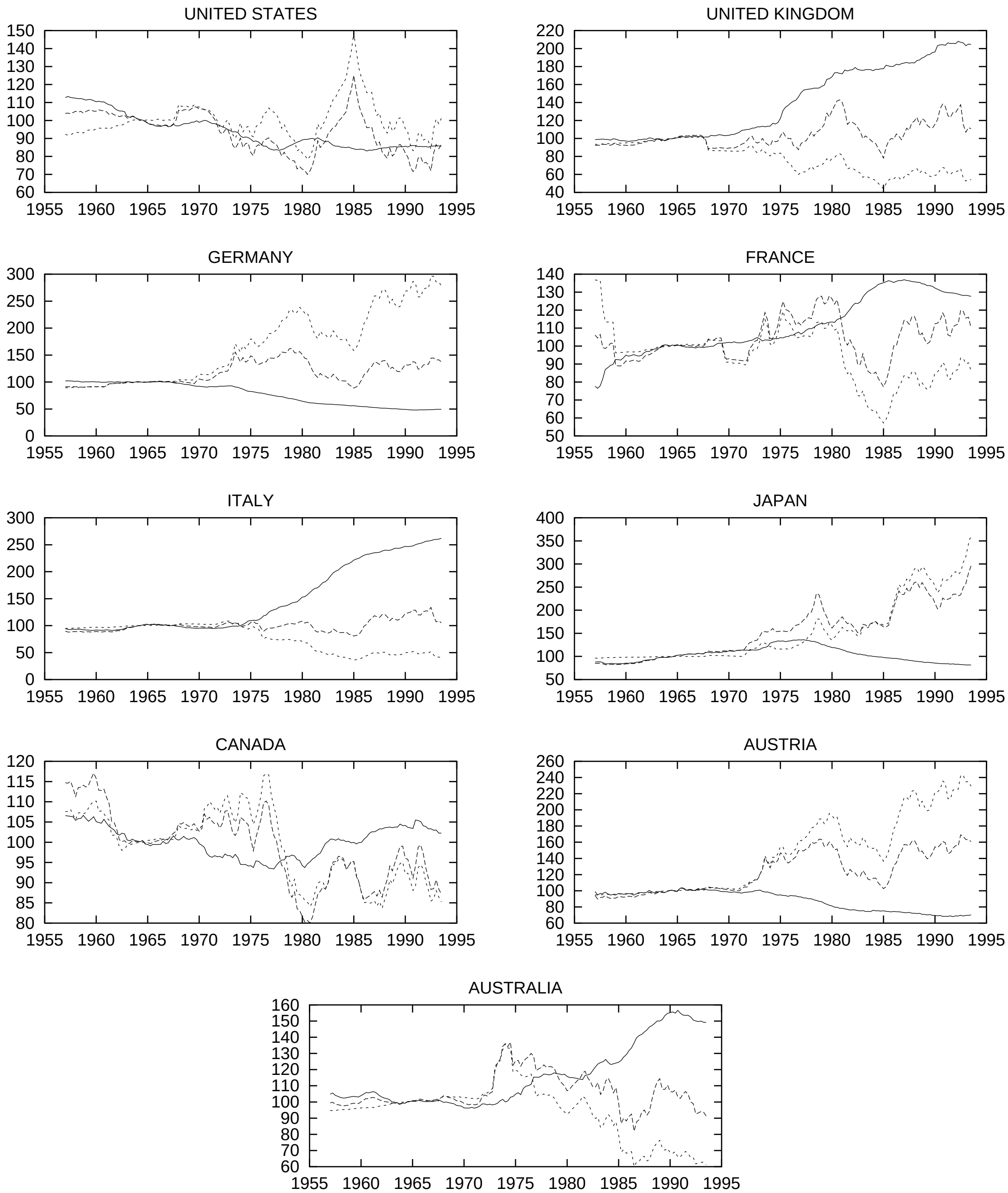


Figure 6: Real exchange rate (*dashed line*), nominal exchange rate (*dotted line*) and ratio of the domestic to rest-of-world price levels (*solid line*) for selected countries, 1957–1993, 1980 = 100. Source: *International Financial Statistics*.

differ from spot rates only to the extent that the expected rates of domestic inflation differ from expected inflation rates abroad. Given rather stable differences in inflation rates across countries, combined with substantial year to year innovations in real exchange rates, it is not surprising that forward discounts predict a very small fraction of future nominal exchange rate movements.

8 The Efficacy of Exchange Rate Target Zones

The results presented above have important implications for the practice of setting target zones within which the authorities commit themselves to maintain future levels of the the nominal exchange rate. Advocacy of target zones has had a long history and the practice has been followed in the European community on and off since the breakdown of the Bretton-Woods system.

Let us define the nominal exchange rate fundamental as the level the nominal exchange rate that would occur at each point in time in response to the underlying full-employment real exchange rate facing the economy together with the desired course of monetary policy. Suppose it happens that the nominal exchange rate fundamental for the next 100 periods behaves as plotted in Figure 7 —that is, as a white noise shock about a constant mean. If a credible target zone extending 7.5% on either side of the exchange rate parity, set as 1.0 in the figure, is established, the exchange rate will remain within that zone. As long as agents believe that the authorities will intervene in the foreign exchange market as necessary to keep the exchange rate within the target zone, the actual exchange rate will not only remain within the zone but will be asymptotic to the upper and lower bounds. As the exchange rate approaches, say, the upper bound, agents will anticipate that given the potential action by the authorities it will be more likely to fall than rise. As a result, the exchange rate will approach the bound asymptotically.[32] Note that it is essential to this argument that the bands be credible—that agents have no doubt that the authorities will behave as they promise.

The question is whether it is a good idea for the authorities to establish target zones and make these commitments. If the shocks to the exchange rate fundamental appearing in Figure 7 are the result of demand for money shocks which the authorities are not offsetting with appropriate adjustments in the money supply, then limiting the extent to which these fluctuations can occur and the resulting shocks to the price level that will result from them is good. But it would be even better for the authorities to fix the exchange rate at its parity level 1.0—the optimal target zone is of zero width. When the shocks to the exchange rate fundamental are purely monetary shocks it is in the public interest to eliminate them entirely. This can be done by fixing the exchange rate—the authorities will thereby be committed to financing all exogenous demand-for-money shocks with adjustments of the

[32]See Paul Krugman, "Target Zones and Exchange Rate Dynamics", *Quarterly Journal of Economics*, 1991 for an elaboration of this point.

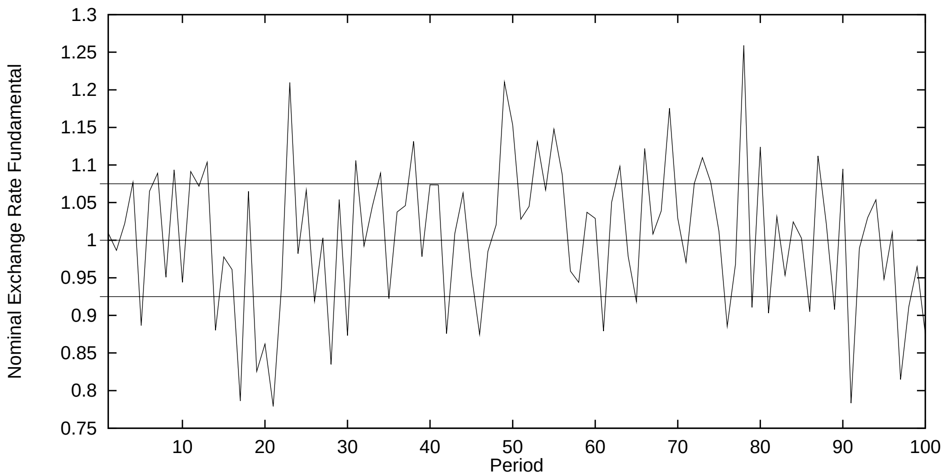


Figure 7: A target zone of $\pm$ 7.5% when the nominal exchange rate is stationary. The exchange rate fundamental is the level of the exchange rate that would be produced by underlying full-employment real exchange rate movements and desired domestic monetary policy. Source: Calculated normal random process with mean = 1 and variance = .2.

money supply.

On the other hand, suppose that the shocks to the exchange rate fundamental appearing in Figure 7 are the result of shocks to the full-employment real exchange rate and the authorities are perfectly smoothing the international portfolio adjustment effects of domestic demand-for-money shocks. In this case it is not in the public interest to in any way limit the resulting nominal exchange rate movements. The exchange rate is insulating the domestic economy from asymmetric real shocks. Capping nominal exchange rate movements will divert these unanticipated shocks onto the domestic price level, increasing the variability of domestic output and employment.

The situation becomes more complicated when there are both monetary and real domestic asymmetric shocks. To not lose the insulating effects of exchange rate flexibility on real shocks one would be inclined to set the target zones outside the limits of fundamental exchange rate variations resulting from asymmetric real shocks so that the additional range of variability resulting from asymmetric monetary shocks would be endogenously eliminated. But if the authorities know the bounds beyond which exchange rate movements will surely be the result of monetary shocks, they can smooth out all shocks that exceed these bounds without setting a target zone by appropriately managing the money supply. Furthermore, there is the problem that monetary as well as real shocks will be causing variability of the exchange rate fundamental within the target zone, so the authorities still have to decide whether it would be better to fix the exchange rate at the unitary par value and give up the insulating advantage of flexible exchange rates in return for the pooling advantage of fixed rates. If the decision is to go for the insulating properties of flexible exchange rates within the bounds of variability due to real shocks, then there would seem to be little advantage in setting a target zone because if the authorities know the appropriate target zone they know enough to eliminate the exchange rate fluctuations in excess of these bounds without setting a target zone. And should an unexpectedly large real shock occur, no credibility will be lost by letting the real exchange rate move beyond its normal bounds to neutralize it.

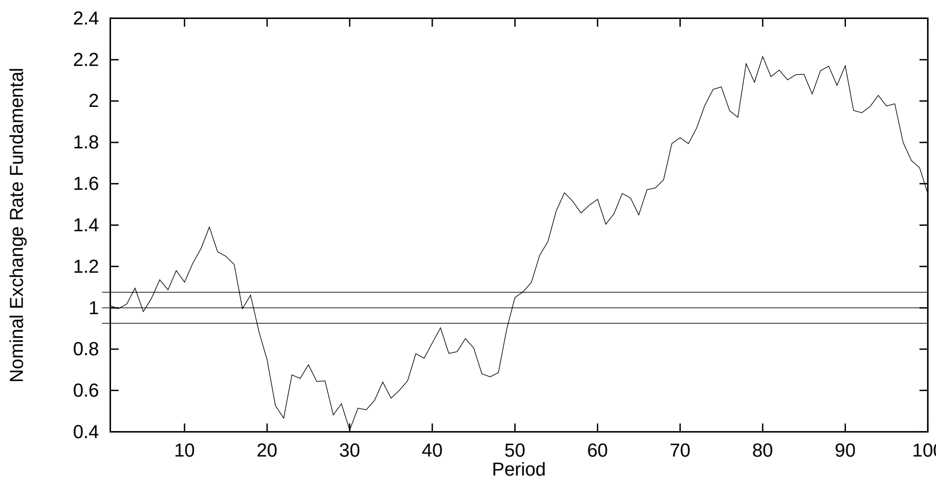


Figure 8: A target zone of ± 7.5% when the nominal exchange rate is a random walk. The exchange rate fundamental is the level of the exchange rate that would be produced by underlying full-employment real exchange rate movements and desired domestic monetary policy. Source: Based on normal random process in Figure 7.

All of the above discussion has been predicated on the assumption that the exchange rate fundamental deviates randomly around a constant mean value. In fact, full-employment real exchange rates are near random walks and, even with appropriate control over long-term domestic inflation rates, exchange rate fundamentals will wander far and wide as indicated in Figure 8. Unless the innovations to the full-employment real exchange rate are extremely small, any reasonable fixed target zone will be quickly violated. The alternative is to adopt a flexible target zone—that is to move the zone periodically so that it will always bracket the expected asymmetric shocks to the full-employment real exchange rate. The flexible target zone so established would have the advantage of neutralizing major monetary shocks while insulating the domestic price level from both current period and cumulative innovations to the real exchange rate. Again, however, if the authorities have enough information to set an appropriate target band beyond which the exchange rate should not be allowed to fluctuate, then they have the information to neutralize the offending asymmetric monetary shocks without setting a target zone.

The only reason for setting a target zone would appear to be to establish credibility in the eyes of the public that there will be limits to the asymmetric monetary shocks and domestic monetary policy shocks that will be allowed to affect the exchange rate. But the central bank does not have to establish credibility with the public that it will neutralize major monetary shocks because it clearly must be able to recognize such shocks if it has sufficient information to continually adjust the flexible target zone as the full-employment real exchange rate evolves and the government has no self-interest in not neutralizing these shocks. It is in situations where the authorities may face tempting political pressures to finance government expenditures by printing money that they need to establish a committment to maintain the nominal exchange rate within a band dictated by the evolving full-employment real exchange rate. As long as they allow themselves to adjust the target zone, as the random walk nature of the real exchange rate requires that they must do from time to time, it is difficult to maintain credibility that the zone will be adjusted only in response to real exchange rate movements and never in anticipation of future inflationary finance. This is especially the case because governments that might reap short-run political gains from inflationary finance can always point to "evidence" of potential real exchange rate movements that led them to inappropriately lower their target zone for the international value of the domestic currency or mistakenly fail to raise that zone when the full-employment real exchange rate increased. To maintain credibility in the face of political pressures to engage in inflationary finance, the only real option is to fix the exchange rate using a currency board or other credible institutional arrangement that will guarantee that the authorities will have no discretion regarding monetary policy. The opportunity cost of this is a price level that will be a near random walk. To eliminate these random walk price level movements, the monetary authorities have to have an appropriate degree of independence from the political pressures for inflationary finance. If they have that independence then there is no reason for them to set a flexible target zone. If they do not have that independence then flexibility of the target zone will make it difficult to establish credibility.

In the context of the present analysis, therefore, target zones do not appear to be useful. To establish credibility in a world where the full-employment real exchange rate is a near random walk, it would be wiser to commit to an inflation rate target zone rather than an exchange rate target zone.

9 The Efficacy of Monetary Unions

The analytical framework developed here is useful in thinking about the costs and benefits to a country of joining a monetary union like the one now under development in Europe. Monetary union means the adoption of a single currency by the countries involved. It can also can be usefully thought of as a system of permanently fixed exchange rates among members.

When would a group of countries want to permanently fix their exchange rates with respect to each other or, in other words, irrevocably adopt a single currency? One can quickly point to obvious advantages—everyone will know henceforth what the relevant exchange rates will be, and the costs of converting from one currency to another will be eliminated. These advantages cannot be achieved by simply fixing the exchange rate as long as the countries' separate currencies remain—when there are separate currencies there is no guarantee that the exchange rate between them will not at some future date be allowed to float in response to market forces. It is irrevocability that distinguishes currency unions from systems of fixed exchange rates.

The advantages and disadvantages to a country of pegging the exchange rate of its currency with respect to the currency of another country or monetary union of countries have already been discussed at length. In short, the main advantage is the pooling of domestic monetary shocks with the monetary shocks of the currency area being pegged to. If the area being pegged to is large relative to the domestic economy the variance of its unanticipated demand for money shocks should be substantially smaller than the variance of the unanticipated domestic demand for money demand shocks. By fixing the rate of exchange of its currency to the currency of a larger area, a country adopts the monetary shocks of that area in lieu of its own monetary shocks. The domestic money supply becomes endogenously determined by the domestic demand for money through the process of intervention by the domestic central bank in the international market for its currency. If the full-employment real exchange rate were an absolute constant, the domestic price level would be directly linked to the price level in the region being pegged to—the foreign and domestic price levels would move in unison.

The problem is that full-employment real exchange rates between countries and regions are not constant. Indeed, they are known to be near random walks. If a country pegs the nominal value of its currency to the currency of another country or region its price level will vary relative to the price level of that region in accordance with movements of its full-employment real exchange rate with respect to that country or region. If this full-employment real exchange rate is a random walk then the ratio of the domestic to the foreign region's price level will also be a random walk. Even when the long-term inflation rate in the larger currency area is stable at a satisfactory level the domestic price level will wander randomly.

The advantage of flexible exchange rates is that they allow movements in full-employment real exchange rates to be reflected in movements in nominal exchange rates rather than the domestic price levels—this is the insulation property of flexible exchange rates. But this insulation comes at a cost of having to conduct domestic monetary policy in the face of a substantially higher variance in the demand for money than in the larger regions to whose currencies the domestic currency could be potentially fixed. As argued earlier, countries have an incentive to try to get the pooling advantages of fixed exchange rates along with the insulation advantages of flexible exchange rates by managing the domestic money supply in such a way as to offset the portfolio pressures on exchange rates that result from differential domestic and foreign demand for money shocks. It was shown that at worst the nominal exchange rate can be managed to follow the real exchange rate with a lag of a period or two. But the necessity of smoothing portfolio shocks or managing the float introduces the prospect of unavoidable errors in monetary policy management that will be reflected in nominal exchange rates and prices. By fixing the currency to that of a larger stable country or region these errors can be avoided, although at opportunity cost of random walk movements in the domestic price level.

Given the insulation properties of flexible exchange rates, why do individual provinces and regions of countries not adopt their own currencies. Is it that full-employment real exchange rate movements within countries are less than than those between countries? There is reason to expect that this will be the case. Since labour can freely move within countries, any tendency of the outputs of particular regions to rise in price relative to the outputs of other regions will cause workers to move to where their marginal products and real wages are higher. Given that capital is also completely mobile in response to net-of-risk differences in returns, this will tend to ensure that resources will be optimally allocated within individual countries—those regions that have more natural resources that can be profitably developed or other natural advantages will tend to employ more capital and labour than regions that have few advantages. And when the output in particular regions within the country becomes less valuable, labour and capital resources will move outward to the rest of the country. This would suggest that full-employment real exchange rate movements within countries should be smaller than those between countries. Nevertheless, relatively depressed regions within countries exist. Because countries have national governments and national tax policies, avenues are available through which booming regions can be forced to help depressed ones. Labour mobility and regionally differentiated national fiscal policies thus represent adjustments to asymmetric real shocks that can substitute for real exchange rate movements that must be allowed to affect either nominal exchange rates or price levels.

When, due to immigration laws and language and cultural differences, labour cannot or will not move between countries, and when institutions are not present that force countries experiencing temporary boom to help those experiencing temporary decline, asymmetric shocks will be more fully reflected in real exchange movements and such movements that are experienced will have to be accommodated either by adjusting national price levels or allowing nominal exchange rates to adjust. In the absence of nominal exchange rate adjustments, declines in domestic real exchange rates will have to be accommodated by declining wages and prices. This can be expected to result in unemployment and political pressures to provide the domestic finance necessary to maintain current nominal wages and prices at the expense of abandoning fixity of the exchange rate. Clearly, in the absence of free immigration and emigration, countries facing big asymmetric real shocks will experience major trend movements in their domestic wages and price levels under fixed exchange rates.

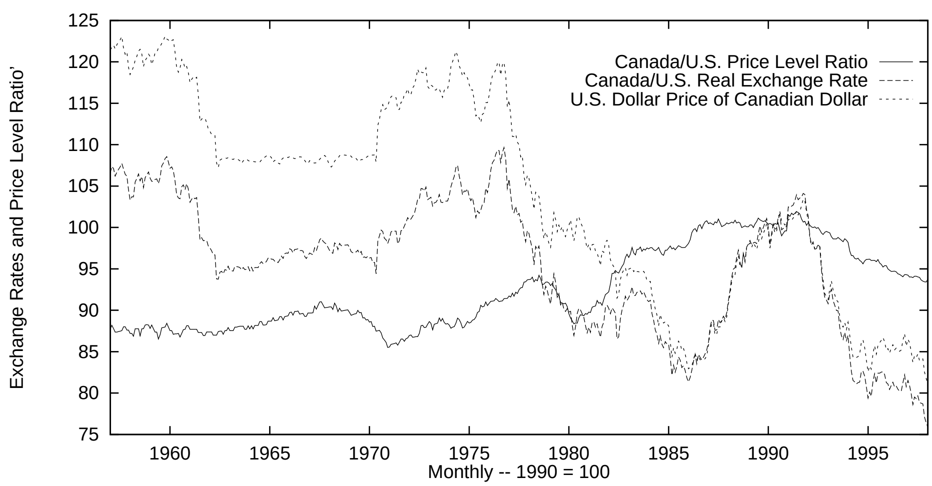


Figure 9: Real and nominal exchange rate movements and relative price level movements, Canada vs. the United States. Source: Calculated from International Monetary Fund, *International Financial Statistics*

Canada is a good example of this. It was shown in Figure 4 that Canada's real exchange rate was as variable during the Bretton Woods period as in the period after 1970—indeed, she was the only country for which this was the case. And Canada broke ranks and adopted a flexible exchange rate for much of this period. Figure (9) shows Canada's real and nominal exchange rates and price level ratio with respect to the United States for the period since 1957. Canada's real exchange rate with respect to the U.S. (given by the heavy dotted line) fell by about 25% between May 1977 and April 1980. It rose by nearly 22% between March 1985 and August 1989 and then fell by about 23% between June 1991 and January 1995. Had the country adopted a fixed exchange rate with respect to the U.S. dollar during this period these real exchange rate movements would have shown up as movements in the Canadian relative to the U.S. price level. Given prospective future movements of the full-employment real exchange rate of magnitudes similar to those that have been experienced during the past two decades, should Canada adopt the U.S. dollar as its currency? It would seem no. Were the U.S. to have a constant price level over the next two decades, these possible prospective real exchange rate movements would see the Canadian inflation rate alternating between, say, +4% for a few years and -4% for the next few years and then back to +4% for the succeeding few years, and so forth.

Eleven of the countries of the European Union have opted for monetary union and in January 1999 adopted a common currency, the Euro, although national currencies will remain in circulation for a couple more years. Figure 10 and Figure 11 plot the movements in the real exchange rates of each these eleven countries with respect to weighted averages of the remaining ten. The averages of the remaining members' real price levels in each case were calculated by weighting indexes of the members' U.S. dollar exchange rate adjusted price levels by the shares of these members in aggregate income in 1990. Figure 12 plots the real exchange rates of European countries who where not members of the Monetary Union with respect to a similarly weighted average of all 11 members. The real exchange rates of Germany, Austria, France, Belgium, Luxembourg, the Netherlands and Ireland have been quite stable with respect to weighted averages of the remaining eleven for the past two decades except for a rather sharp rise in the real exchange rates of Germany, Austria, Belgium, Luxembourg and the Netherlands in 1992–95. This real shock, which was greatest for Germany (over 20%), is usually attributed to German unification. The real exchange rates of Ireland and France remained stable throughout this period, with the rises in the previously noted countries coming at the expense of Italy, Finland, and Spain. Of the European countries who did not join the EMU, only Denmark

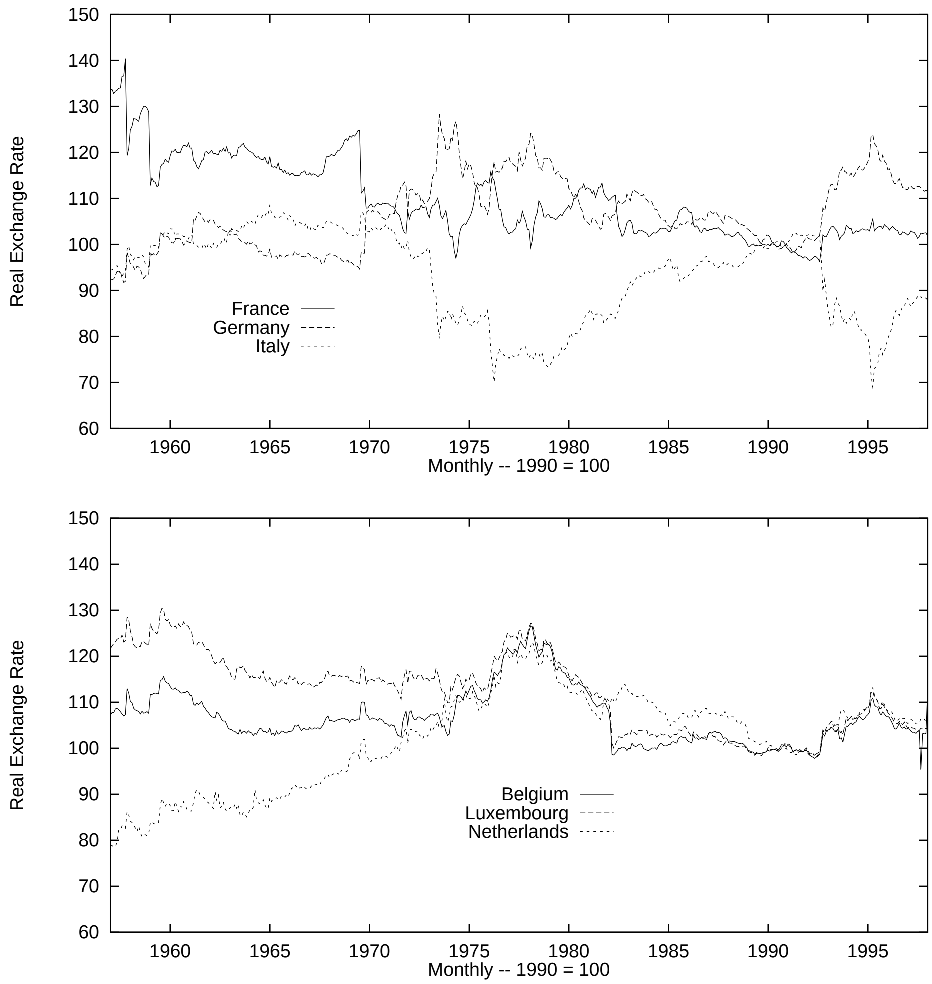


Figure 10: Real exchange rates of selected European Monetary Union members with respect to weighted averages of the other members. The weights are the countries' shares in the total 1990 incomes of the other members. Source: Calculated from International Monetary Fund, *International Financial Statistics.*

and Norway, the latter which is not even in the European Union, had two decades of stable real exchange rates with respect to the eleven members of the Monetary Union. Britain's real exchange rate with respect to the eleven has trended downward since 1981 after showing a 48% increase between January 1979 and May 1981. It also fell about 25% between July 1985 and January 1987, rose by 22% by January 1989, experienced an 11% dip

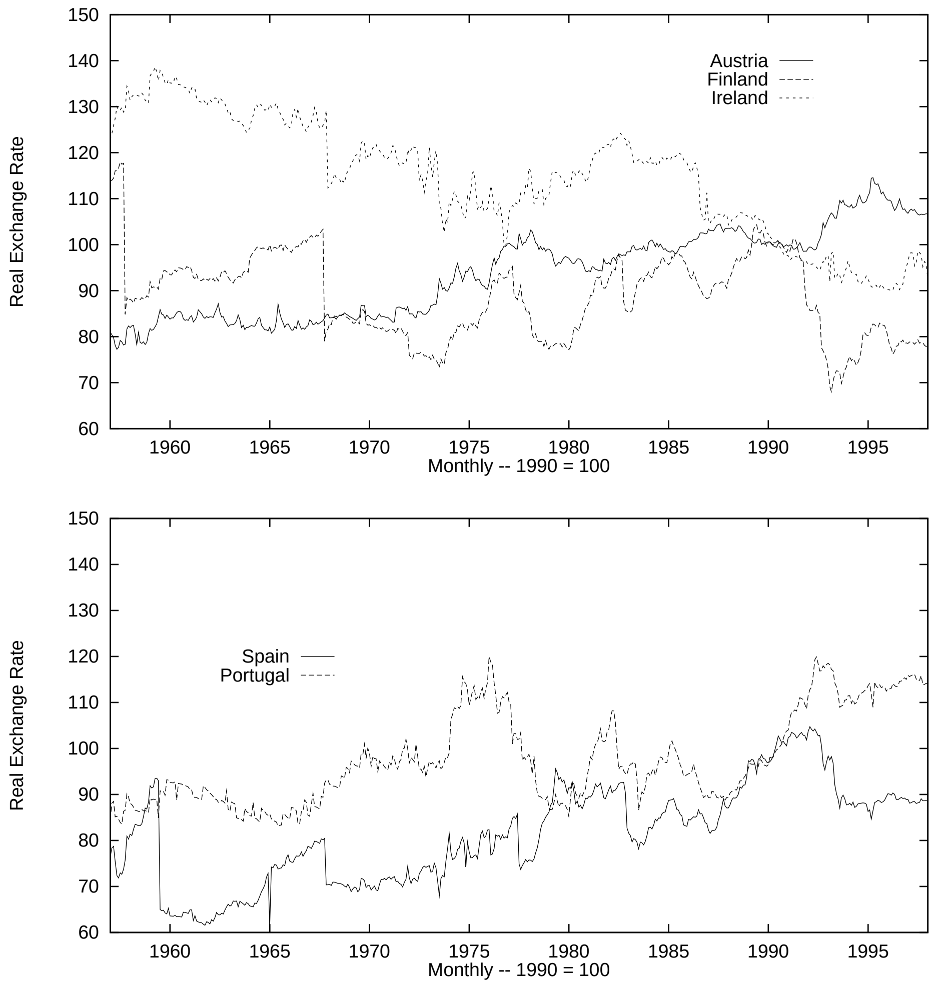


Figure 11: Real exchange rates of the European Monetary Union members not included in Figure 10 with respect to weighted averages of the other 10 members. The weights are the countries' shares in the total 1990 income of the other members. Source: Calculated from International Monetary Fund, *International Financial Statistics.*[A

in the subsequent year and then fell about 13% between June and October 1992. This latter fall, accompanied by a fall in the value of the Pound, drove Britain out of the European Union's Exchange Rate Mechanism, which she had joined only two years earlier. Britain's real exchange rate stayed rather low until late 1996 and then rose by over 30% by July 1998 when the data series ends. While some of these real exchange rate movements are sharp

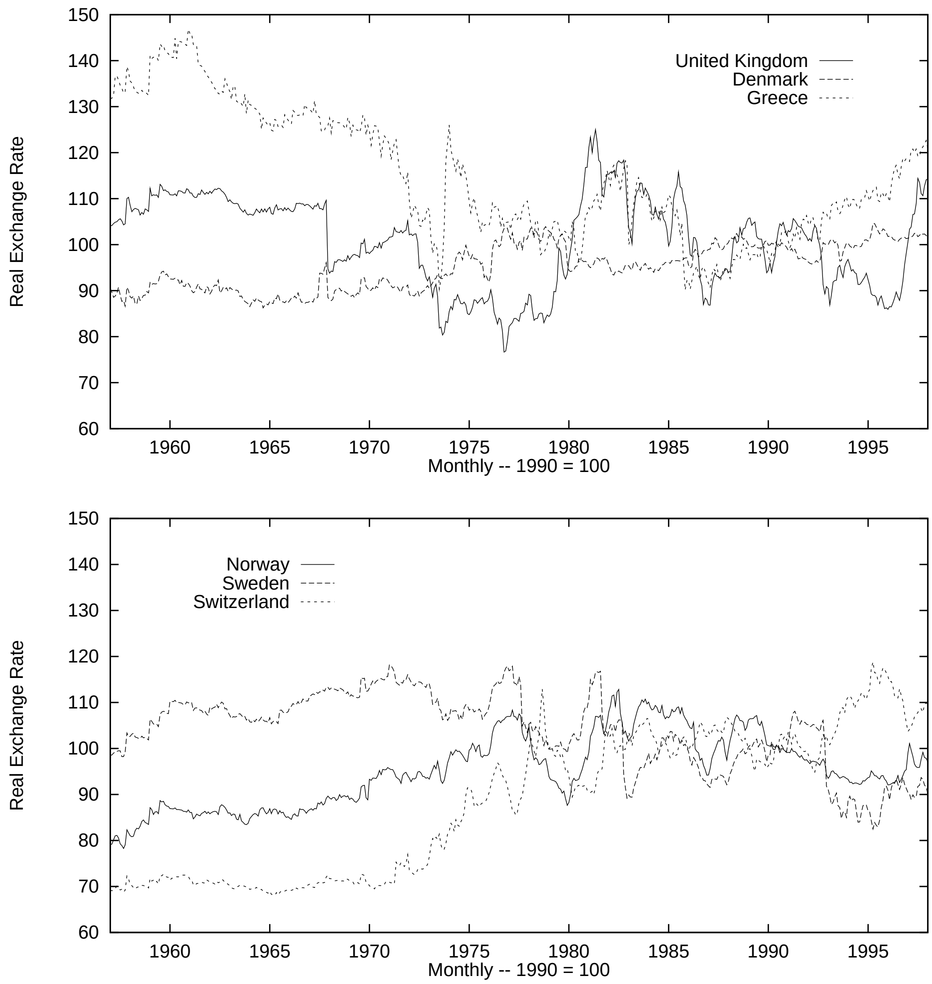


Figure 12: Real exchange rates of the European countries who are not members of the European Monetary Union members, calculated with respect to a weighted average of the 11 members of the European Monetary Union. The weights are the countries' shares in the total 1990 income of all members. Source: Calculated from InternationalMonetary Fund, *International Financial Statistics*.

enough to have been driven by portfolio shocks to the nominal exchange rate under conditions where domestic wages and prices did not have enough time to adjust there were three movements over two years in length that exceeded 20%. It is therefore not surprising that there is much opposition in Britain to joining the EMU.

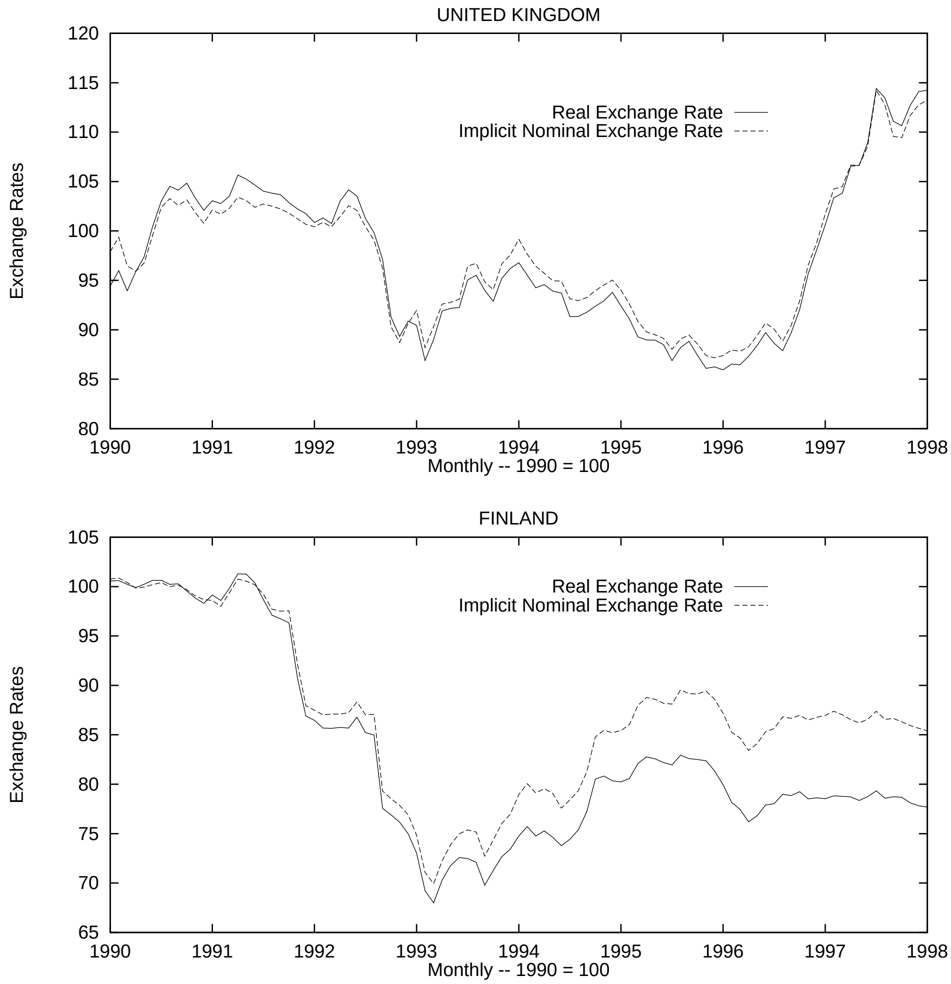
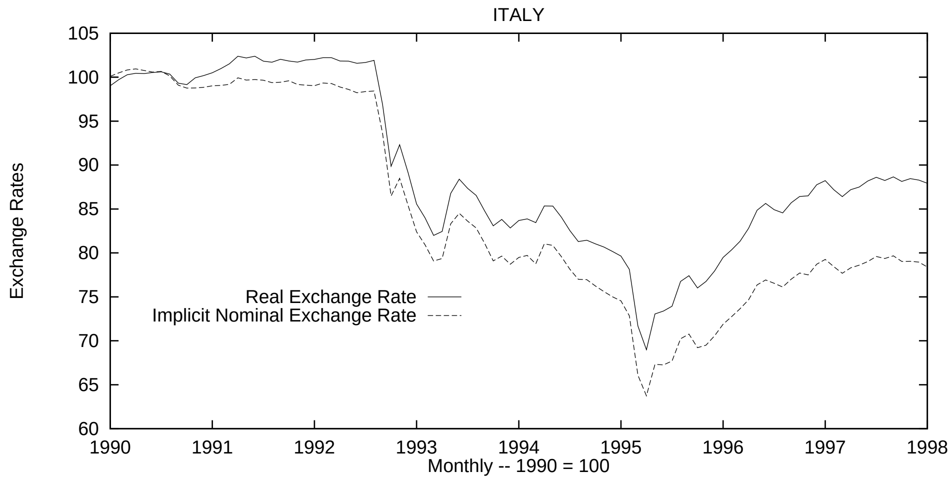


Figure 13: Real and implicit nominal exchange rates of the United Kingdom and Finland, calculated with respect to a weighted average of the 11 members of the European Monetary Union, 1990–1998 The weights are the countries' shares in the total 1990 income of all members. Source: Calculated from InternationalMonetary Fund, *International Financial Statistics*.

It is worth noting, however, that Italy also suffered a nearly 30% real exchange rate decline between March 1992 and April 1998, about half of which had been recovered by the end of the data set in July 1998. Yet that country chose to join the Monetary Union. The experience of Spain was somewhat similar. Figure 13 and Figure 14 show that nominal exchange rates followed the real exchange rates closely in those countries, namely



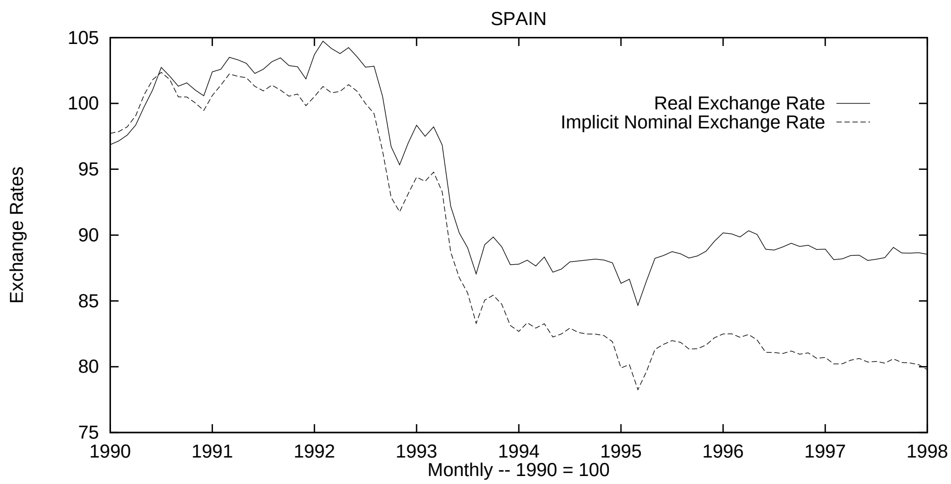


Figure 14: Real and implicit nominal exchange rates of Italy and Spain, calculated with respect to a weighted average of the other members of the European Monetary Union, 1990–1998 The weights are the countries' shares in the total 1990 income of all members. Source: Calculated from InternationalMonetary Fund, *International Financial Statistics*.

Italy, Spain, the United Kingdom and Finland, whose real exchange rates moved substantially in the 1990s.

Figure 15 plots the real exchange rates of the United States, Canada and Japan with respect to the eleven EMU members. The real exchange rate movements here are sufficiently large as compared to the intra-European real exchange rate movements that the range of the vertical scale had to be

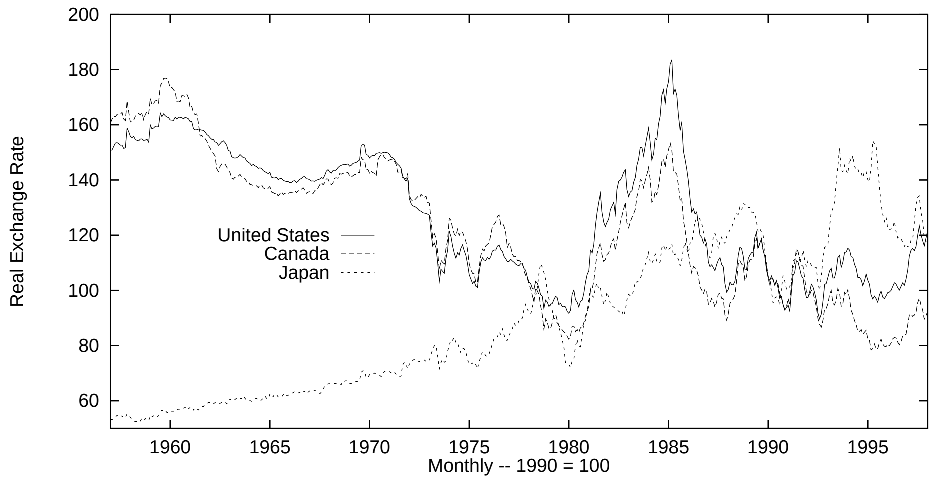


Figure 15: Real exchange rates of the United States, Canada and Japan, calculated with respect to a weighted average of the 11 members of the European Monetary Union. The weights are the countries' shares in the total 1990 income of all members. Source: Calculated from International Monetary Fund, *International Financial Statistics*.

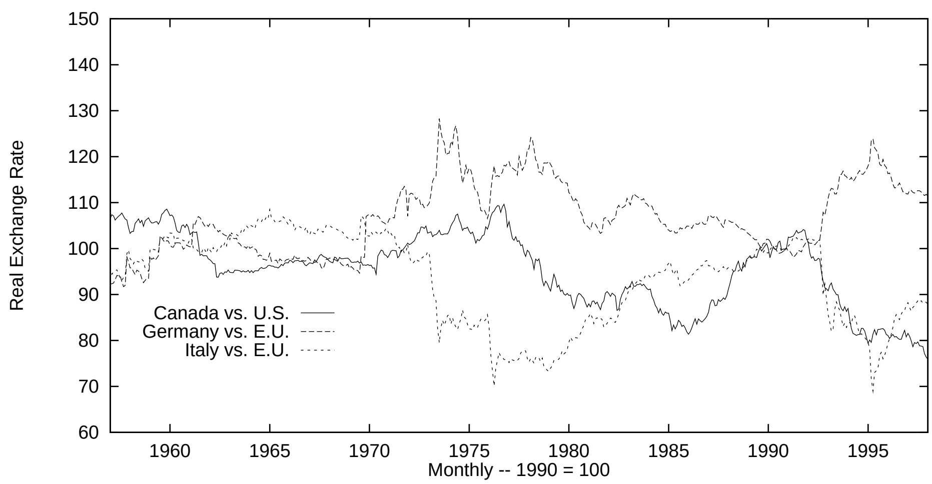


Figure 16: Comparison of the real exchange rate of Canada with respect to the United States and the real exchange rates of Germany and Italy, calculated with respect to a weighted average of the 11 members of the European Monetary Union. The weights are the countries' shares in the total 1990 income of all members. Source: Calculated from International Monetary Fund, *International Financial Statistics*.

increased.

Finally, the movements of Canada's real exchange rate with respect to the U.S. are compared in Figure 16 with the movements of the real exchange rates of Germany and Italy with respect to other EMU members. Canada's real exchange rate movements with respect to the U.S. since 1957 appear to be a bit greater than those of Germany with respect to the rest of the ECU, though they are not obviously larger than the movements with respect to the rest of the ECU experienced by Italy. This raises the interesting question of what will happen in Europe if major movements of intra-union real exchange rates occur before separate currencies are abandoned in 2002.

10 Implications for Estimating the Effects of Monetary Policy

Over the past decade a substantial literature has developed dealing with the process by which monetary policy is transmitted to the economy and the timing of its effects on economic activity. One important strand of that literature uses vector autoregression analysis to empirically measure the response of various aggregates such as output, employment, the price level, long-term interest rates, etc., to monetary shocks defined in various ways.[33] Another strand has been investigating the nature of the mechanism by which monetary shocks are transmitted, focusing almost entirely on the effects of monetary shocks on investment and arguing that this mechanism extends well beyond the simple response of investment to interest rate changes incorporated in the model developed here.[34] The vector autoregression literature has been developed largely within a closed economy framework and applied principally to the United States, although open-economy extensions are well under way.[35] Most detailed transmission mechanism investigations have been essentially closed-economy focused, incorporating open-economy issues, if at all, in an ad-hoc fashion. All of this reflects the U.S. centered intellectual environment of most researchers as well as the historical practice of developing new concepts first in a closed-economy environment and then gradually extending them to incorporate open-economy considerations.

The analytical framework developed here has important implications for

[33] This literature is very ably surveyed by Lawrence J. Christiano, Martin Eichenbaum, and Charles Evans in "Monetary Policy Shocks: What Have We Learned and to What End?" Unpublished Manuscript, Northwestern University, 1997. It builds on a number of important papers by Chrisopher Sims, among them "Macroeconomics and Reality," *Econometrica*, Vol. 48, 1980, pp. 1–48, and "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy," *European Economic Review*, Vol. 36, No. 5, 1992, pp. 975–1000. See also Eric M. Leeper, Christopher A. Sims and Tao Zha, "What Does Monetary Policy Do?" *Brookings Papers on Economic Activity*, Vol. 2, 1996, pp. 1–63, and David B. Gordon and Eric M. Leeper, "The Dynamic Impacts of Monetary Policy: An Exercise in Tentative Identification,", *Journal of Political Economy*, Vol. 102, 1994, pp. 1228–1247.

[34] This literature is surveyed by Ben Bernanke and Mark Gertler in "Inside the Black Box: The Credit Channel of Monetary Transmission," *Journal of Economic Perspectives*, Vol. 9, No. 4, Fall 1995, pp. 27–48. A number of other interesting articles on the transmission process are also contained in this issue of *Economic Perspectives*.

[35] See, for example, David O. Cushman and Tao Zha, "Identifying Monetary Policy in a Small Open Economy Under Flexible Exchange Rates," *Journal of Monetary Economics*, Vol. 39, 1997, pp. 433–448, and Richard Clarida and Mark Gertler, "How the Bundesbank Conducts Monetary Policy', in Christina D. and David H. Romer, eds., *Reducing Inflation: Motivation and Strategy*, University of Chicago Press, pp. 363–406.

the interpretation and future development of these lines of research. It is clearly inappropriate to apply closed economy techniques directly to small open economies. A more complex question is whether these theoretical and empirical models are even appropriate for analyzing the (big?) U.S. economy in an environment where countries' monetary policies are stochastically interdependent. How do we interpret the empirical results that are emerging from these studies within the framework developed here? The vector autoregression results for the U.S. and at least some of the results for Canada and other smaller countries are consistent with the standard implications of traditional closed-economy-focused macroeconomics. The onus is therefore on the model developed here to explain both this evidence and its consistency with traditional ideas.

One feature of contemporary research on the operation and transmission of monetary policy that seems especially at variance with the approach developed in this paper is the widespread view that the main instruments of monetary policy are the federal funds rate in the U.S. and the correspondingly defined interest rates in other countries, which are alleged to operate through their effects on a wide class of short-term, and ultimately long-term, interest rates and hence on investment. Although exchange rate effects of monetary policy are also considered, they are not given the same emphasis as the interest rate effects. This would seem at variance with the theory developed here in that we postulate a single world interest rate from which individual countries' interest rates will differ only by risk premia. It makes little sense to argue that monetary policy operates by changing risk premia since the latter, according to modern capital asset pricing theory, are determined by the covariance structure of output and asset returns and not by the most recent shocks to the demand or supply of money. Moreover, the notion that monetary policy affects interest rates through its effects on actual relative to expected exchange rates ignores the well-known fact that the real exchange rate is a near random walk and the best forecasts of tomorrow's real exchange rate tends to be today's rate.[36] Observed interest rates and exchange rates also reflect inflation expectations as well as the effects of monetary policy, making it difficult to disentangle the latter's influence. Identification problems are further compounded by the difficulty of disentangling the effects of long-term real forces on exchange rates from the influence of monetary shocks.

[36]On these points, see John B. Taylor, "The Monetary Transmission Mechanism: An Empirical Framework," *The Journal of Economic Perspectives*, Vol. 9, No. 4, Fall 1995, pp. 11–26.

10.1 A Basic Restrictive Case

We begin the discussion with three extreme assumptions that will be relaxed as we proceed:

a) The world consists of many small countries that are identical except for scale.

b) All countries' monetary authorities perfectly smooth the effects of portfolio shocks on their nominal exchange rates.

c) There are no exogenous shocks to the full-employment real interest rate.

A number of basic results follow from these assumptions. First, monetary tightness can be measured by the level of the world real interest rate which in every country will equal the nominal interest rate minus that country's expected inflation rate. The degree of monetary tightness is thus given by (25) with $Dr^*_{ft} = 0$.

Second, all individual countries' outputs will respond cyclically to the world interest rate with the result that, as long as expected inflation rates do not behave counter-cyclically, properly constructed single-country closed-economy-style empirical models will correctly characterize the effects of world monetary tightness, as reflected in the observed world interest rate movements, on every country's output, employment and prices. For each (unstarred) country, the relationship will be

$$y_t - y_{ft} \;=\; y^*_t - y^*_{ft} \;=\; -\,\phi^*(r^*_t - r^*_{ft}), \tag{63}$$

although the correspondence between r^*_t and countries' observed nominal interest rates will be contaminated by changes in inflationary expectations.

Third, the observed relationship between domestic interest rates and output and employment in each country does not yield the response of output and employment to independent domestic monetary policy, should the government choose to exercise it. Such policy would affect output only through its effect on the exchange rate. The effects of domestic nominal money supply changes that are unanticipated by the private sector can be obtained by substituting (32) into (1) and setting all exogenous shocks but Dm_t equal to zero:

$$Dy_t \;=\; \frac{\theta\,\gamma}{1+\epsilon\,\theta\,\gamma}\,Dm_t + \lambda\,Dy_{t-1} \tag{64}$$

Fourth, observed unanticipated shocks to individual countries' money supplies will, under assumption b) above, reflect shocks to their demands for money and will have no measurable effects on output and employment. When perfectly smoothing portfolio shocks on the exchange rate, each country's authorities have to supply at all points in time the quantities of money that their domestic residents wish to hold, given the country's core inflation rate.

Fifth, under the above assumptions, there will also be no unambiguously positive effects of the world money stock on output and employment in any country. Every country's money stock will depend on its demand for money, given its underlying core inflation rate, so the aggregate of these demands will equal the world money stock. Consider a situation where the demand for money increases simultaneously in a significant fraction of countries. Those countries will experience upward pressure on the external values of their currencies which they will offset by expanding their money supplies. The other countries will experience downward pressure on the values of their currencies which they will offset by contracting their money supplies. The monetary contractions in these latter countries will offset some of the expansion in the countries whose demand for money increased, making the expansion of the world money supply less than the initial expansion of the world demand for money. The real interest rate will rise relative to its full-employment level and there will be a world-wide contraction of output and employment. Monetary expansion will thus be associated with output contraction.

If, in the above case, the countries experiencing downward pressures on their currency values do not contract their money supplies, those countries whose demand for money is increasing will be led to increase their money supplies by the full increase in their residents' demand for money, so the world money supply will expand to fully accommodate the increase in the demand for money and an upward deviation of the world real interest rate from its full-employment level will be prevented. The monetary expansion will then be unrelated to output and employment.

An increase in the full-employment level of output will lead to an increase in the quantity of money demanded in the same way as if the relevant country's demand function for money had shifted to the right. If the world real interest rate rises in response to increases in the full-employment output levels in a significant fraction of countries, outputs in all countries will decline relative to their (in some cases increasing) full-employment output levels. Again, output declines will be associated with observed increases in the world money supply.

Finally, world business cycles will be caused by shocks to the world demand for money resulting from shocks either to aggregate world full-employment output or to the demand function for money. These shocks will operate through their effects on the world real interest rate.

10.2 One Big Country and Many Small Countries

Now relax assumption a) and consider a world consisting of many small countries with one country large enough for shocks to its domestic money supply, output and demand function for money to affect the world interest rate. Assume that this latter country's authorities conduct monetary policy without reference to the effects of that policy on exchange rates or on output, employment and prices in any country but their own. The country, which might be taken to represent the U.S. under current real-world conditions, effectively becomes a key-currency country. If we maintain all the remaining assumptions, there will still be an negative empirical relationship between the world interest rate and output and employment in all countries. Since all countries other than the key-currency country perfectly smooth the portfolio effects of the latter's monetary policy on the rates of exchange of their currencies with the key-currency, monetary shocks in the key-currency country will be matched by equivalent induced monetary supply shocks everywhere else—the key-currency country will effectively determine the world money supply. The smoothing actions of the peripheral countries' authorities will eliminate any relationship between the key-currency country's monetary shocks and nominal exchange rates. But there will now be a positive relationship between *excess* money supply shocks in the key-currency country and output and employment in every peripheral country.

If the peripheral countries choose not to smooth the effects of key-currency country monetary shocks on their exchange rates the magnitude of the key-currency country's effect on the real interest rate will depend on the size of the key-currency country in relation to the rest of the world. In this case, positive (negative) monetary shocks in the key-currency country will lead to a devaluation (appreciation) of the key-currency in terms of all peripheral currencies. Econometric estimates of the effect of the key-country's interest rate on its output and employment will provide a valid indication of that country's policy options. The observed empirical relationships between other countries' interest rates, which will equal the world interest rate, and their levels of output and employment will reflect the effects of key-currency policy on their economies and will not measure the effects of an independent domestic policy, were one to be followed. As before, unanticipated excess money supply shocks in the peripheral countries will operate on their economies by affecting the exchange rate and will have no measurable effect on the world, and their domestic, real interest rate.

10.3 Imperfect Smoothing

Next we relax assumption b) and assume that countries other than the key currency country, if one exists, do an imperfect job of smoothing the effects of portfolio shocks on their exchange rates. This means that there will be shocks to individual countries' outputs additional to those operating through changes in the level of world real interest rates. Risk and inflation premia will now be incorporated in the interest rates of individual countries and will change with the ebb and flow of individual countries' smoothing errors. Central banks can measure domestic monetary tightness, which will be coterminous with world monetary tightness, with reference to domestic nominal interest rates, but this measure will be contaminated by changes in the risk premia and in the expected domestic inflation rate. And small countries still cannot affect their output and employment levels by operating on domestic interest rates because the latter are determined in the rest of the world. Only by inducing changes in the external values of their currencies can they affect their domestic output, employment and price levels.

The domestic authorities might be able to control for short periods the interest rate on overnight borrowing of reserves by commercial banks from each other. And they can encourage commercial banks to borrow from the central bank by adjusting the "bank rate" charged on those borrowings relative to interest rates in general and thereby induce changes in the level of the domestic money supply. But they cannot affect those domestic interest rates relevant for determining the level of domestic investment.

The key-currency country's authorities can determine the level of the domestic interest rates relevant for determining that country's investment, and in doing so they determine the whole structure of world interest rates when the key-currency country is large, with the effects being stronger to the extent that the peripheral countries are at least partially successful in smoothing the portfolio effects of key-currency country policy on their exchange rates. The individual peripheral countries' exchange rates will be empirically unrelated to their unanticipated monetary shocks if they are sufficiently successful at portfolio smoothing. To the extent that all the peripheral countries successfully smooth the portfolio effects of asymmetric domestic relative to foreign monetary shocks on their exchange rates, the external value of the key-currency will also be independent of the key-currency country's unanticipated monetary shocks. The long-run effect of persistent monetary expansion in the key-currency country will, of course, be a nominal but not real devaluation of its currency. And long-run persistent monetary expansion in peripheral countries will also lead to nominal but not real devaluations of their currencies.

10.4 Shocks to the Full-employment Real Interest Rate

Finally, we relax assumption c) and allow for the presence of exogenous shocks to the full-employment world real interest rate. This destroys the previously clear relationship between the level of the world real interest rate and monetary tightness. Since changes in the full-employment interest rate cannot be observed even when the expected rate of inflation is known to be constant, it is impossible to determine the deviation of the world interest rate from its full-employment level. Hence, it is impossible to determine whether money is tight or easy by observing current interest rates even when there are no reasons for risk premia and expected inflation rates to have changed. An unobserved rise in the full-employment real interest rate will create unobserved monetary ease. If the shocks to world full-employment real interest rate levels are small, however, and substantial excess money supply shocks are occurring in the key-currency country, a negative statistical relationship between reasonably measured world real interest rates and that country's output and employment would be observed. A less precisely measurable negative relationship between the world interest rate and individual peripheral countries' output and employment levels might also be expected.

10.5 Unfinished Business

It turns out that, when we take into consideration the arguments just noted, there is little inconsistency between the closed economy empirical VAR results for the U.S. and the theory developed in this paper as long as we assume that the United States authorities do not conduct domestic monetary policy with reference to its effects either on the exchange rate or on economic conditions in the rest of the world. The area where potential problems arise is in the interpretation of observed effects of U.S. policy on the exchange rate. Researchers have made substantial efforts to obtain evidence that a monetary contraction in the U.S. leads to an appreciation of the U.S. dollar in world markets, and also, that monetary contractions in other countries lead to appreciations of their currencies. Cushman and Zha,[37] for example, find that observed shocks to the Canadian money supply result in seemingly permanent negative effects on Canada's real exchange rate. These results are, on the surface, inconsistent with the notion that peripheral countries' vary their money supplies endogenously to smooth the effects of portfolio shocks on their real and nominal exchange rates. They are also inconsistent with evidence from my own empirical research that purely unanticipated money supply shocks have no apparent effect on current nominal exchange rates using quarterly data for nine countries including the U.S.[38] How could there be longer-term effects of monetary policy on the real exchange rates in the absence of immediate short-run effects?

One possibility is that world-wide monetary shocks, possibly initiated in the United States, by differentially affecting output and employment in different countries, could lead to deviations of real exchange rates from their full-employment levels as a consequence of consumption smoothing. This would imply, however, that some country's currencies will depreciate in real terms in response to world contractions. Another possibility is that current research efforts have not yet correctly modeled the real exchange rate. VAR models which include the exchange rate in the set of variables the monetary authority looks at in determining its policy actions incorporate all exchange rate changes, including those resulting from underlying random-walk full-employment real exchange rate movements. This may lead to misspecification of the relationship between monetary shocks and real exchange rates. To test the theory developed here we need to use measures of portfolio pressure on the exchange rate rather than the level of the rate itself. In any case, work in this area has just begun.

10.6 Credit-Channel Transmission of Money Shocks

Although this present paper puts forward a very rudimentary and stylized view of the mechanism by which exogenous money shocks are transmitted to output and prices, the addition of a credit channel to this transition mechanism would appear to create no inconsistency as long as monetary tightness or ease is viewed in world-wide terms. The details of the mechanism by which exchange rate changes transmit small-country exogenous monetary

[37] David O. Cushman and Tao Zha, "Identifying Monetary Policy in a Small Open Economy Under Flexible Exchange Rates," *Journal of Monetary Economics*, Vol. 39, 1997, pp. 433–448.

[38] See, J. E. Floyd, "Monetary Policy and the Exchange Rate: Some Evidence, Unpublished Paper, University of Toronto, 1998.

shocks to output, employment and prices have to this point been given little attention by the profession. How the transmission of these exchange rate shocks into the demands for domestic import- and export-goods production might interact with credit channels, if at all, remains an open question.

APPENDIX A: Data Sources for Figure 4

The series for the period 1957–1995 for all countries except France before 1982 and Canada were obtained from *International Monetary Fund: International Financial Statistics*. Implicit GDP deflators (GNP in the case of Germany) were used as the price series for all countries and time periods. For the U.S. the source was Romer[39] for 1880–1929 and Friedman and Schwartz[40] for 1930–1956. The U.K. series are from Deane[41] for the period 1880–1914, and Friedman and Schwartz[42] for 1915–1956. The German Data for the period before 1957 are from *European Historical Statistics*.[43] For Italy the pre-1957 data are from compilations by Michael D. Bordo using *European Historical Statistics* and other sources. The French data for the period 1880–1982 are from Toutain.[44] The Canadian data are from Green and Urquhart[45] for the period 1880–1926 and from *CANSIM* thereafter. All of these data except for those from the IMF were brought together in the form used here by Trevor J. O. Dick.

[39] Christina D. Romer, "The Pre-war Business Cycle Reconsidered: New Estimates of Gross National Product, 1869–1908," *Journal of Political Economy*, February 1989.

[40] Milton Friedman and Anna J. Schwartz, *A Monetary History of the United States, 1876–1960*, NBER, Princeton University Press, 1963.

[41] Phyllis Deane, "New Estimates of Gross National Product for the United Kingdom, 1838–1914," *Review of Income and Wealth*, June 1968, 95–112.

[42] Milton Friedman and Anna J. Schwartz, *Monetary Trends in the United States and the United Kingdom*, National Bureau of Economic Research, University of Chicago Press, 1982.

[43] B.R. Mitchell, *European Historical Statistics, 1750–1970*, MacMillan, 1975.

[44] Jean-Claude Toutain, "Le Produit Interieur Brut de la France de 1789 à 1982," in Francois Perroux, ed. *Economies et Societies, Cahiers de l' I.S.M.E.A., Serie Historie Quantitative de l'Economie Francaise*, Paris, 1987.

[45] Alan Green and M.C. Urquhart, "New Estimates of Output Growth in Canada: Measurement and Interpretation," in Douglas McCalla, ed. *Perspectives on Canadian Economic History*, Toronto: Copp, Clark and Pitman, 182–189.

APPENDIX B: The Lucas Supply Curve

This appendix presents in detail the derivation of the Lucas supply curve.[46] Since the domestic and rest-of-world Lucas supply curves have the same derivation we will restrict our attention here to the domestic one.

The Lucas supply curve represents the aggregate response of suppliers who observe current-period conditions in their own industries and markets but have imperfect knowledge of what is happening in the economy as a whole. Consider a supplier in the local market z. The quantity supplied, in logarithms, is decomposed into a trend or secular component common to all markets, y_{ft}, and a cyclical component y^z_{ct} specific to the local market. Thus we can write

$$y^z_t \quad = \quad y_{ft} + y^z_{ct} \tag{65}$$

The cyclical component varies in response to changes in the output price in the local market, p^z_t relative to output prices in other markets as represented by what is perceived to be the price level in the economy as a whole. Representing the perceived overall price level by $E_{t-1}p_t$ we can express the cyclical component as

$$y^z_{ct} \quad = \quad \gamma\,(p^z_t - E_{t-1}p_t) + \lambda\, y^z_{c(t-1)} \tag{66}$$

where the term $\lambda\, y^z_{c(t-1)}$ is added to incorporate persistence. For y^z_{ct} to be stationary it is necessary that $\lambda < 1$.

Based on their observation of previous period's levels of p, agents in all local markets have a prior distribution on p_t that is normal with mean $\bar{p}_t$ and constant variance σ^2.

In the individual market, the actual output price p^z_t differs from the economy-wide average p_t by an amount u^z_t which is normally distributed independently of p_t with mean zero and constant variance τ^2. The variate u^z_t represents all the factors affecting industry z that are independent of what is happening in the economy as a whole. The observed price p^z_t (in logarithms) is thus the sum of two independent normal variates,

$$p^z_t \quad = \quad p_t + u^z_t. \tag{67}$$

Agents, being unable to observe p_t, must estimate it using their knowledge of $\bar{p}_t$ and the currently observed level of p^z_t. One can imagine them

[46] See Robert E. Lucas Jr., "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review*, 63, June 1973, pp. 326–334.

running a simple regression of p_t on p_t^z for an immediately preceding period of some length,

$$p_t = \hat{a} + \hat{b}\, p_t^z + v_t. \tag{68}$$

According to the standard derivation appearing in all statistics textbooks covering simple regression analysis the slope coefficient $\hat{b}$ equals

$$\hat{b} = \frac{\Sigma(p_t^z - \bar{p}_t^z)(p_t - \bar{p}_t)}{\Sigma(p_t^z - \bar{p}_t^z)^2}. \tag{69}$$

Substituting (67) into this, we obtain

$$\begin{aligned}
\hat{b} &= \frac{\Sigma[p_t + u_t^z - \bar{p}_t - 0)(p_t - \bar{p}_t]}{\Sigma[(p_t^z - \bar{p}_t)^2]} \\
&= \frac{\Sigma\{[p_t - \bar{p}_t) + (u_t^z - 0)](p_t - \bar{p}_t)\}}{\Sigma[(p_t - \bar{p}_t + (u_t^z - 0)]^2} \\
&= \frac{\Sigma[(p_t - \bar{p}_t)^2 + (u_t^z - 0)(p_t - \bar{p}_t)]}{\Sigma[(p_t - \bar{p}_t)^2 + (u_t^z - 0)^2 + 2\,(p_t - \bar{p}_t)(u_t^z - 0)]}.
\end{aligned} \tag{70}$$

Dividing both the numerator and the denominator by the number of observations and using the definition of variance, we get

$$\hat{b} = \frac{\sigma^2}{\sigma^2 + \tau^2}, \tag{71}$$

where we utilize the fact that, because p_t and u_t^z are uncorrelated,

$$\Sigma[(u_t^z - 0)(p_t - \bar{p}_t)] = 0.$$

The constant term $\hat{a}$ equals

$$\begin{aligned}
\hat{a} &= \bar{p}_t - \hat{b}\,\bar{p}_t^z \\
&= \bar{p}_t - \hat{b}\,\bar{p}_t \\
&= \left[1 - \frac{\sigma^2}{\sigma^2 + \tau^2}\right] \bar{p}_t \\
&= \frac{\tau^2}{\sigma^2 + \tau^2}\;\bar{p}_t.
\end{aligned} \tag{72}$$

The predicted price level in the economy as a whole thus equals

$$E_{t-1}p_t = \theta\,\bar{p}_t + (1-\theta)\,p_t^z. \tag{73}$$

where

$$\theta = \frac{\tau^2}{\sigma^2 + \tau^2}.$$

The variance of the predicted price level equals the expectation of the square of the residuals.

$$\begin{aligned}
E_{t-1}\{(p_t - E_{t-1}p_t)^2\} &= E_{t-1}\{(P_t - \theta\,\bar{p}_t - (1-\theta)\,p_t^z)^2\} \\
&= E_{t-1}\{[P_t - \theta\,\bar{p}_t - (1-\theta)\,p_t - (1-\theta)\,u_t^z]^2\} \\
&= E_{t-1}\{[\theta\,(p_t - \bar{p}_t) - (1-\theta)\,u_t^z]^2\} \\
&= E_{t-1}\{\theta^2\,(p_t - \bar{p}_t)^2 - (1-\theta)^2\,(u_t^z)^2 \\
&\qquad -2\,\theta\,(1-\theta)(p_t - \bar{p}_t)\,u_t^z\} \\
&= \theta^2\,\sigma^2 - (1-\theta)^2\,\tau^2 \qquad (74)
\end{aligned}$$

Here again we have taken advantage of the fact that because p_t and u_t^z are uncorrelated

$$E_{t-1}\{(p_t - \bar{p}_t)\,u_t^z\} = 0.$$

Substituting the expressions for θ and $(1-\theta)$ we can reduce (74) to

$$\begin{aligned}
E_{t-1}\{(p_t - E_{t-1}p_t)^2\} &= \frac{\tau^2\,\tau^2\,\sigma^2 + \sigma^2\,\sigma^2\,\tau^2}{(\sigma^2+\tau^2)^2} \\
&= \frac{\tau^2\,\sigma^2\,(\tau^2+\sigma^2)}{(\sigma^2+\tau^2)^2} \\
&= \frac{\tau^2\,\sigma^2}{(\sigma^2+\tau^2)} \\
&= \theta\,\sigma^2. \qquad (75)
\end{aligned}$$

Now we can substitute (73) into (66) to obtain

$$\begin{aligned}
y_{ct}^z &= \gamma\,(p_t^z - \theta\,\bar{p}_t - (1-\theta)\,p_t^z) + \lambda\,y_{c(t-1)}^z \\
&= \theta\,\gamma\,(p_t^z - \bar{p}_t) + \lambda\,y_{c(t-1)}^z. \qquad (76)
\end{aligned}$$

Then we can take a weighted average of the separate equations (76) for all the individual markets z where $(z = 1, 2, \ldots, Z)$ to obtain

$$y_{ct} = \theta\,\gamma\,(p_t - \bar{p}_t) + \lambda\,y_{c(t-1)}. \qquad (77)$$

Finally, substitution of (65) gives us

$$y_t = y_{ft} + \theta\,\gamma\,(p_t - \bar{p}_t) + \lambda\,(y_{t-1)} - y_{f(t-1)}), \qquad (78)$$

which, given that $E_{t-1}p_t = \bar{p}_t$, is identical to the Lucas supply curve (1).

It is crucial to understand the role of the parameter θ. As the variance of the domestic price level around its perceived mean gets smaller—that is, as σ^2 falls— θ gets larger, approaching unity as σ^2 approaches zero. In this limiting case, an observed change in p_t^z that results from a (totally unexpected) change in p_t will be viewed as a change in the relative price of local output in terms of output in the rest of the economy. The elasticity of the Lucas supply curve will equal λ, the elasticity of response of local output to the relative price of locally produced goods. In the opposite case where the variance of the domestic price level about its perceived mean gets larger and larger, θ gets smaller and smaller, approaching zero as σ becomes infinite. At this extreme, all of the observed variability of p_t^z is interpreted as coming from variations in the general price level, with the relative price of local output in terms of rest-of-economy output being unaffected. Suppliers in all local markets will produce their normal or trend levels of output and the Lucas supply curve will be vertical. The Lucas supply curve will also be vertical—that is, θ will equal zero—when the variance of the shocks to the relative price of local in terms of rest-of-economy output, given by τ^2, is zero. In this event, all shocks are regarded as economy-wide and as long as σ^2 is non-zero θ will be zero and unanticipated shocks to the domestic price level will have no effect on current output.