ECO 426 (Market Design) - Lecture 11

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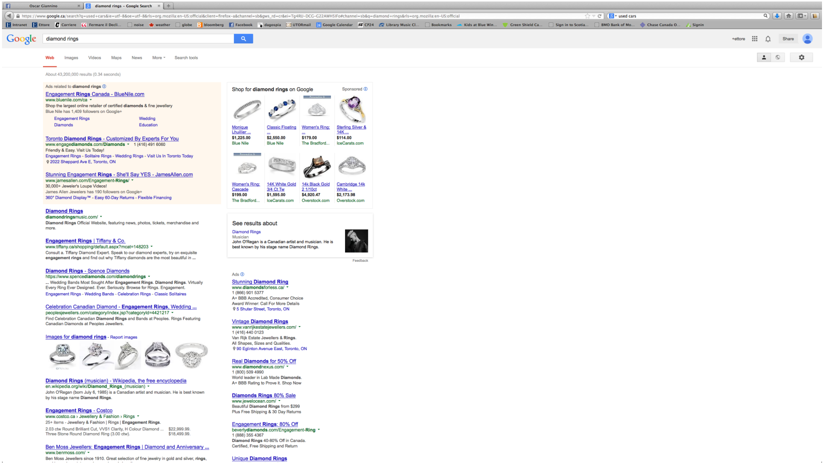
Sponsored search auctions

- Google, Yahoo etc.. sell ad spaces linked to keyword searches



- Google advertising revenue: USD 42.5bn in 2012

sponsored search auctions

- Bidders (advertisers) submit bids on a keyword (e.g. diamond ring, used cars, legal services, auto insurance, etc.)
- A bid typically is a
  - price per "click," or
  - price per "impression,"
- Multiple ads are shown after each keyword search (multi-unit auction)
  - Bidders only submit one bid (unit demand)
  - Bids order determine the ad position within the web-page
- An auction is for **one** query of **one** keyword

efficient allocation

Example:

- Two positions on a web-page:
  - *A* generates 200 clicks per-day,
  - *B* generates 100 clicks per-day.
- Three advertisers, have different per-click "values"
  - Firm 1 value is \$10 per-click
  - Firm 2 value is \$4 per-click
  - Firm 3 value is \$2 per-click
- Efficient allocation?
  - Firm 1 gets position *A*
  - Firm 2 gets position *B*
- Total value $= 200 \times \$10 + 100 \times \$4 = \$2,400$

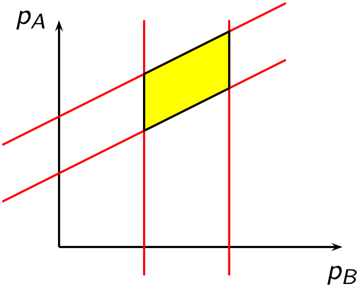
competitive equilibrium prices

- In a "competitive equilibrium" the two position prices, p_A and p_B, and such that demand = supply
  - Exactly one firm demands position A and one firm demands position B
- Competitive equilibrium prices?
  - $p_A = \$4$ and $p_B = \$2$ YES
    - Firm 3 demands nothing
    - Firm 2 demands B
    - Firm 1 demands A $\quad (10-4)*200 > (10-2)*100$
  - $p_A = \$5$ and $p_B = \$3$ YES
    - Firm 3 demands nothing
    - Firm 2 demands B
    - Firm 1 demands A
  - $p_A = \$7$ and $p_B = \$3$ NO
    - Firm 3 demands nothing
    - Firm 2 demands B
    - Firm 1 demands B $\quad (10-7)\times 200 < (10-3)\times 100$

competitive equilibrium prices

Finding all competitive equilibrium prices

- Competitive equilibrium allocation are efficient
  - Firm 3 must demand nothing
    - $p_A, p_B \geq 2$
  - Firm 2 must demand position B
    - $p_B \leq \$4$
    - $(4 - p_B) \times 100 \geq (4 - p_A) \times 200 \Rightarrow p_A \geq 2 + p_B/2$
  - Firm 1 must demand position A
    - $p_A \leq 10$
    - $(10 - p_A) \times 200 \geq (10 - p_B) \times 100 \Rightarrow p_A \leq 5 + p_B/2$



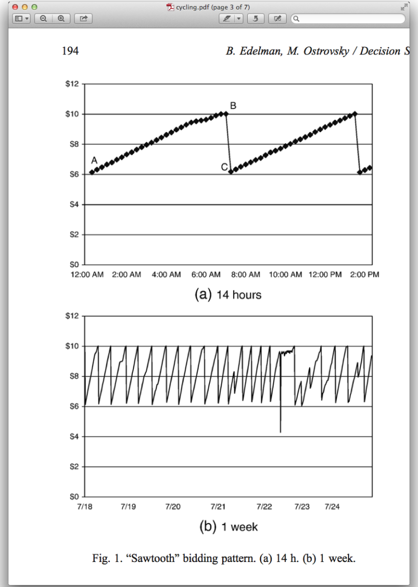
pay-your-bid auction

- Example. Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
- Pay-your-bid auction
  - Firm 3 bids up to \$2 per-click
  - Firm 2 can get position B for \$2.01
  - Firm 1 can get position A for \$2.02
  - Firm 2 would want to top 1's offer and get A (e.g. \$2.03)

  ⋮

  - Price escalates until it reaches \$3.01 at which point firm 2 wants to revert back to paying \$2.01 for position B
  - Firm 1 wants to lower its bid to \$2.02
  - ....start over....
- pay-your-bid auctions were used in the 1990's (Overture, Yahoo, MSN)

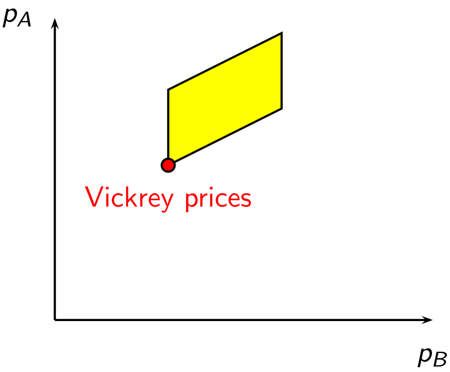
pay-your-bid auction



- Example. Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
- In a Vickrey auction it is a dominant strategy to bid own valuation. (i.e. Firm 1 bids \$10, firm 2 bids \$4 and firm 3 bids \$2)
  - Allocation: Firm 1 gets A firm 2 gets B, firm 3 gets nothing (efficient)
  - Prices:
    - Firm 3 pays nothing
    - Firm 2 displaces firm 3 for 100 clicks $\Rightarrow$ pays $\$2 \times 100 = \$200$
    - Firm 1 displaces firm 3 for 100 clicks and firm 2 for 100 clicks $\Rightarrow$ pays $\$2 \times 100 + \$4 \times 100 = \$600$
  - Revenue $= \$800$

Vickrey auction

- Vickrey prices are **lowest** competitive equilibrium prices



Google, generalized second price auction

- Google GSP auction
  - Bidders submit per-click bids
  - Ad positions are allocated following the order of bids (top bidder gets top position, second bidder gets second position, . . .)
  - Each bidder pays a price equal to the next lower bid (i.e. top bidder pays second highest bid, second bidder pays third highest bid, . . .)

GSP auction equilibria

- Bidding own value is **NOT** a dominant strategy
  - Example: Two positions 100 and 200 clicks. Value 10 per click. If competing bids are 5 and 9, winning second position at price 5 generates more profit than winning first position at price 9.
- Bidding own value **can** be an equilibrium
  - Example. Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
  - The three firms bidding own value is a Nash equilibrium
  - Allocation is efficient
  - Prices are $p_A = \$4$, $p_B = \$2$ ($>$ Vickrey prices)
  - Revenue $= \$1,000 >$ Vickrey

GSP auction equilibria

- Example. Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
  - Profile of bids (\$2, \$3, \$10) is also a Nash equilibrium (Vickery prices) - revenue \$800
  - Profile of bids (\$3, \$5, \$8) is also a Nash equilibrium - revenue \$1,300
  - Profile of bids (\$3, \$7, \$3.30) is also a Nash equilibrium (inefficient)

General model

- Model
  - K positions with click rate $x_1 > x_2 > \cdots > x_K$
  - N bidders with per-click values $v_1 > v_2 > \cdots > v_N$
  - Full information
  - Auction format: GSP
- Efficiency: an allocation is efficient if it is “positive assortative” (i.e. the highest **value** bidder gets top position, and so on...)

- A strategy profile is a vector of bids $b = (b_1, \ldots, b_N)$
  - b^k is the k^{th} highest bid
  - v^k is the per-click value of the k^{th} highest bidder
- Given a strategy profile b, payoff to the k^{th} highest bidder is

$$v^k x^k - b^{k+1} x_k = (v^k - b^{k+1}) x_k$$

- A strategy profile b is a **Nash equilibrium** if for each k

$$(v^k - b^{k+1}) x_k \geq (v^k - b^{m+1}) x_m \quad \text{for } m > k$$

$$(v^k - b^{k+1}) x_k \geq (v^k - b^m) x_m \quad \text{for } m < k$$

- Definition: An equilibrium is local envy-free if no bidder can increase his payoff by "swapping" bids with the player who bid just above him.
    - k^{th} highest bidder swapping bids with $k-1^{th}$ highest bidder
        - wins $k-1^{th}$ position (click rate x_{k-1})
        - pays the k^{th} highest bid b^k
        - payoff $(v^k - b^k)x_{k-1}$
    - local envy free requires

$$(v^k - b^{k+1})x_k \geq (v^k - b^k)x_{k-1}$$

local envy-free and stability

- Two-sided one-to one matching analogy
  - advertisers: payoff = profit
  - positions: payoff = revenue
    - Given a strategy profile b, position $k-1$ is assigned to the $k-1^{th}$ highest bidder who pays the k^{th} highest bid (b^k). Revenue
$$b^k x_{k-1}$$
- If an equilibrium is **not** local envy-free
$$(v^k - b^{k+1})x_k < (v^k - b^k)x_{k-1}$$
  - Paying a little more than b^k for position $k-1$, the bidder is still strictly better off, and position $k-1$ gets more revenue
  - k^{th} bidder and position $k-1$ block the matching (unstable)
- local envy-free is sufficient for stability

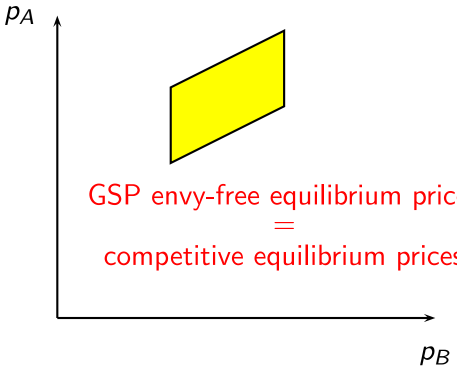
- Stable assignment (specify both matching and payments "transfers")
  - matching is efficient: higher value bidders get higher positions (positive assortative matching)
  - position prices $(p_1, \ldots, p_K)$ satisfy

$$(v_k - p_k)x_k \geq (v_k - p_m)x_m \quad \text{for all } m$$

    otherwise "bidder k" and position m block the assignment

- same as competitive equilibrium
  - efficient allocation
  - demand=supply

- Theorem: The outcome of a locally envy-free equilibrium of the GSP auction is a stable assignment (= competitive equilibrium allocation). Further, provided $N > K$, any stable assignment (=competitive equilibrium allocation) is an outcome of a locally envy-free equilibrium of the GSP auction.



equilibrium ranking

- Theorem: There is a "bidder-optimal" competitive (GSP envy-free) equilibrium and a "seller-optimal" competitive (GSP envy-free) equilibrium.
  - "bidder-optimal" equilibrium generates same revenue as Vickrey auction
  - "seller-optimal" equilibrium