ECO 426 (Market Design) - Lecture 7

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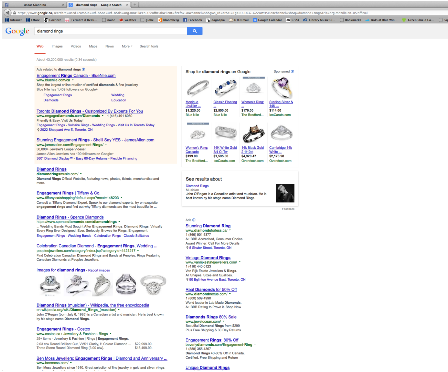
700 MHz Spectrum Auction

- Started: January 14, 2014
- Ended: February 19, 2014
- Rounds of bidding: 108
- Licenses allocated: 98
- Total Revenue: CAD 5.27bn

Google AdWords

- online advertising service that places advertising copy at the top or bottom of, or beside, the list of results Google displays for a particular search query.



- An auction determines the order of the ads and the payment to Google (per click or per impression)
- Google advertising revenue: USD 42.5bn in 2012

Auctions

- Examples of common auctions
  - Bus routes (London, England)
  - Fine wines
  - Art and collectibles
  - Treasury bills
  - Natural resources (timber, oil, radio spectrum)
  - CO_2 emission permits
  - Procurement contracts: construction, defense

  ⋮
- Auctions are used to buy/sell goods that are hard to price (e.g. the willingness to buy/sell for varies across individuals and is not observed (private information))
- The rules of the auction affect the outcome, for example
  - revenue to the seller, or
  - allocation efficiency
- Auction design: choose the auction format that best achieve the designer's objective

Selling a single object

- Key ideas:
  - Seller does not know how much potential buyers are willing to pay for the object
  - Potential buyers know what they would pay but are not telling (private information)
- Auction serves as a "price discovery" mechanism
- Look at different auction formats

Independent private values - two bidders example

- Potential buyers
  - Two bidders, 1 and 2
  - Each bidder $i = 1, 2$ values the object v_i (i.e. the most bidder i is willing to pay to acquire the object)
  - The valuation v_i is known to bidder i only (private information), the other bidders and the seller only know v_i's distribution
  - Each v_i is drawn independently from the uniform distribution on the interval [0,1].
- The seller "designs" (i.e. sets the rules) of the auction

Ascending price auction

- Price starts at 0 and rises slowly (small increments or, as a theoretical modelling abstraction, "continuously")
- At each price, bidders indicate if they want to continue bidding (e.g. by pushing on a button, or keeping their hand raised) or exit the auction (no re-entry)
- Auction ends, and price stops rising, when only one bidder remains
- Auction outcome
  - Allocation: the object is assigned to the last bidder remaining
  - Price: the last bidder remaining pays the final auction price

Ascending price auction

- What should bidders do?
- Each bidder i has an optimal (i.e. dominant) strategy
  - Stay in the auction as long as the price is smaller than i's valuation, v_i
- Outcome
  - Allocation: the object goes to the bidder with the highest valuation (the outcome is efficient)
  - Price: the price paid equals the second highest valuation among all bidders
- Example: four bidders with valuations (0.2, 0.33, 0.6, 0.8),
  - first bidder exits when the price hits 0.2
  - second bidder exits when the price hits 0.33
  - third bidder exits when price hits 0.6
  - only one bidder remains, price stops and last bidder receives the object after paying 0.6 (revenue to the seller)

Ascending price auction

- Question: on average, how much revenue can the seller expect to raise from the auction?
- Two bidders - two independent draws from $U[0,1]$
  - on average, the highest draw will be $\frac{2}{3}$
  - on average, the second highest draw will be $\frac{1}{3}$
  - average revenue: $\frac{1}{3}$
  - winner surplus average: $\frac{1}{3}$
- With N bidders - N independent draws from $U[0,1]$
  - on average, the highest draw will be $\frac{N}{N+1}$
  - on average, the second highest draw will be $\frac{N-1}{N+1}$
  - average revenue $\frac{N-1}{N+1}$
  - winner surplus average: $\frac{1}{N+1}$
- seller revenue grows with the number of bidders
- winner surplus decreases with the number of bidders

Ascending price auction

- What is the expected surplus to a bidder?
- Take bidder 1, with a valuation $v_1 = v$
  - The probability that 1 wins equals the probability that $v_2 < v$

$$Pr(v_2 < v) = v$$

  - When winning, 1 pays a price equal to the value of v_2. On average that is

$$\mathbb{E}(v_2 | v_2 < v) = \frac{v}{2}$$

  - Bidder 1 with a valuation of v, expects a profit of

$$(1 - v)0 + v(v - v/2) = \frac{v^2}{2}$$

- **Before** observing his valuation, bidder 1 expected profit is

$$\mathbb{E}\left(\frac{v_1^2}{2}\right) = \frac{1}{6}$$

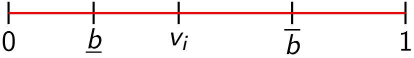
Second price auction

- Each bidder submits a sealed bid
- Bids are open
  - Bidder who submitted the highest bid wins the object
  - Winner pays a price to the seller equal to the second highest submitted bid
- What should bidders do?

Second price auction

- Each bidder i has an optimal (i.e. dominant) strategy
  - Place a bid equal to his valuation (i.e. bid v_i)

- consider three bids
  $\underline{b} < v_i < \overline{b}$



- second highest bid $b^{(2)} < \underline{b}$
- $b^{(2)} \in [\underline{b}, \mathrm{v}_i]$
- $b^{(2)} \in [v_i, \overline{b}]$
- $b^{(2)} > \overline{b}$

- all bids win, payoff $v_i - b^{(2)}$
- v_i and $\overline{b}$ win, payoff $v_i - b^{(2)} > 0$
- only $\overline{b}$ wins, payoff $v_i - b^{(2)} < 0$
- all bids lose, payoff 0

- Regardless of second highest bid, bidding true valuation always does best

Second price auction

- Every bidder bids his valuation
- Outcome
    - Allocation: the object is assigned to the bidder with the highest valuation
    - Price: the winner pays a price equal to the second highest valuation
- Identical to the ascending price auction

First price auction

- Each bidder submits a sealed bid
- Bids are open
  - Bidder who submitted the highest bid wins the object
  - Winner pays a price to the seller equal to his own bid
- What should bidders do?
- Bidders do not have an optimal strategy
- What is best for a bidder depends on what the other bidders are doing

Nash equilibrium

- Each bidders chooses a "bidding strategy," describing his bid as a function of his valuation

$$\beta_i(v_i)$$

- Definition: A profile of bidding strategies is a **Nash Equilibrium** if each bidder's strategy maximizes his payoff given the strategies of the others.
  - For each possible valuation v_i, bidder i's bid must maximize his "payoff"
  - Each bidder does not know the opponents' values (i.e. incomplete information game)
  - Each bidder's equilibrium strategies maximizes his **expected payoff** given the bidder's belief about the distribution of the opponents' values

First price auction: equilibrium

- Two bidders. With valuations v_1 and v_2 uniformly distributed on [0,1].
- Suppose the equilibrium bidding strategy of bidder 2 is linear in the valuation

$$b_2 = \beta v_2$$

- In equilibrium, bidder 1 correctly conjectures the **bidding strategy** of 2, but does not know 2's bid because he does not observe 2's valuation
- If bidder 1 bids b, he wins when $b > \beta v_2$ (i.e. $v_2 < b/\beta$), which has probability b/β
- By bidding b, bidder 1 expected profit is

$$(b/\beta)(v_1 - b)$$

First price auction: equilibrium

- Bidder 1 optimal bidding problem

$$\max_b \; (b/\beta)(v_1 - b)$$

- First order condition

$$0 = (1/\beta)(v_1 - b) - (b/\beta)$$

- Solving for b, we get $b = (1/2)v_1$
- Same argument holds for bidder 2
- Symmetric Nash equilibrium
  - $b_1 = v_1/2$ and $b_2 = v_2/2$
- With N bidders, symmetric Nash equilibrium
  - $b_i = \frac{N-1}{N} v_i$ for $i = 1, \ldots, N$

First price auction: equilibrium

- Equilibrium outcome
  - The bidder with the highest valuation wins the auction (efficient allocation)
  - The winner pays a price equal $1/2$ of his valuation
- What is, on average, the seller revenue?
  - On average the highest valuation (with two bidders) is $2/3$
  - On average the revenue is $1/3$
  - Same as in ascending and second price auctions

  - On average the highest valuation (with N bidders) is $N/(N+1)$
  - On average the revenue is $(N-1)/(N+1)$
  - Same as in ascending and second price auctions
- The revenue can be different for specific realization of the valuations, it is the same on average
  - Example: two bidders with valuations 0.4 and 0.6

Descending price auction

- Price starts very high (higher than the maximum possible valuation) and decreases slowly (small increments or, as a theoretical modelling abstraction, "continuously")
- At any price a bidder can claim the object (e.g. raising his hand or pushing a button)
- Auction ends and price stops as soon as one bidder claims the object
- Auction outcome
  - Allocation: the object is assigned to the bidder who claimed it
  - Price: the winner pays the price at which he/she claimed the object

Descending price auction

- Strategically equivalent to a first price auction
  - The only strategically relevant choice is the highest price at which to claim the object
  - You win if the highest price at which to claim the object is higher than those of your opponents and lose otherwise
  - The winner pays the price at which he claimed the object
- Same equilibrium and same revenue as in a first price auction

- Four auction formats: DP, SP, FP and AP
  - Same allocation: object is assigned to the bidder with highest valuation (i.e. efficient allocation)
  - Same expected revenue to the seller
  - Same expected profit to the buyers
- Is this a coincidence?