ECO 426 (Market Design) - Lecture 9

Ettore Damiano

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Common Value Auction

- In a private value auction:
  - the valuation of bidder i, v_i, is independent of the other bidders' value
- In a common value auction:
  - bidders' valuation are identical (i.e $v_1 = v_2 = \cdots = v_N = v$)
  - Examples
    - Wallet auction
    - Jar of pennies auction
- How much information each bidder has about the common value v matters
  - Example: auctioning off a wallet with a SPA, how much would you bid if:
    - Everybody gets to see the content of the wallet
    - Just one person does
    - Nobody does

Private Information in common value auction

- Interesting case is when each bidder has some private information about the common value of the object for sale
- Example:
  - Two bidders with common value v
  - Bidder 1 observes $s_1 = v + \epsilon_1$
  - Bidder 2 observes $s_2 = v + \epsilon_2$
  - The two terms ϵ_1 and ϵ_2 are (independent) error terms
  - s_i is bidder i's private "estimate" of the common valuation
  - second price auction
  - If all information were observed the public estimate of the common valuation would be

$$\frac{s_1 + s_2}{2} = v + \frac{\epsilon_1 + \epsilon_2}{2}$$

- Question: Should a bidder bid his private estimate of v?

Common Value Auction

- Bidding own "estimate" not longer a dominant strategy.
  - price paid when winning never larger than own "estimate" of v
  - winning is "bad news" the winner was more optimistic about v than his opponents
  - knowing your opponents have a lower estimate of v that you do decreases your estimate of v
- winning the object reduces how much you think it is worth "winner's curse"
- similarly, losing the auction may increase how much you think it is worth "loser's curse"
- equilibrium bidding must reflect the information contained in the event you are winning

Bidding in a common value second price auction

- Claim: In the equilibrium of the second price auction, bidders use the strategy $b(s_i) = \mathbb{E}[v|s_i, s_j = s_i]$
  - bidders bid an estimate of v obtained: i) using their private information; and ii) assuming their opponent observes exactly the same signal.
  - estimate v assuming a "tie" when winning
- Sketch of the argument
  - In equilibrium bidders cannot gain from marginally lowering or increasing their bid (i.e. bidding $b(s) + \epsilon$ or bidding $b(s) - \epsilon$)
  - Marginal changes in a bid only matter if there is a tie (i.e. if my opponent has my same signal)
  - If $b(s_i) < \mathbb{E}[v|s_i, s_j = s_i]$, can gain by marginally raising bid
  - If $b(s_i) > \mathbb{E}[v|s_i, s_j = s_i]$, can gain by marginally lowering bid

Common value auction winner's and loser's curse

- Do bidders bid more or less than their private estimate of v in equilibrium?
- Example 1: Second price auction, 1 object for sale, $N > 2$ bidders
  - Equilibrium bidding strategy

$$b(s) = \mathbb{E}[v|s \text{ is tied for highest estimate}] < \mathbb{E}[v|s]$$

  - winner's curse
- Example 2: Lowest price auction, $N - 1$ objects for sale, $N > 2$ bidders
  - Equilibrium bidding strategy

$$b(s) = \mathbb{E}[v|s \text{ is tied for lowest estimate}] > \mathbb{E}[v|s]$$

  - loser's curse

Revenue comparison

- Revenue equivalence no longer holds
- Expected revenue comparison
  - Ascending price $>$ Second Price $>$ First price
  - Milgrom-Weber: an open auction does better than a sealed bid auction with correlated estimates of a common value
- Broader result: "Linkage principle"
  - Suppose the seller can give bidders access to better information. Then the revenue is increased on average by making the information publicly available
  - public information will move everyone's bid in the same direction (i.e. up if good news, down if bad news)
  - public info will on average be good news when the high bidder has high value, reducing the winner's profit when it is high

Examples of common value auctions

- Treasury bill auctions
  - common value is resale price in the secondary market
- Natural resources
  - Timber auctions: quality and type of timber available in the tract auctioned off is uncertain
  - Oil Lease auction
    - quantity of oil available in the tract auction off is unknown
    - bidders do independent seismic studies - private information on the amount of oil reserves in the tract

⋮

- The US Government auctions off the right to drill for oil on the outer continental shelf

Outer continental shelf auctions

- No one knows how much oil there is in a tract being auctioned off
- Before the auction, bidders conduct seismic studies to obtain an estimate of the amount of oil available
- Seismic studies results are valuable private information, which bidders do not share with each other
- Two different type of tracts are auctioned off
  - "Wildcat sale": new territory being sold
  - "Drainage sale": territory adjacent to already developed tracts
- Question: What is different between these two types of sales?

Wildcat vs. Drainage

TABLE 1 — SELECTED STATISTICS ON WILDCAT AND DRAINAGE TRACTS[a]

	Wildcat	Drainage
Number of Tracts	1056	144
Number of Tracts Drilled	748	124
Number of Productive Tracts	385	86
Average Winning Bid	2.67	5.76
	(0.18)	(1.07)
Average Net Profits	$\boxed{1.22}$	$\boxed{4.63}$
	(0.50)	(1.59)
Average Tract Value	5.27	13.51
	(0.64)	(2.84)
Average Number of Bidders	3.46	2.73

[a] *Source:* Kenneth Hendricks, Robert Porter, and Bryan Boudreau (1987). Dollar figures are in millions of \$1972. The numbers in parentheses are standard deviations of the sample means.

- drainage sales more profitable than wildcat sale (for the bidders)

Drainage sales closer look

TABLE 3—SAMPLE STATISTICS ON TRACTS WON BY EACH TYPE OF FIRM[a]

	Wins by Neighbor Firms		Wins by Non-Neighbor Firms		
	A	Total	B	C	Total
No. of Tracts	35	59	19	36	55
No. of Tracts Drilled	23	47	18	33	51
No. of Productive Tracts	16	36	12	19	31
Average Winning Bid	3.28	6.04	2.15	6.30	4.87
	(0.56)	(2.00)	(0.67)	(1.31)	(0.92)
Average Gross Profits	10.05	12.75	−0.54	7.08	4.45
	(3.91)	(3.21)	(0.47)	(2.95)	(1.99)
Average Net Profits	6.76	6.71	−2.69	0.78	−0.42
	(3.02)	(2.69)	(0.86)	(2.64)	(1.76)

[a]Dollar figures are in millions of $1972. The numbers in parentheses are the standard deviations of the sample means. Column A refers to tracts which received no non-neighbor firm bid, column B refers to tracts which received no neighbor bid, and column C to those in which a neighbor firm bid, but a non-neighbor firm won the tract.

- Drainage sales are only profitable to "insiders"
- Asymmetric information matters

Common value auctions with asymmetric information

- Common value v
- Two bidders
  - Insider knows v
  - Outsider believes that v is $U[0,1]$
- Ascending price auction equilibrium?
  - insider stays in until price hits v (dominant strategy)
  - outsider drops immediately
  - seller revenue $= 0$
- First price auction equilibrium?

First price auction equilibrium

- Equilibrium properties
  - Outsider cannot play a pure strategy, b_o, in equilibrium
    - If $b_o = 0$, the insider's best response would be a small bid larger than 0, $b_i = \epsilon$.
    - Not an equilibrium: the outsider can profitably deviate to a small bid $b_o = 2\epsilon$.
    - If $b_o > 0$, the insider's best response would be to bid just above when $b_o < v$ and below it when $b_o > v$
    - Not an equilibrium: the outsider only wins when $b_o > v$, making negative profits

First price auction equilibrium

- Outsider randomizes across many bids
  - loses for sure at lowest bid $\Rightarrow$ lowest bid must be zero
  - wins for sure at highest bid, $\overline{b}$
  - expected payoff from each bid must be zero
  - expected payoff from $\overline{b}$ is

$$\mathbb{E}[v|win, \overline{b}] - \overline{b} = \frac{1}{2} - \overline{b} \Rightarrow \overline{b} = \frac{1}{2}$$

  - For each bid value, between 0 and $1/2$, the indifference condition implies

$$Prob(win|b)(\mathbb{E}[v|win, b] - b) = 0 \Rightarrow \mathbb{E}[v|win, b] = b$$

  - Winning means the insider's value is below a certain value, $\tilde{v}(b)$ (monotone strategies), hence $\mathbb{E}[v|win, b] = \tilde{v}(b)/2$
    - the threshold value must be $\tilde{v}(b) = 2b$
    - the insider bidding strategy must be $b_i(v) = v/2$

First price auction equilibrium

- The outsider randomizes among bids in the interval $[0, 1]$
  - The probability that the outsider places a bid smaller than x is

$$F(x) = 2x$$

- The insider plays a pure strategy
  - The insider places a bid equals to half of his valuation

$$b_i(v) = v/2$$

- The outsider strategy is a best response to $b_i(v)$
  - By construction, outsider is indifferent between any bid in [0,1/2]
  - no need to bid more that $1/2$ since at $1/2$ wins for sure
- Consider an insider with valuation v, bidding b has an expected payoff

$$Prob(win|b)(v - b) = 2b(v - b),$$

which is maximized at $b = v/2$

First price auction

- Comparing bids
  - Both insider and outsider bids are distributed uniformly on the interval [0,1/2]
  - It is equally likely that insider and outsiders win, but
    - insider wins more often when v is high
    - outsider wins more often when v is low
    - given a valuation v the insider wins with probability v
- The distribution of information across bidders is crucial