

ECO 426 (Market Design) - Lecture 11

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Sponsored search auctions

- Google, Yahoo etc.. sell ad spaces linked to keyword searches

The screenshot shows a Google search for "diamond rings". The organic results on the left include:

- Engagement Rings Canada - BlueNix.com**: Shop the largest online retailer of certified diamonds & fine jewelry. BlueNix has 1,453 reviews on Google+.
- Toronto Diamond Rings - Customized By Experts For You**: Friendly & Easy. Visit Us Today! Engagement Rings, Solitaire Rings, Wedding Rings - Visit Us in Toronto Today. 9 2022 Sheppard Ave E, Toronto, ON.
- Buying Engagement Rings - She's So YER - JamesAllen.com**: 35,000+ Jewelers' Choice Winner! James Allen Jewelers has 101 followers on Google+.
- Diamond Rings**: diamondrings.com - Diamond Rings Offer: Solitaires, Wedding rings, phobias, labrets, merchandise and more.
- Engagement Rings | Tiffany & Co.**: Consult a Tiffany Diamond Expert. Speak to our diamond experts, by an exclusive engagement rings and find out why Tiffany diamonds are the most beautiful in...
- Diamond Rings - Source Diamonds**: Wedding Bands Must Bought After Engagement Rings. Diamond Rings, Virtually every Ring Fine Diamond, Fine Jewelry, Services for Rings, Engagement Rings, Engagement Rings, Wedding Bands, Celebration Rings - Classic Solitaires.
- Collaborator: Canadian Diamond - Engagement Rings, Wedding ...**: Find Collaborator Canadian Diamond Rings and Bands at Peoples. Rings Featuring Canadian Diamonds of Peoples Jewellers.
- Icons for diamond rings - heart icons**: [Images of various diamond rings]
- Diamond Rings Insider - Wikipedia, the free encyclopedia**: John O'Hagan is a Canadian artist and musician. He is best known for his stage name Diamond Rings.
- Engagement Rings - Costco**: 20+ Items - Jewelry & Fashion | Rings | Engagement Rings. 222 John Road - Brentwood City Mall, Chicago, IL. \$22,899.99. Three Stone Round Diamond Ring (\$28.00).
- See More Jewellers, Engagement Rings | Diamond and Anniversary ...**: Best Ring Jewellers along 915. Great selection of the jewelry in gold and silver. Rings.

The sponsored section on the right is titled "Shop for diamond rings on Google" and features:

- A grid of diamond ring images with prices and retailer names.
- A table of diamond prices:

Women's Ring	14K White Gold	14K Black Gold	Convertible 14K
Diamond	\$2,527.00	\$2,127.00	\$2,174.00
Men's Ring	\$1,899.00	\$1,699.00	\$1,773.96

Below the table, it says "See results about Diamond Rings" and lists John O'Hagan as a Canadian artist and musician.

- Google advertising revenue: USD 42.5bn in 2012

- Bidders (advertisers) submit bids on a keyword (e.g. diamond ring, used cars, legal services, auto insurance, etc.)
- A bid typically is a
 - price per “click,” or
 - price per “impression,”
- Multiple ads are shown after each keyword search (multi-unit auction)
 - Bidders only submit one bid (unit demand)
 - Bids order determine the ad position within the web-page
- An auction is for **one** query of **one** keyword

Example:

- Two positions on a web-page:
 - A generates 200 clicks per-day,
 - B generates 100 clicks per-day.
- Three advertisers, have different per-click “values”
 - Firm 1 value is \$10 per-click
 - Firm 2 value is \$4 per-click
 - Firm 3 value is \$2 per-click
- Efficient allocation?
 - Firm 1 gets position A
 - Firm 2 gets position B
- Total value = $200 \times \$10 + 100 \times \$4 = \$2,400$

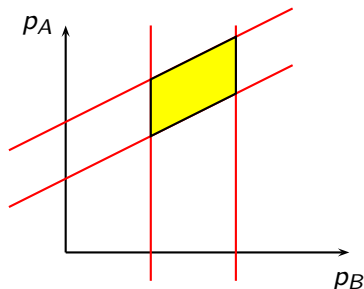
competitive equilibrium prices

- In a “competitive equilibrium” the two position prices, p_A and p_B , and such that demand = supply
 - Exactly one firm demands position A and one firm demands position B
- Competitive equilibrium prices?
 - $p_A = \$4$ and $p_B = \$2$ **YES**
 - Firm 3 demands nothing
 - Firm 2 demands B
 - Firm 1 demands A $(10 - 4) * 200 > (10 - 2) * 100$
 - $p_A = \$5$ and $p_B = \$3$ **YES**
 - Firm 3 demands nothing
 - Firm 2 demands B
 - Firm 1 demands A
 - $p_A = \$7$ and $p_B = \$3$ **NO**
 - Firm 3 demands nothing
 - Firm 2 demands B
 - Firm 1 demands B $(10 - 7) * 200 < (10 - 3) * 100$

competitive equilibrium prices

Finding all competitive equilibrium prices

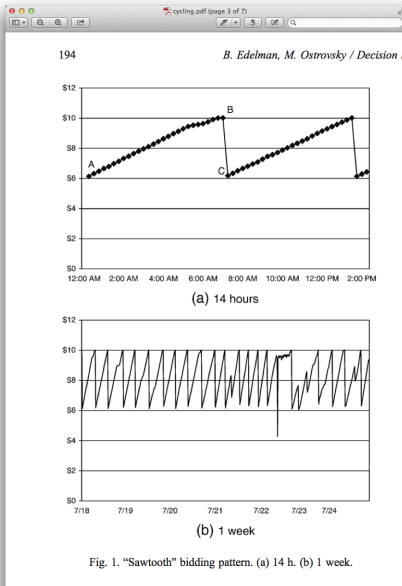
- Competitive equilibrium allocations are efficient
 - Firm 3 must demand nothing
 - $p_A, p_B \geq 2$
 - Firm 2 must demand position B
 - $p_B \leq \$4$
 - $(4 - p_B) \times 100 \geq (4 - p_A) \times 200 \Rightarrow p_A \geq 2 + p_B/2$
 - Firm 1 must demand position A
 - $p_A \leq 10$
 - $(10 - p_A) \times 200 \geq (10 - p_B) \times 100 \Rightarrow p_A \leq 5 + p_B/2$



pay-your-bid auction

- **Example.** Two positions: *A* generates 200 clicks per-day, *B* generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
- Pay-your-bid auction
 - Firm 3 bids up to \$2 per-click
 - Firm 2 can get position *B* for \$2.01
 - Firm 1 can get position *A* for \$2.02
 - Firm 2 would want to top 1's offer and get *A* (e.g. \$2.03)
 - ⋮
 - Price escalates until it reaches \$3.01 at which point firm 2 wants to revert back to paying \$2.01 for position *B*
 - Firm 1 wants to lower its bid to \$2.02
 -start over....
- pay-your-bid auctions were used in the 1990's (Overture, Yahoo, MSN)

pay-your-bid auction

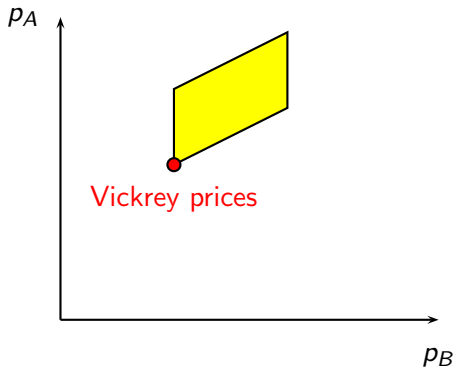


Vickrey auction

- **Example.** Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
- In a Vickrey auction it is a dominant strategy to bid own valuation. (i.e. Firm 1 bids \$10, firm 2 bids \$4 and firm 3 bids \$2)
 - Allocation: Firm 1 gets A firm 2 gets B , firm 3 gets nothing (efficient)
 - Prices:
 - Firm 3 pays nothing
 - Firm 2 displaces firm 3 for 100 clicks \Rightarrow pays $\$2 \times 100 = \200
 - Firm 1 displaces firm 3 for 100 clicks and firm 2 for 100 clicks \Rightarrow pays $\$2 \times 100 + \$4 \times 100 = \$600$
 - Revenue = \$800

Vickrey auction

- Vickrey prices are **lowest** competitive equilibrium prices



- Google GSP auction
 - Bidders submit per-click bids
 - Ad positions are allocated following the order of bids (top bidder gets top position, second bidder gets second position, ...)
 - Each bidder pays a price equal to the next lower bid (i.e. top bidder pays second highest bid, second bidder pays third highest bid, ...)

- Bidding own value is **NOT** a dominant strategy
 - Example: Two positions 100 and 200 clicks. Value 10 per click. If competing bids are 5 and 9, winning second position at price 5 generates more profit than winning first position at price 9.
- Bidding own value **can** be an equilibrium
 - **Example.** Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
 - The three firms bidding own value is a Nash equilibrium
 - Allocation is efficient
 - Prices are $p_A = \$4$, $p_B = \$2$ ($>$ Vickrey prices)
 - Revenue = \$1,000 $>$ Vickrey

- **Example.** Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
 - Profile of bids (\$2, \$3, \$10) is also a Nash equilibrium (**Vickery prices**) - revenue \$800
 - Profile of bids (\$3, \$5, \$8) is also a Nash equilibrium - revenue \$1,300
 - Profile of bids (\$3, \$7, \$3.30) is also a Nash equilibrium (**inefficient**)

- Model
 - K positions with click rate $x_1 > x_2 > \dots > x_K$
 - N bidders with per-click values $v_1 > v_2 > \dots > v_N$
 - Full information
 - Auction format: GSP
- Efficiency: an allocation is efficient if it is “positive assortative” (i.e. the highest **value** bidder gets top position, and so on...)

- A strategy profile is a vector of bids $b = (b_1, \dots, b_N)$
 - b^k is the k^{th} highest bid
 - v^k is the per-click value of the k^{th} highest bidder
- Given a strategy profile b , payoff to the k^{th} highest bidder is

$$v^k x^k - b^{k+1} x_k = (v^k - b^{k+1}) x_k$$

- A strategy profile b is a **Nash equilibrium** if for each k

$$(v^k - b^{k+1}) x_k \geq (v^k - b^{m+1}) x_m \quad \text{for } m > k$$

$$(v^k - b^{k+1}) x_k \geq (v^k - b^m) x_m \quad \text{for } m < k$$

- **Definition:** An equilibrium is **local envy-free** if no bidder can increase his payoff by “swapping” bids with the player who bid just above him.
 - k^{th} highest bidder swapping bids with $k - 1^{\text{th}}$ highest bidder
 - wins $k - 1^{\text{th}}$ position (click rate x_{k-1})
 - pays the k^{th} highest bid b^k
 - payoff $(v^k - b^k)x_{k-1}$
 - local envy free requires

$$(v^k - b^{k+1})x_k \geq (v^k - b^k)x_{k-1}$$

local envy-free and stability

- Two-sided one-to one matching analogy
 - advertisers: payoff = profit
 - positions: payoff = revenue
 - Given a strategy profile b , position $k - 1$ is assigned to the $k - 1^{th}$ highest bidder who pays the k^{th} highest bid (b^k).

Revenue

$$b^k x_{k-1}$$

- If an equilibrium is **not** local envy-free

$$(v^k - b^{k+1})x_k < (v^k - b^k)x_{k-1}$$

- Paying a little more than b^k for position $k - 1$, the bidder is still strictly better off, and position $k - 1$ gets more revenue
 - k^{th} bidder and position $k - 1$ block the matching (**unstable**)
- local envy-free is sufficient for stability

- Stable assignment (specify both matching and payments “transfers”)
 - matching is efficient: higher value bidders get higher positions (positive assortative matching)
 - position prices (p_1, \dots, p_K) satisfy

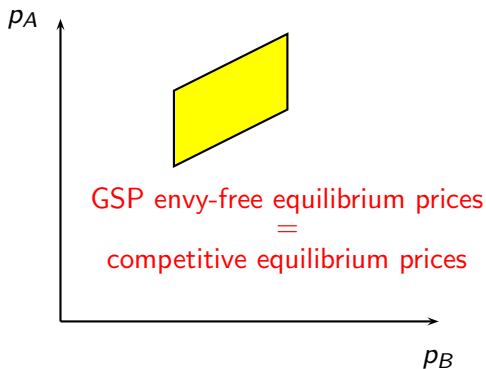
$$(v_k - p_k)x_k \geq (v_k - p_m)x_m \quad \text{for all } m$$

otherwise “bidder k ” and position m block the assignment

- same as competitive equilibrium
 - efficient allocation
 - demand=supply

stability and competitive equilibrium

- **Theorem:** The outcome of a locally envy-free equilibrium of the GSP auction is a stable assignment (= competitive equilibrium allocation). Further, provided $N > K$, any stable assignment (=competitive equilibrium allocation) is an outcome of a locally envy-free equilibrium of the GSP auction.



equilibrium ranking

- **Theorem:** There is a “bidder-optimal” competitive (GSP envy-free) equilibrium and a “seller-optimal” competitive (GSP envy-free) equilibrium.
 - “bidder-optimal” equilibrium generates same revenue as Vickrey auction
 - “seller-optimal” equilibrium

