Exchange Rate Overshooting

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1 Introduction

In the standard Fleming-Mundell small country model with flexible exchange rates and perfect capital mobility, the equilibrium response to an increase in the supply of money (or decrease in the demand for money) is a rightward shift of the LM curve and increase in output. The IS curve shifts to the right endogenously as a result of nominal and real exchange rate adjustments to pass through the new intersection of the LM curve with the rZ line as shown in Figure 1. The excess supply of money causes domestic residents to try to reestablish portfolio equilibrium by purchasing assets abroad. This creates an incipient balance of payments deficit, causing the nominal and real exchange rates to devalue. The resulting increase in exports relative to imports shifts IS to the right, raising output and employment. Equilibrium is reestablished when the excess supply of money has been eliminated—this occurs at output Y_1. In the long-run, prices rise in proportion to the increase in the nominal money stock and LM shifts back to its original level, dragging IS with it. This formulation assumes that output can adjust immediately in response to the devaluation of the real exchange rate. That is unlikely to be the case because trade and output adjustments take time.

Suppose that it takes some time after the monetary expansion for exports and imports, and hence output, to adjust to the devaluation. The situation can be analyzed by looking at the asset equilibrium equations,

$$M = P\,L(r + E_P, Y) \tag{1}$$
$$r = r^* + \rho - E_Q, \tag{2}$$

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where M is the nominal money stock, P is the domestic price level, $L(\ldots)$ is the demand function for money, Y is real output and income, E_P is the expected rate of change of the price level, r and r^* are the domestic and foreign real interest rates and E_Q is the expected rate of change in the real exchange rate. The latter is defined as

$$Q \quad = \quad \frac{\Pi\, P}{P^*} \tag{3}$$

with Q being the real exchange rate, Π the nominal exchange rate, defined as the foreign currency price of domestic currency, and P^* the price level in the rest of the world.

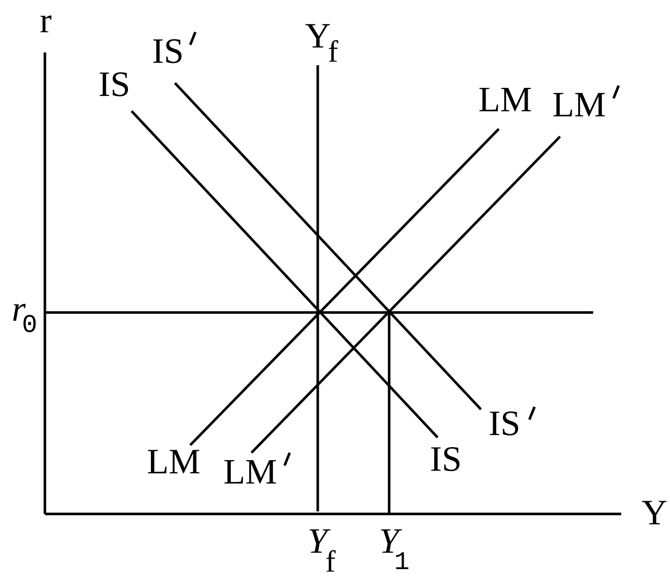

Figure 1: A Fleming-Mundell analysis of the effect of an increase in the nominal money supply.

If we make the crude Keynesian assumption that the price level is fixed and assume in addition that the risk premium on domestic assets and the expected rate of change of the real exchange rate are unaffected, there is no mechanism by which the right side of equation (1) can adjust to the left side in the face of an exogenous shock to the nominal money stock or the demand function for money. Output cannot increase immediately because it takes a while for exports and imports to respond to the change in the exchange rate. The rate of interest is fixed in the rest of the world and the expected inflation rate is determined by past inflation rates and current "news" about future developments. It is unlikely that the risk premium on domestic assets would change in response to a one-time monetary shock since risk premia are related to the covariance structure of domestic asset returns and will depend on the entire past history of monetary change. So, unless something happens that is not yet in the model, the exchange rate will explode.

2 Two Avenues of Adjustment

A little thought suggests two potential avenues of adjustment. First, we should express the domestic price level in terms of its components, one of which is the nominal exchange rate:

$$P \;=\; \tilde{P}^{\alpha}\,(P^*/\Pi)^{1-\alpha} \tag{4}$$

The price level is expressed as geometrically weighted index of prices of the non-traded components of domestic output, given by $\tilde{P}$, and the prices of the traded components in domestic currency, given by ΠP^*, with α being the share of non-traded components in domestic output. It is obvious from the above equation that a fall in the nominal exchange rate (devaluation of the domestic currency) will increase the price level even if the prices of domestic non-traded components of output and all nominal prices abroad are fixed. The devaluation in response to an increase in the money stock will be bounded if the share of traded components in output, $(1-\alpha)$, is not zero. The nominal exchange rate will devalue only until P has increased proportionally with M. This will imply that[1]

$$\frac{\Delta\Pi}{\Pi} \;=\; -\frac{1}{1-\alpha}\,\frac{\Delta M}{M}. \tag{5}$$

As long as the share of non-traded goods in domestic absorption is not zero, the nominal exchange rate will decline more than proportionally with the increase in excess money holdings. We know that in the long-run after the prices of non-traded components have adjusted and full employment has been reestablished the domestic prices of both non-traded and traded output components must increase, and the nominal exchange rate must decline, in the same proportion as the increase nominal money stock or decline in the demand for money. In the short run, therefore, the nominal exchange rate will overshoot its long-run equilibrium level.

[1]This can be seen by taking the logarithm of (4) and noting that $\Delta Log(X) = \Delta X/X$.

We are thus-far assuming that the expected rate of change in the real exchange rate is unaffected by this short-run overshooting movement in the nominal exchange rate. This would be reasonable if there is a lot of noise in the nominal and real exchange rates and agents act as though the real exchange rate is a random walk. When this assumption does not hold and expectations are rational a second mechanism of adjustment will arise. Suppose agents realize that the nominal exchange rate is overshooting its long-run level. This means that the real exchange rate must have declined relative to its long-run level because the proportionally greater fall in Π than rise in P in the short run reduces Q whereas the equi-proportional fall in Π and rise in P in the long run restores Q to its initial level. Agents will therefore expect Q to return to its equilibrium level and E_Q will become positive. As can be seen from equation (2), this will cause the domestic nominal interest rate to fall. The quantity of money demanded will thus increase in equation (1), offsetting some of the shock to the excess supply of money and moderating the resulting overshooting movement of the exchange rate. This second mechanism of adjustment originates with Dornbusch[2]. It cannot operate if the exchange rate is viewed by agents as a random walk.

3 A General Model

To check the validity of the above intuition, we must derive these adjustment mechanisms from a more complete model. Suppose that non-traded goods prices remain constant in the period in which a demand shock occurs and then adjust fully to their new long-run equilibrium in subsequent periods. We can define the time unit arbitrarily to represent a day, week, month, year or several years. Let us express all variables but interest rates in logarithms, denoting the logarithms of these variables by lower-case letters. Also, let us define the units of rest-of-world output so that the foreign price level is unity (and the logarithm of that price level is zero). The asset equations (1) and (2) can now be written

$$m_t = h_t + p_t + \epsilon\, y_t - \eta\, r_t - \eta\, (E\{p_{t+1}\} - p_t) \qquad (6)$$

$$r_t = r^*_t + \rho - (E\{q_{t+1}\} - q_t) \qquad (7)$$

where we are linearizing the demand function for money in logarithms. The income elasticity of demand for money is given by ϵ, the absolute value

[2] Rudiger Dornbusch, "The Theory of Flexible Exchange Rate Regimes and Macroeconomic Policy", *The Scandinavian Journal of Economics*, Vol. 78, No. 2, 1976, 255-275.

of the interest semi-elasticity of demand for money (interest rates are not in logarithms) is denoted by η and an additional variable h_t is introduced to denote a proportional shock the the demand function for money. The logarithm of equation (3) defining the real exchange rate becomes

$$q_t \quad = \quad p_t + \pi_t. \tag{8}$$

When all prices are flexible in response to market forces and the economy is at full employment, y_t will be at some technologically determined level $\hat{y}_t$. The expected rate of change in the real exchange rate $(E\{q_{t+1}\} - q_t)$ will be zero if the real exchange rate is perceived to be a random walk, or equal to some value determined by the technology if rational agents have information about its time path. The expected rate of change of the price level $(E\{p_{t+1}\} - p_t)$ will be determined by past experience and current information about the future course of monetary policy. To simplify matters, assume that both these expected rates of change are zero under conditions of full-employment when the nominal money supply is a known constant quantity.

Suppose that there is an unexpected once-and-for-all shock to the nominal money supply in time period $t + 1$ starting from a full-employment situation in period t and that the adjustment of non-traded goods prices to this monetary shock is completed by period $t + k$. The situation in period $t + k$ as compared to period t can be seen by subtracting (6) and (7) from themselves advanced k periods:

$$\begin{aligned} m_{t+k} - m_t \quad &= \quad (h_{t+k} - h_t) + (p_{t+k} - p_t) + \epsilon\,(y_{t+k} - y_t) - \eta\,(r_{t+k} - r_t) \\ &\qquad - \eta\,(E\{p_{t+k+1}\} - p_{t+k}) + \eta\,(E\{p_{t+1}\} - p_t) \end{aligned} \tag{9}$$

$$r_{t+k} - r_t \quad = \quad (r^*_{t+k} - r^*_t) - (E\{q_{t+k+1}\} - q_{t+k}) + (E\{q_{t+1}\} - q_t) \tag{10}$$

Since the economy is fully employed in period t and again in period $t + k$, $(E\{q_{t+1}\} - q_t)$ and $(E\{q_{t+k+1}\} - q_{t+k})$ are both zero by construction—all effects of the monetary shock on the real exchange rate will be eliminated by the end of period $t + k$. Along with the assumption that the real interest rate in the rest of the world is constant, this implies from (10) that

$$r_{t+k} - r_t = r^*_{t+k} - r^*_t = 0.$$

Since the money shock is a one-time unexpected shock whose effects on the price level will be completed by the end of period $t + k$, $(E\{p_{t+1}\} - p_t)$ and $(E\{p_{t+k+1}\} - p_{t+k})$ will also both be zero. Letting the full-employment output level be constant at $\hat{y}_t$, we have $y_{t+k} = y_t = \hat{y}_t$ (since any effects of the money shock on output are completed by (t+k)). Assume as well that $h_{t+k} = h_t$. Equation (9) thus reduces to

$$p_{t+k} - p_t \;\; = \;\; m_{t+1} - m_t \tag{11}$$

which implies that

$$p_{t+k} \;\; = \;\; p_t + \Delta_m \tag{12}$$

where $\Delta_m = m_{t+1} - m_t$ is the shock to the nominal money supply in period $t+1$.

Subtracting (8) from itself advanced k periods we have

$$q_{t+k} - q_t \;\; = \;\; p_{t+k} - p_t + \pi_{t+k} - \pi_t. \tag{13}$$

Since we can assume that any adjustment of the real exchange rate to the monetary shock will be reversed by $t+k$ (so that $q_{t+k} = q_t$), substitution of (11) into (13) yields the following expressions for the change in the nominal exchange rate from period t to period $t+k$:

$$\pi_{t+k} - \pi_t \;\; = \;\; -(m_{t+1} - m_t) \tag{14}$$

$$\pi_{t+k} \;\; = \;\; \pi_t - \Delta_m \tag{15}$$

Note that we are also assuming here for convenience that the technologically determined long-run equilibrium real exchange rate is unchanged. It should be evident that all the results here will follow identically for a shock to h except, of course, that the effects of a shock to h will be in the opposite direction to the effects of a shock to m.

An increase in the nominal money supply (or equivalent reduction in the demand for money) leads to an equi-proportional rise in the price level and fall in the nominal exchange rate once full adjustment has occurred in period $t+k$. The analysis of overshooting requires that we determine π_{t+1} and compare it with π_{t+k}—if $\pi_{t+1} < \pi_{t+k}$ there is overshooting.

Under less-than-full-employment conditions with constant technology domestic output will respond to changes in the domestic real interest rate and to changes in the logarithm of the real exchange rate. This relationship can be expressed in terms of the deviations of real output and the real interest rate from their full-employment levels as follows:

$$y_t - \hat{y}_t \;\; = \;\; -\phi\,(r_t - \hat{r}_t) - \theta\,(q_t - \hat{q}_t) \tag{16}$$

where ϕ is the absolute value of the interest semi-elasticity of the response of output to deviations of the interest rate from its full employment level $\hat{r}$ and and θ is the absolute value of the elasticity of output with respect to deviations of the real exchange rate from its full-employment level $\hat{q}$. The logarithm of equation (4) becomes

$$p_t \;=\; \alpha\,\tilde{p}_t - (1-\alpha)\,\pi_t. \tag{17}$$

The system of equations (6), (7), (8), (16) and (17) can now be solved at each time period for the five variables r_t, y_t, p_t, π_t and q_t, given the predetermined levels of m_t, h_t, $\tilde{p}_t$, $E\{p_{t+1}\}$ and $E\{q_{t+1}\}$. Equation (6) gives the condition that the supply of money equal the demand and (7) gives the relationship between domestic and foreign interest rates that must hold. Together, these two equations define asset equilibrium. Equation (16) gives the condition that the aggregate demand for output must equal the quantity produced and equations (8) and (17) give the definitions of the real exchange rate and the domestic price level.

Substitution of (16) into (6) to eliminate y_t and rearranging the terms yields

$$\begin{aligned} p_t \;=\;& m_t - h_t + (\epsilon\,\phi + \eta)\,r_t + \epsilon\,\theta\,q_t + \eta\,(E\{p_{t+1}\} - p_t) \\ & \qquad - \epsilon\,\hat{y}_t - \epsilon\,\phi\,\hat{r}_t - \epsilon\,\theta\,\hat{q}_t. \end{aligned} \tag{18}$$

Then substitution of (7) into this expression to eliminate the domestic interest rate yields the following equation for the equilibrium price level:

$$\begin{aligned} p_t \;=\;& m_t - h_t + (\epsilon\,\phi + \eta)\,r^*_t + (\epsilon\,\phi + \eta)\,\rho - (\epsilon\,\phi + \eta)\,(E\{q_{t+1}\} - q_t) \\ & + \epsilon\,\theta\,q_t + \eta\,(E\{p_{t+1}\} - p_t) - \epsilon\,\hat{y}_t - \epsilon\,\phi\,\hat{r}_t - \epsilon\,\theta\,\hat{q}_t. \end{aligned} \tag{19}$$

Now, subtract (19) from itself advanced one period, noting that $h_{t+1} = h_t$, $r^*_{t+1} = r^*_t$, $\hat{r}_{t+1} = \hat{r}_t$, $\hat{y}_{t+1} = \hat{y}_t$, and $\hat{q}_{t+1} = \hat{q}_t$, to obtain

$$\begin{aligned} p_{t+1} - p_t \;=\;& m_{t+1} - m_t - (\epsilon\,\phi + \eta)\,(E\{q_{t+2}\} - q_{t+1}) \\ & + (\epsilon\,\phi + \eta)\,(E\{q_{t+1}\} - q_t) + \epsilon\,\theta\,q_{t+1} - \epsilon\,\theta\,q_t \\ & + \eta\,(E\{p_{t+2}\} - p_{t+1}) - \eta\,(E\{p_{t+1}\} - p_t). \end{aligned} \tag{20}$$

The expected rate of change in the real exchange rate and the expected rate of inflation in period t are by assumption both zero. Assuming that a fraction λ of the adjustment to final equilibrium will occur between periods $t+1$ and $t+2$,

$$E\{p_{t+2} - p_{t+1}\} = \lambda\,(p_{t+k} - p_{t+1}),$$

which from (12) yields

$$E\{p_{t+2} - p_{t+1}\} = \lambda\,[\Delta m - (p_{1+1} - p_t)].$$

Equation (20) thus reduces to

$$\begin{aligned} p_{t+1} - p_t &= (1 + \eta\,\lambda)\,\Delta_m - (\epsilon\,\phi + \eta)\,(E\{q_{t+2}\} - q_{t+1}) \\ &\quad + \epsilon\,\theta\,(q_{t+1} - q_t) - \eta\,\lambda\,(p_{t+1} - p_t). \end{aligned} \tag{21}$$

From this point on the results depend upon whether agents regard the real exchange rate as a random walk or have information about its long-run equilibrium level.

3.1 The Random Walk Case

When agents view the real exchange rate as a random walk the expected rate of change in it between periods $t+1$ and $t+2$ will be zero. Taking this into account and utilizing (8), equation (21) reduces to

$$\begin{aligned} p_{t+1} - p_t &= (1 + \eta\,\lambda)\,\Delta_m + \epsilon\,\theta\,(q_{t+1} - q_t) - \eta\,\lambda\,(p_{t+1} - p_t) \\ &= (1 + \eta\,\lambda)\,\Delta_m + \epsilon\,\theta\,(p_{t+1} + \pi_{t+1} - p_t - \pi_t) \\ &\qquad - \eta\,\lambda\,(p_{t+1} - p_t), \\ &= (1 + \eta\,\lambda)\,\Delta_m + \epsilon\,\theta\,(p_{t+1} - p_t) + \epsilon\,\theta\,(\pi_{t+1} - \pi_t) \\ &\qquad - \eta\,\lambda\,(p_{t+1} - p_t), \end{aligned} \tag{22}$$

which further reduces to

$$(1 - \epsilon\,\theta + \eta\,\lambda)(p_{t+1} - p_t) = (1 + \eta\,\lambda)\,\Delta_m + \epsilon\,\theta\,(\pi_{t+1} - \pi_t). \tag{23}$$

From equation (17), taking into account that $\hat{p}_{t+1} = \hat{p}_t$, we have

$$p_{t+1} - p_t = -\,(1 - \alpha)(\pi_{t+1} - \pi_t) \tag{24}$$

which substituted into (23) yields

$$\begin{aligned} -(1 - \epsilon\,\theta + \eta\,\lambda)(1 - \alpha)(\pi_{t+1} - \pi_t) &= (1 + \eta\,\lambda)\,\Delta_m \\ &\quad + \epsilon\,\theta\,(\pi_{t+1} - \pi_t). \end{aligned} \tag{25}$$

Upon rearrangement this reduces to

$$-[(1 - \epsilon\,\theta + \eta\,\lambda)(1 - \alpha) + \epsilon\,\theta](\pi_{t+1} - \pi_t) = (1 + \eta\,\lambda)\;\Delta_m, \tag{26}$$

which implies that

$$
\begin{aligned}
\pi_{t+1} - \pi_t &= -\frac{(1+\eta\,\lambda)}{(1+\eta\,\lambda-\epsilon\,\theta)(1-\alpha)+\epsilon\,\theta}\,\Delta_m \\
&= -\frac{(1+\eta\,\lambda)}{(1+\eta\,\lambda)(1-\alpha)-\,\epsilon\,\theta\,(1-\alpha)+\epsilon\,\theta}\,\Delta_m \\
&= -\frac{(1+\eta\,\lambda)}{(1+\eta\,\lambda)(1-\alpha)+\alpha\,\epsilon\,\theta}\,\Delta_m \\
&= -\frac{1}{(1-\alpha)+\Psi\,\alpha}\,\Delta_m \;=\; -\frac{1}{1-\alpha\,(1-\Psi)}\,\Delta_m \qquad (27)
\end{aligned}
$$

where

$$
\Psi = \frac{\epsilon\,\theta}{1+\eta\,\lambda}. \qquad (28)
$$

Overshooting will occur if the coefficient of Δ_m in (27) is less than -1. Utilizing equation (15) we can rewrite (27) as

$$
\begin{aligned}
\pi_{t+1} - \pi_{t+k} - \Delta_m &= -\frac{1}{1-\alpha\,(1-\Psi)}\,\Delta_m \\
\pi_{t+1} - \pi_{t+k} &= \left[1 - \frac{1}{1-\alpha\,(1-\Psi)}\right]\Delta_m. \qquad (29)
\end{aligned}
$$

Substitution of (24) yields

$$
p_{t+1} - p_t = \frac{1-\alpha}{1-\alpha\,(1-\Psi)}\,\Delta_m. \qquad (30)
$$

Using (12), we can rewrite the above expression as

$$
\begin{aligned}
p_{t+1} - p_{t+k} &= -\left[1 - \frac{1-\alpha}{1-\alpha\,(1-\Psi)}\right]\Delta_m \\
&= -\frac{\alpha\,\Psi}{1-\alpha\,(1-\Psi)}. \qquad (31)
\end{aligned}
$$

And from the definition of the real exchange rate (8) we can express the deviation of the real exchange rate from its initial and final equilibrium as

$$
q_{t+1} - q_t = q_{t+1} - q_{t+k} = -\frac{\alpha}{1-\alpha\,(1-\Psi)}\,\Delta_m. \qquad (32)
$$

It is clear from (32) that, as long as output contains some non-traded components whose prices are fixed in the short-run, the real exchange rate will always devalue as a result of monetary expansion. The nominal exchange rate will devalue beyond, or overshoot, its long-run equilibrium level when the coefficient of Δ_m in (29) is negative. This happens when $\Psi < 1$ — that is, when $\epsilon\,\theta < 1 + \eta\lambda$. Here, $\epsilon\,\theta$ is the elasticity of the quantity of money demanded as a result of output expansion in the current period in response to a current period real exchange rate devaluation. The degree of overshooting will be greater, the smaller the expansion of current output in response to a current-period devaluation (i.e., the smaller is θ), the smaller the income elasticity of demand for money ϵ, the larger in absolute value the interest semi-elasticity of demand for money η and the larger is λ. The expansion of output as a result of the devaluation temporarily reduces the excess supply of money and thereby reduces the fraction of the total required price adjustment that occurs in the initial period, making future expected increases in the price level positive. A higher interest elasticity of demand for money increases the degree of overshooting because it makes the reduction in the demand for money as a result of this expected inflation of the price level along the path to long-run equilibrium larger, thereby increasing the excess supply of money that has to be eliminated by a fall in the exchange rate. If θ is zero so that no output expansion occurs in the current period, Ψ will also be zero, causing the price level to move to its long-run equilibrium level immediately and overshooting to necessarily occur—the exchange rate overshoots because it has to account in the short run for more than its ultimate share of the movement of the price level to long-run equilibrium, given the temporarily sticky prices of the non-traded components of output. For undershooting to occur, output must be sufficiently responsive to a current period real exchange rate devaluation to make $\epsilon\,\theta > 1 + \eta\lambda$.

3.2 The Informed Agents Case

When agents correctly perceive the equilibrium real exchange rate and expect the adjustment towards that equilibrium between $t+1$ and $t+2$ to be a fraction λ of the distance from equilibrium,

$$E\{q_{t+2} - q_{t+1}\} = -\lambda\,(q_{t+1} - q_t)$$

and equation (21) reduces to

$$\begin{aligned} p_{t+1} - p_t &= (1 + \eta\,\lambda)\,\Delta_m + \lambda\,(\epsilon\,\phi + \eta)\,(q_{t+1} - q_t) + \epsilon\,\theta\,(q_{t+1} - q_t) \\ &\quad - (\eta\,\lambda)\,(p_{t+1} - p_t) \\ &= (1 + \eta\,\lambda)\,\Delta_m + [(\epsilon\,\phi + \eta)\,\lambda + \epsilon\,\theta]\,(q_{t+1} - q_t) \\ &\quad - (\eta\,\lambda)\,(p_{t+1} - p_t) \end{aligned} \tag{33}$$

Substituting the definition of the real exchange rate (8), we obtain

$$\begin{aligned} p_{t+1} - p_t &= (1 + \eta\,\lambda)\,\Delta_m + [(\epsilon\,\phi + \eta)\,\lambda + \epsilon\,\theta]\,(p_{t+1} - p_t) \\ &\quad + [(\epsilon\,\phi + \eta)\,\lambda + \epsilon\,\theta]\,(\pi_{t+1} - \pi_t) \\ &\quad - (\eta\,\lambda)\,(p_{t+1} - p_t) \end{aligned} \tag{34}$$

which simplifies to

$$\begin{aligned} [1 - \epsilon(\phi\,\lambda + \theta)](p_{t+1} - p_t) &= (1 + \eta\,\lambda)\,\Delta_m \\ &\quad + [(\epsilon\,\phi + \eta)\,\lambda + \epsilon\,\theta]\,(\pi_{t+1} - \pi_t). \end{aligned} \tag{35}$$

Substitution of (24) yields

$$\begin{aligned} -[1 - \epsilon(\phi\,\lambda + \theta)](1 - \alpha)(\pi_{t+1} - \pi_t) &= (1 + \eta\,\lambda)\,\Delta_m \\ &\quad + [(\epsilon\,\phi + \eta)\,\lambda + \epsilon\,\theta]\,(\pi_{t+1} - \pi_t). \end{aligned} \tag{36}$$

which simplifies as follows,

$$-\{[1 - \epsilon(\phi\,\lambda + \theta)](1 - \alpha) + [(\epsilon\,\phi + \eta)\,\lambda + \epsilon\,\theta]\}(\pi_{t+1} - \pi_t) = (1 + \eta\,\lambda)\,\Delta_m$$

$$-\{1 + \eta\,\lambda - \alpha + \alpha\,[\epsilon\,(\phi\,\lambda + \theta)]\}\,(\pi_{t+1} - \pi_t) = (1 + \eta\,\lambda)\,\Delta_m, \tag{37}$$

implying that

$$\begin{aligned} \pi_{t+1} - \pi_t &= -\frac{(1 + \eta\,\lambda)}{1 + \eta\,\lambda - \alpha\,[1 - \epsilon\,(\phi\,\lambda + \theta)]}\,\Delta_m \\ &= -\frac{1}{1 - \alpha\,(\Omega - \Psi)}\,\Delta_m \\ &= -\frac{1}{1 - \alpha\,\Phi}\,\Delta_m \end{aligned} \tag{38}$$

where

$$\Omega = \frac{1 - \epsilon\,\phi\,\lambda}{1 + \eta\,\lambda} \qquad \text{and} \qquad \Phi = \Omega - \Psi = \frac{1 - \epsilon\,(\phi\,\lambda + \theta)}{1 + \eta\,\lambda}.$$

Using (15) we can write (38) as

$$\pi_{t+1} - \pi_{t+k} - \Delta_m = -\frac{1}{1 - \alpha\,\Phi}\,\Delta_m,$$

whence

$$\pi_{t+1} - \pi_{t+k} = \left[1 - \frac{1}{1 - \alpha\,\Phi}\right]\Delta_m, \tag{39}$$

Applying (24) converts (38) to

$$p_{t+1} - p_t \quad = \quad \frac{1-\alpha}{1-\alpha\,\Phi}\,\Delta_m \tag{40}$$

which using (12) becomes

$$\begin{aligned} p_{t+1} - p_{t+k} \quad &= \quad -\left[1 - \frac{1-\alpha}{1-\alpha\,\Omega+\alpha\,\Psi}\right]\Delta_m \\ &= \quad -\frac{\alpha\,(1-(\Omega-\Psi))}{1-\alpha\,(\Omega-\Psi)}\,\Delta_m \\ &= \quad -\frac{\alpha\,(1-\Phi)}{1-\alpha\,\Phi}\,\Delta_m. \end{aligned} \tag{41}$$

Finally, from (38), (40) and the definition of the real exchange rate (8),

$$q_{t+1} - q_t = q_{t+1} - q_{t+k} \quad = \quad -\frac{\alpha}{1-\alpha\,\Phi}\,\Delta_m. \tag{42}$$

Overshooting occurs when the coefficient of Δ_m in (39) is negative. This happens when Φ is positive, which occurs when $\epsilon\,(\phi\lambda+\theta) < 1$ —that is, when the output growth as a result of the devaluation is sufficiently small. If $\epsilon\,(\phi\lambda+\theta) > 1$, so that Φ is negative, the coefficient of Δ_m in (39) will be positive. The exchange rate will not fall all the way to its long-run equilibrium level π_{t+k} in period $t+1$ and undershooting will occur. The price level in period $t+k$ will be above the price level in period $t+1$ if $\Phi < 1$, which will occur whenever ϕ, θ and η are not all zero and the income elasticity of demand for money is positive.[3] Equation (42) implies that the real exchange rate will always fall below its initial and long-run equilibrium level as a result of a monetary shock.[4] Since $\Omega < 1$, it will always be the case that

$$1-\alpha\,\Phi = 1-\alpha\,(\Omega-\Psi) > 1-\alpha\,(1-\Psi)$$

so the influence of agents' having information that overshooting is taking place will always be to reduce the degree of overshooting.

[3] The condition reduces to

$$-\epsilon\,(\phi\,\lambda+\theta) < \eta\,\lambda.$$

[4] The requirement is that $\alpha\,\Phi < 1$, which occurs when

$$1-\alpha+\eta\,\lambda > -\alpha\,\epsilon\,(\phi\,\lambda+\theta).$$

In the case where the balance of trade and output are completely unresponsive to a real exchange rate shock and the prices of traded as well as non-traded components are rigid as a result of pricing to market, so that $\alpha = 1$, (42) becomes

$$q_{t+1} - q_t = q_{t+1} - q_{t+k} = -\frac{1 + \eta\,\lambda}{\eta\,\lambda}\;\Delta_m. \tag{43}$$

A positive (in absolute value) interest semi-elasticity of demand for money is required to bound the downward movement of the real exchange rate during the adjustment period.

While there is no informative way of modelling the details of the dynamics of adjustment, it is not unreasonable to assume that the path of adjustment from $t+1$ to $t+k$ will be monotonic—that is, that the most extreme overshooting will occur in the first period. To see this, simply increase the length of the time unit. Since all the parameters that control the degree of overshooting get larger with time, the degree of overshooting one year from the shock will be less than that one quarter from it, which will in turn be less than the degree of overshooting one month from the shock, and so forth. And any knowledge agents have about the degree of overshooting would be expected to increase as time passes, further moderating the decline in overshooting as the size of the time unit increases.

4 Some Plausible Quantitative Magnitudes

Assume a positive money supply shock and suppose that there is no response of the balance of trade to changes in the real exchange rate or of output to changes in real interest rates—i.e., $\theta = \phi = 0$. When agents perceive the real exchange rate to be a random walk the increase in the logarithm of the real exchange rate from period t to period $t+1$ equals $1/(1-\alpha)$ which, of course, exceeds unity. This follows from equation (27) where θ is set equal to zero. As α, the share of non-traded goods in domestic output, gets larger the degree of overshooting increases. If we assume that the share of non-traded components in total output is 0.8, the nominal exchange rate will fall by five times, and the real exchange rate by four times, the relative increase in the money supply in the first period and the price level will rise by exactly the relative increase in the money supply. When long-run equilibrium is established nominal exchange rate will have fallen in the same proportion as the nominal money supply increased and the real exchange rate will have returned to its pre-shock level. The price level will remain unchanged after period $t+1$, having already completed its entire adjustment.

Now suppose that agents know the period $t+2$ real exchange rate—that is, they know what the adjustment will be between $t+1$ and $t+2$. When there is no response of output to the real exchange rates and real interest rate (ϕ and θ are zero) the relative decline in the nominal exchange rate between period t and period $t+1$ becomes $(1+\eta\,\lambda)/(1+\eta\,\lambda-\alpha)$ times the relative increase in the nominal money supply.[5] If the interest semi-elasticity of demand for money, η, is also zero, the relative decline in the nominal and real exchange rates and the relative rise in the price level will be the same as in the random walk case. If we assume that a one percentage point increase in the interest rate will reduce the quantity of money demanded by 5%, so that $\eta = .05$, and that agents expect 50% of the adjustment to final equilibrium to occur between $t+1$ and $t+2$, so that $\lambda = .5$, the relative decline in the nominal exchange rate will be 4.56 times the relative increase in the nominal money stock. The price level will now rise by .91 of the relative increase in the nominal money supply and the real exchange rate will decline by 3.64. Agent's knowledge of the next period movement towards equilibrium modestly reduces the degree of overshooting. Suppose, however, that agents expect only 20% of the full adjustment to occur between $t+1$ and $t+2$. This will increase the relative decline in the respective nominal and real exchange rates to 4.91 and 3.85 times the relative increase in the money supply and increase the first period price increase to $.96\,\Delta_m$. The slower agents expect the adjustment to occur after $t+1$, the greater will be the degree of overshooting. Even if we increase η to .10, returning λ to .5, the respective numbers for the nominal and real exchange rate and the price level will still be -4.2, -3.36 and .84 times the relative increase in the nominal money supply. Suppose that in addition we allow the relative increase in output in response to a one percentage point decline in the real interest rate in the informed agents case to be 0.1, a seemingly large value. This only reduces the above three numbers to -3.62, -2.90 and .72 respectively. It has no effect in the random walk case. With these assumptions, the degree of overshooting is considerably reduced but still remains substantial.

Now let us extend the calculations to allow for a response of real output to the real exchange rate. Suppose we let the response vary as indicated by the left column in the table below. Assuming the same values for the other parameters as immediately above, the relative changes in the nominal and real exchange rate and price level as proportions of the relative change in

[5] Ψ becomes zero and

$$\Omega = \frac{1}{1+\eta\,\lambda}.$$

the nominal money supply are given by the remaining columns.

θ	Random Walk Nominal Exchange Rate	Random Walk Price Level	Random Walk Real Exchange Rate	Informed Agents Nominal Exchange Rate	Informed Agents Price Level	Informed Agents Real Exchange Rate
0.0	-5.00	1.00	-4.00	-3.62	0.72	-2.90
0.1	-3.62	0.72	-2.90	-2.84	0.57	-2.27
0.2	-2.84	0.57	-2.27	-2.33	0.47	-1.87
0.5	-1.72	0.34	-1.28	-1.52	0.33	-1.22
1.0	1.04	0.21	-0.83	-0.96	0.19	-0.77
1.5	0.74	0.15	-0.60	-0.70	0.14	-0.56

Thinking in terms of monthly data, it would seem unreasonable for a 10% devaluation of the real exchange rate to increase output by more than 2% within the current month. Were this to be true we would expect a real devaluation of of somewhat more than twice the percentage increase in the nominal money supply if agents regard the real exchange rate as a random walk and a bit less than twice the percentage increase in the nominal money stock if agents expect half the disequilibrium to be eliminated in the next month. The nominal exchange rates will decline by much larger percentages. Even if we assume that a 10 percent devaluation would result in a 5 percent increase in output the fall in the real exchange rate will still be substantially greater than the percentage increase in the money supply and the nominal exchange rate will still overshoot by a factor of one-half. These are quite large magnitudes. Moreover, if we think of the nominal exchange rate response within days or a week or two of the monetary expansion the degree of overshooting will be much larger.

5 Some Important Implications

The results above have important implications for how monetary policy is conducted and for interpreting and quantifying the effects of monetary policy using statistical models. It is difficult to believe that output adjustments to real exchange rate and real interest rate movements can occur immediately. So shocks to the demand or supply of money—i.e., to the excess supply of money—are certain to have very substantial overshooting effects on the exchange rate in the very short run—say, a week. This will be the case regardless of the mechanism and extent of long-run adjustment. The shorter the initial observation period, the greater the overshooting.

In a world where there are stochastic shocks to the demand function for money and where central banks face the usual political pressures to maintain stability of the financial system, it is difficult to imagine that the authorities would not move quickly to smooth overshooting exchange rate effects of fluctuations in the demand for money by providing appropriate adjustments of the stock of high-powered money. Moreover, any attempt by the central bank to conduct significant activist monetary policy to combat recession or inflation will immediately result in overshooting nominal exchange rate adjustments. It would seem from their concern with keeping markets 'orderly' that central banks would abhor overshooting nominal exchange rate movements.

It is clear that the routine maintenance of 'orderly markets' will result in loss of central bank control over the price level, since the authorities will end up responding to demand for money shocks, financing whatever quantity of money the public wants to hold. Attempts by the central bank to influence agents' inflation expectations by talk and symbolic movements in the bank rate take on considerable importance in such an environment—low expected inflation will cause low actual inflation to be a natural consequence of orderly-markets-style exchange rate smoothing. The consequences of constant money growth in an environment where there are stochastic shocks to the demand for money are obvious. This probably explains why small-country central bankers' eyes gloss over when confronted with arguments supporting a Friedman rule.

If central banks abhor overshooting exchange rate movements and conduct orderly-markets/exchange-rate-smoothing policies as a result, we will rarely observe exchange rate overshooting in the data. Observed shocks to the money supply will represent central bank financed shocks to the demand for money, not exogenous shocks motivated by central bank activism. This endogeneity of the money supply will seriously hamper attempts to find and quantify a central bank's policy stance through time. Moreover, exchange smoothing policies create links between countries' monetary policies that render attempts to use closed economy analysis nugatory, even for relatively large countries like the United States.

What kind of assumptions do we have to buy into to make this analysis valid? Really only one—that portflolios adjust faster than commodity markets. Although a very rudimentary model is used in the analysis above, it is hard to imagine that the results are model specific. Any model in which output adjusts to real interest rate and real exchange rate changes and money is reasonably neutral in the long run will suffice. Such output adjustments in response to real exchange rate and interest rate movements ensure the existence of a long-run adjustment mechanism. When these output adjustments are non-existent or perverse, the case for overshooting and its implications for central bank policy is even stronger.

The implications of the model also do not depend on the reasons for price stickiness—any detailed formulation of the behavior of price-setting agents will deliver the results as long as prices do not adjust immediately. In this respect, limitations on the response of the domestic price level to exchange rate movements due to 'pricing to market' will weaken the bounds on the nominal exchange rate in the face of exogenous money shocks and increase overshooting.[6] Overshooting will necessarily occur when portfolios adjust faster than everything else.

[6] These models and their implications are surveyed in Michael B. Devereux, "Real Exchange Rates and Macroeconomics: Evidence and Theory," *The Canadian Journal of Economics*, Vol. 30, No. 4a, November, 1997, 773–808.