ECO 426 (Market Design) - Lecture 10

Ettore Damiano

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Multi-item auctions

- In the auction models considered so far, there is a single object for sale
- In a multi-item auction: two or more objects are for sale
  - bidders might want just a single object (unit demand);
    - an ad on a google search page
  - bidders might be willing to buy more multiple objects
    - a combination of radio spectrum licenses
- Examples of multi-item auctions abound
  - treasury bills
  - radio spectrum
  - CO_2 emissions
  - landing slots
  - MBA classes
  - wine auctions
  - oil and gas leases
  - bus routes
  - sponsored search
  - condo sales
  - auto auctions
  - ⋮

multi-unit pricing

- When there are multiple objects sold simultaneously (rather than one at the time in sequence), the auction could specify that:
  - each object is sold at the same price

    uniform price auction
  - price per object might be different

    discriminatory price auction

Emissions trading

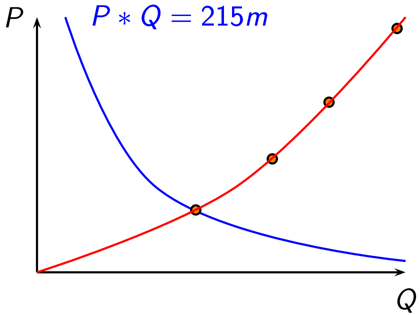
- **UK Emissions Trading Scheme.**
  - a voluntary emissions trading system created as a pilot prior to the mandatory European Union Emissions Trading Scheme
  - Started in 2002
  - The voluntary trading scheme recruited participants from UK industries and organisations
  - Participants promised to make reductions in their carbon emissions, in return they received a share of a £215 million "incentive fund" from the Department for Environment, Food and Rural Affairs (DEFRA)
  - How much each participant would reduce their emission and how much money they would receive in exchange was determined through a "clock auction"

Clock auction

- Auction starts with a sufficiently high price
  - **price** is the amount of money a firm would receive per unit of reduction in emissions (unit = 1 metric ton of CO_2 emissions)
- Bidders state the amount of CO_2 emissions they would abate at current price
- The auctioneer calculate total cost (i.e. tons of CO_2 abatement times price)
  - If total cost $>$£215m, the price is lowered
  - If total cost $\leq$£215m, price (and auction) stops
- When price stops, allocation is implemented
  - bidders reduce emissions by their stated amount
  - bidders receive a subsidy = emissions reduction$*$final price
- Auction results:
  - total emissions reduction = 4m metric tons of CO_2 emissions
  - price per metric ton £53.75
  - 34 firms received a subsidy (and reduced their emissions)

Clock auction

- A clock auction traces out the aggregate supply of emission reductions



- Clock auction price = competitive price

$$Q \text{ demanded } = Q \text{ supplied}$$

Sealed bid supply function auction

- Each bidder bids a “supply function”
  - a schedule of a quantity of emission reductions at each price
- Bids are open
- Individual supply functions are aggregated
- Auction price = market clearing price
- Emission reduction amounts are assigned to each bidder given the auction price and their submitted supply functions.

- Question: Are the clock auction and the sealed bid supply function auction strategic equivalent?
  - Yes if, in the clock auction, bidders only observe current price
  - No if, in the clock auction, bidders observe more information (e.g. competitor supply decisions at each price)

$(K+1)^{th}$ price auction

- There are K identical objects to sell (i.e. bidders do not care about the exact object they receive)
- Bidders have unit demand (i.e. they only want to purchase one object)
- There are $N > K$ bidders
- (Independent) private values
- Each bidder submits a sealed bid
- Bids are open
  - K highest bidders each receive one object
  - Winning bidders pay a price equal to $(K+1)^{th}$ highest bid
- Question: What should bidders bid?
  - Theorem: It is a dominant strategy, for a bidder with unit demand, to bid her valuation.
  - Same argument as in second price private value auction

Demand reduction

- Question: What if bidders want more than one unit?
- Example: Three (identical) objects for sale, four bidders
  - Bidder 1: wants one object, values it \$120
  - Bidder 2: wants one object, values it \$110
  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$105
- If every bidder bids own value bids are: 120, 110, 100, 105, 105 (bidder 4 submits a 105 bid for two objects)
  - Bidders 1, 2, and 4 win one object each
  - Auction price is 105
  - Bidder 4 profit is 0
- If Bidder 4 reduces his demand to one object bids are: 120, 110, 100, 105
  - Same allocation
  - Auction price is 100
  - Bidder 4 profit is 5

Discriminatory pricing

- Question: Is there an "auction mechanism" that
  1. makes it a dominant strategy for the bidders to submit truthful bids (i.e. honestly report their valuations)
  2. lead to an efficient allocation of the objects (i.e. objects are assigned to the bidders who value them the most)?
- Vickrey action:
  - Objects are assigned to highest bidders
  - Each bidders pay a price equal to the value of the displaced bidder/s
    - (using submitted bids) calculate the aggregate value of the auction allocation to all bidders but n, $\mathcal{V}_{-n}$
    - calculate total value of the allocation of an hypothetical auction where bidder n **does not participate**, $\hat{\mathcal{V}}_{-n}$
    - price paid by bidder n $= \hat{\mathcal{V}}_{-n} - \mathcal{V}_{-n}$
- bidders pay different prices (discriminatory pricing)
- the price each bidder pays equals to the negative externality his participation imposes on all other bidders
- bidding truthfully is a dominant strategy

Vickrey auction - example

- Example: Three (identical) objects for sale, four bidders
  - Bidder 1: wants one object, values it \$120
  - Bidder 2: wants one object, values it \$110
  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$105
- Bidding own values, bids are: 120, 110, 100, 105, 105
  - Bidders 1, 2, and 4 win one object each
  - Removing bidder 1: 2 wins one object and 4 wins two objects
    - Bidder 1 price is 105 (displaces one unit of bidder 4)
  - Removing bidder 2: 1 wins one object and 4 wins two objects
    - Bidder 1 price is 105 (displaces one unit of bidder 4)
  - Removing bidder 3: same allocation
    - Bidder 3 price is 0 (displaces nobody)
  - Removing bidder 4: 1, 2 and 3 win one object each
    - Bidder 4 price is 100 (displaces bidder 3)
- Bidder 4 has no incentive to reduce demand

Vickrey auction - example

- Example: Three (identical) objects for sale, four bidders
  - Bidder 1: wants one object, values it \$120
  - Bidder 2: wants one object, values it \$110
  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$115
- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)
  - Bidder 4 pays a total of 210 (displaces bidder 2 and 3)
  - No incentive to change bids
- Uniform price (4^{th} price auction)
  - price is 110
  - bidder 4 gains by reducing demand to 1 unit

Multi-item auctions with different goods

- Multiple objects for sale, K
- Objects are different
  - spectrum licenses in different regions
  - landing slots at different time of the day
  - different bus routes
  - different MBA classes

    ⋮
- N bidders
- A bidder n
  - has a vector of valuations for each of the objects, $(v_n^1, \ldots, v_n^K)$
  - only wants one of the objects (unit demand)

a simple example of ascending auction

- Example: Three bidders, three objects
  - Bidder A valuations: 3.1, 5.2, 7.3
  - Bidder B valuations: 4.3, 3.2, 6.1
  - Bidder C valuations: 6.2, 2.1, 5.3
- propose a price for an object to maximize profit
  - A and B offer 0 for good 3, C offers zero to good 1
- sellers "hold" one highest price offer and rejects all others
  - good 3 rejects B's offer
- rejected bidders make new proposals (min price increase: 1)
  - B offers $1 to good 3 $\Rightarrow$ good 3 rejects A's offer
  - A offers $1 to good 3$\Rightarrow$ good 3 rejects B's offer
  - B offers $0 to good 1$\Rightarrow$ good 1 rejects B's offer
  - B offers $2 to good 3$\Rightarrow$ good 3 rejects A's offer
  - A offers $2 to good 3$\Rightarrow$ good 3 rejects B's offer
  - B offers $1 to good 1$\Rightarrow$ good 1 rejects C's offer
  - C offers $1 to good 1$\Rightarrow$ good 1 rejects B's offer
  - B offers $0 to good 2
- A gets 3 at $2, B gets 2 at $0, C gets 1 at $1

Ascending auction - preference lists

- More generally,
  - Bidders have preferences over (object-price) pairs
    Example: Bidder 1's valuations 3.1, 5.2, 7.3 $\Rightarrow$ preferences
    $(3, \$0) \succ (3, \$1) \succ (3, \$2) \succ (2, \$0) \succ (3, \$3) \succ (2, \$1) \succ (3, \$4) \succ (2, \$2) \succ (1, \$0) \succ \dots$
  - Sellers prefer the highest offer
  - Bidders propose to objects following their preference list
  - Sellers "hold" the highest offer and reject all others
  - Rejected bidders make next offer on their preference list
  - Process ends when no offer is rejected
- Have we seen this before?
  - Deferred acceptance with bidders proposing

Ascending auction - properties

- Unit demand $\Rightarrow$ one-to-one matching
  - each bidder receives at most one object
  - each object is assigned at most to one bidder
- With one-to-one matching, the DA bidder proposing algorithm
  - yields a stable matching
  - is strategy proof for the bidders (i.e. it is a dominant strategy to report preference list honestly / bid truthfully)
- Stable matching means that at the final auction prices
  - each object is demanded by at most one person
  - not demanded objects have zero price (price only grows if more than one person demands it)
- Auction prices are such that
  - either demand = supply, or
  - demand=0<1=supply and price is zero
  - competitive equilibrium
  - same as Vickrey prices (strategy proof)