# MODULE 2

# Interest Rates and Asset Values

This lesson starts by explaining what interest rates are and then introduces the concept of present value. This leads to discussions of the meaning of the term assets and of the relationship between the prices of assets and the rate of interest. The effects of unanticipated inflation on the wealth positions of debtors and creditors is then analyzed and the concept of the realized real interest rate is introduced. The lesson then considers the effects of anticipated inflation on market interest rates and explains the relationship between real and nominal interest rates and the expected rate of inflation. After a brief discussion of the concept of indexing, the relationships between interest rates and actual inflation rates in a number of countries are plotted and analyzed.

## 1. Interest Rates

The interest rate is the price of using funds today rather than in the future—indirectly, it is the price of having goods now rather than in the future. Interest rates are always positive because capital is productive—by not consuming an amount today and adding it instead to the stock of capital one can have tomorrow both that additional capital and the income produced by it. No one will lend funds at less than the interest rate that could be obtained by purchasing real capital goods instead.

A sum $A_0$ invested today at an annual interest rate of $r$ percent per year will grow to

$$A_1 = A_0(1 + r) \tag{1}$$

by the end of the year. And a sum $A_1$ to be received at the end of the year will be worth

$$A_0 = \frac{A_1}{(1 + r)} \tag{2}$$

today. $A_0$ is the discounted or present value of $A_1$ and $r$ is the discount rate. The *right to receive* $A_1$ dollars at the end of the year is an *asset* for which one would be willing to pay $A_0$ dollars today.

In general, a sum $A_0$ will accumulate to

$$A_n = A_0(1 + r)^n \tag{3}$$

after $n$ years. And the present value of $A_n$ to be received in $n$ years is

$$A_0 = \frac{A_1}{(1 + r)^n}. \tag{4}$$

## 2. Interest Rates and Asset Values

The value of an asset consisting of the right to receive a set of future annual payments, $R_1$, $R_2$, $R_3$, .... ,$R_n$ is the present value of those payments, which is

$$PV = \frac{R_1}{(1 + r)} + \frac{R_2}{(1 + r)^2} + \frac{R_3}{(1 + r)^3} + .... + \frac{R_n}{(1 + r)^n}. \tag{1}$$

In the case where the future payments are constant (i.e., $R_i = R$ for all $i$), this expression becomes

$$PV = \Omega R \tag{2}$$

where

$$\Omega = a + a^2 + a^3 + .... + a^n \tag{3}$$

with $a = 1/(1+r)$. A standard result from the mathematics of geometric series is

$$1 + \Omega = 1 + a + a^2 + a^3 + .... + a^n = \frac{1}{1-a}. \tag{4}$$

This implies that $\Omega = 1/r$ which, in turn, implies that

$$PV = \frac{R}{r} \tag{5}$$

An asset with a fixed coupon yield in perpetuity and no provision for repayment of principal is called a *consol*. Its present value is given by (5). A more typical bond yields an annual coupon yield plus repayment of the principal in the last year. Its value is

$$PV = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + .... + \frac{R+A}{(1+r)^n} \tag{6}$$

where $A$ is the principal repaid. Another type of bond is a *treasury bill* which has no coupon yield and pays a fixed amount at the end of a specific period, usually 30 or 90 days. Its value is given by

$$PV = \frac{T}{1+r} \tag{7}$$

where $T$ is the amount to be received and $r$ is now interpreted as the interest rate for the period to maturity of the bill.

It should be noted that the present value of each of the above bonds is inversely related to the interest rate—higher bond prices and lower interest rates go hand in hand. This is the case with all debt instruments that have fixed nominal coupon yields and fixed nominal values at maturity.

## 3. Effects of Unanticipated Inflation: Realized Real Interest Rates

Inflation that is unanticipated by borrowers and lenders reduces the real amount of principal and interest repaid on loans whose principal and interest is fixed in nominal amount. This redistributes income from lenders (creditors) to borrowers (debtors). For example, unanticipated inflation reduces the real value of nominally fixed pensions and the real value of mortgage payments faced by homeowners. Unanticipated deflation has the opposite effect.

The real interest rate actually realized on loans whose principal and interest is fixed in nominal amount is approximately equal to the agreed-upon interest rate per annum minus the year-to-year percentage rate of inflation.

Distribution effects of both unanticipated and fully anticipated inflation can also occur through the tax system. When the tax system is progressive and taxes are based on nominal incomes, inflation of prices, nominal wages and nominal incomes puts individuals in higher tax brackets. This increases the real amount of taxes paid. Also, when firms' depreciation allowances for tax purposes are based in historic cost, inflation reduces the real amount of depreciation allowances they can include in measuring their costs and their taxable income. As a result, the real returns to shareholders are reduced.

## 4. Effects of Fully Unanticipated Inflation: The Fisher Equation

When inflation is fully anticipated borrowers and lenders will add to the real interest rate they agree upon an inflation premium equal to the expected rate of inflation. Thus

$$i = r + \tau^e \tag{1}$$

were $i$ is the market (nominal) interest rate, $r$ is the *contracted* or *ex ante real interest rate* and $\tau^e$ is the expected rate of inflation. This is the Fisher equation.

Corresponding to equation (1) is an expression involving the *realized* or *ex post real interest rate* and the actual rate of inflation.

$$i = r_R + \tau \tag{2}$$

where $r_R$ is the realized real interest rate and $\tau$ is the actual inflation rate. This equation together with (1) yields

$$r_R - r = \tau^e - \tau \tag{3}$$

When the actual inflation rate exceeds the expected inflation rate the realized real interest rate falls below the contracted real interest rate—wealth is redistributed from the lender to the borrower. When the actual inflation rate falls short of the expected rate the opposite occurs. Wealth is redistributed only when the inflation rate turns out to be greater or smaller than anticipated.

## 5. Indexing

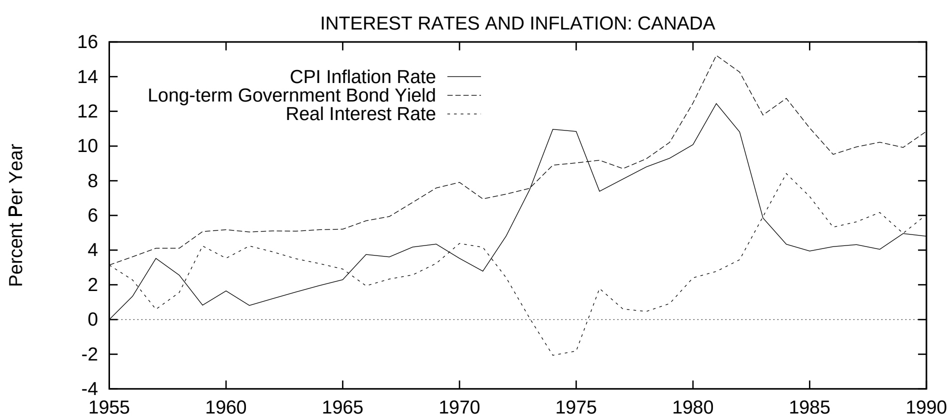
One way to avoid the wealth redistribution effects of unanticipated inflation is to hold one's wealth in assets such as machinery, real estate and equity shares whose nominal earnings vary roughly in proportion with the price level. The desirability of holding wealth in nominally fixed assets for certain purposes and of making other nominally fixed contracts, however, makes it costly to forego these types of financial commitments in order to avoid the distribution effects of inflation.

Indexing is a way of reducing the wealth redistribution effects of unanticipated inflation (and anticipated inflation as well). Indexed bonds and contracts have their monetary values expressed in constant dollars and, as a result, their real values are unaffected by inflation. To express magnitudes in constant dollars, however, one must use some index of prices. The same index will not always be appropriate for all signatories to a contract a since each individual consumes a different bundle of goods—one of the parties may be worse off with an indexed contract than a non-indexed one. Indexing thus carries its own risks. This is one reason why indexing tends to be widespread when high and variable inflation rates are being experienced and rare when inflation rates tend to be low.

## 6. Inflation and Interest Rates: Some Evidence

Figures 6.1 and 6.2 plot government long-term bond yields, the CPI inflation rates and the associated *realized* real interest rates for Canada and the United States. In both cases an upward trend in the inflation rates between the late 1960s and 1980 was accompanied by an upward trend in nominal interest rates. And a decline in the inflation rates in the 1980s was accompanied by a decline in nominal interest rates. This suggests that expected inflation rates adjusted to reflect actual inflation rates and became incorporated into nominal interest rates as the theory developed above predicts. Furthermore, the peaks in the inflation rates in 1973-74 and 1979-80 were accompanied by declines in the realized real interest rates, reflecting the tendency for expected inflation rates to adjust to actual inflation rates with a lag. Since there is no evidence of a trend in realized real interest rates for the years before 1980 it is reasonable to believe that there was also no trend in contracted real interest rates. Realized real interest rates tended to be high in the 1980s relative to the 1960s. This was due either to a continuation of high expected inflation rates in the face of declines in actual inflation rates or to high contracted real interest rates.

**FIGURE 6.1:**




The experience of Italy, shown in Figure 6.3, is similar to that of Canada and the United States. The New Zealand evidence, shown in Figure 6.4, indicates a somewhat different pattern but one also consistent with the theory. There were three peaks in the inflation rate and a sharp temporary downward movement in 1983-84 followed by a resumption of high inflation. Unlike the other three countries, the nominal interest rate remained high after 1980 in anticipation by asset holders of a continuance of high inflation.

**FIGURE 6.2:**

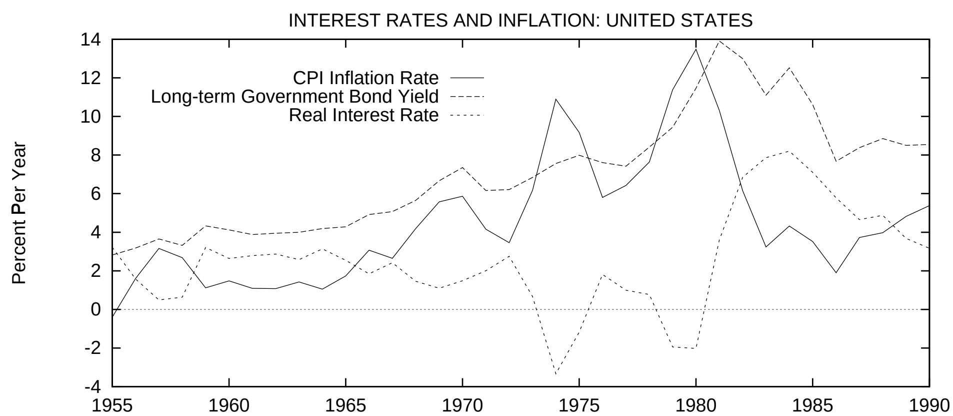



**FIGURE 6.3:**

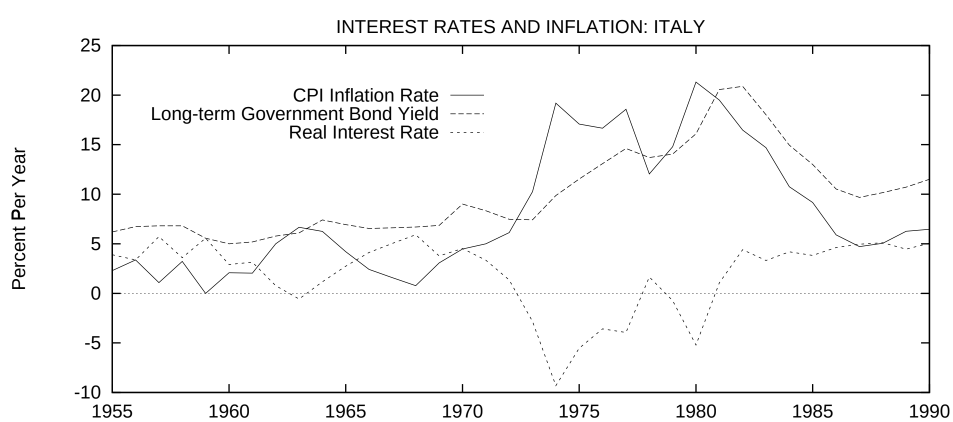


**FIGURE 6.4:**

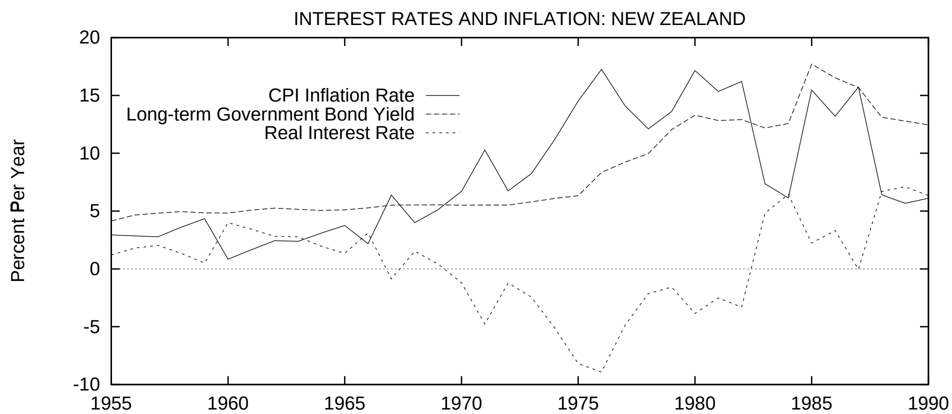
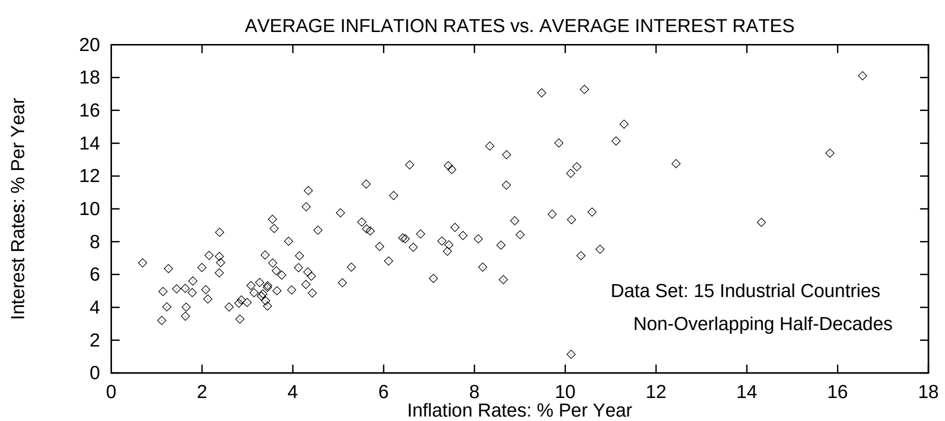



Figure 6.5 presents a scatter-plot of the relationship between 5-year average bond yields and 5-year average inflation rates. The 5-year periods are the non-overlapping half-decades 1955-59, 1960-64, 1965-69, 1970-74, 1975-79, 1980-84, and 1985-89, and the data are for a set of countries consisting of Austria, Belgium, Denmark, France, Germany, Italy, Norway, Sweden, Switzerland, United Kingdom, Japan, Australia, New Zealand, Canada and the United States. The evident positive relationship suggests that contracted real interest rates have tended to be similar across countries and through time—differences in nominal interest rates in large part reflect differences in rates of inflation.

**FIGURE 6.5:**



## Study Questions

1. You won't have studied economics very long before someone who is aware of your intellectual background will ask you whether they should renew their mortgage for five years at the current five-year rate of, say, 9 percent or for one year at the current one-year rate of, say, 7 percent. The issue, of course, is whether interest rates will rise in a couple of years so that being locked into a contract for five years at 9 percent will be advantageous, or whether interest rates will fall in a year or two so that the one-year and five-year rates at that time will be less than 7 percent. Why might interest rates on five-year mortgages be higher than interest rates on one-year mortgages? What is the most likely factor that will determine whether interest rates rise or fall substantially over the next few years?

2. Many years ago a 4th year economics student was sitting in the dentist chair getting his teeth fixed. At one point the dentist said, "Tell me Jack. You're an honours economics student. Since 1929 the prices of nearly everything we buy have doubled. But the interest rate, which is the price of money, hasn't doubled. Why?" What should have been Jack's reply? Under what circumstances would the interest rate have doubled?

3. Suppose that your father retires early at age 55 and is supporting the family on income from investments. One day he happily announces at the breakfast table that "things are looking up with the increase in interest rates because we now earn $40,000 per year from our accumulated assets instead of the $30,000 we were earning a couple of years ago." Should you risk getting yelled at by saying something? If you want to run that risk, what should you say?

## References

Robert J. Barro, *Macroeconomics*, 4th Edition, John Wiley and Sons, 1993, pp. 172-182. (Quite advanced)

John R. Durbin, *Precalculus*, John Wiley and Sons, 1986, Ch. 14, especially p. 525. (Provides Math Background)

N. Gregory Mankiw, *Macroeconomics*, Worth Publishers, 1992, pp. 155-158, or 3rd. Edition, 1997, pp. 159-164.

Richard G. Lipsey, Douglas D. Purvis and Peter O. Steiner, *Economics*, 7th Canadian Edition, HarperCollins, 1991, pp. 564-566 and 724-25.

Richard G. Lipsey, Paul N. Courant and Douglas D. Purvis, *Economics*, 8th Canadian Edition, HarperCollins, 1994, pp. 529-535 and 682-86.

Jeffrey D. Sachs and Felipe Larrain, *Macroeconomics in the Global Economy*, Prentice Hall, 1993, pp. 349-352.