

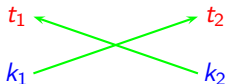
# ECO 426 (Market Design) - Lecture 5

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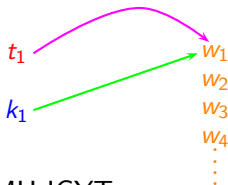
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# Exchanging Kidneys

- Two types of kidney exchanges
  - Pairwise kidney exchange:** exchange kidney with another patient-donor pair



- Exchange to list:** donate kidney to patient on waiting list in exchange of a better spot on waiting list



- Looks similar to YRMH-IGYT

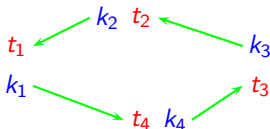
# Kidney exchange problem

- A Kidney exchange problem consists of:
  - A set of donor-patient pairs  $\{(t_1, k_1), \dots, (t_n, k_n)\}$
  - For each patient,  $t_i$ , a set of compatible kidneys  $K_i \subseteq K = \{k_1, \dots, k_n\}$
  - For each patient,  $t_i$ , a (strict) preference ordering over the set of compatible kidneys  $K_i$  **and** the option of exchanging own kidney,  $k_i$  for priority  $w$  on the waiting list
- **Question:** How do we organize a kidney exchange program such that
  - The outcome is Pareto efficient, **it is not possible to improve further the welfare of all**
  - For each patient, the outcome is never worse than not participating in the mechanism, **ensures broad participation, no donor kidney is un-necessarily "wasted"**
  - The mechanism is strategy proof, **patients have incentive to disclose their preferences honestly**

# Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)



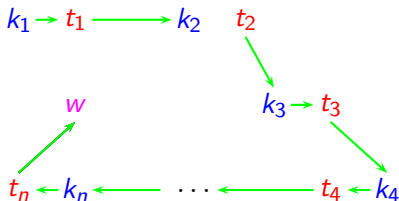
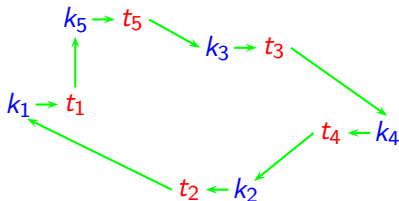
- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)
- Strict preferences:** No patient is indifferent between any two (compatible) alternatives (i.e. strict preferences over: compatible kidneys + trading donor for wait list + remaining with own donor)
- Interpreting live donors' kidneys as "owned" by their respective patients, the problem resembles one of house allocation with existing tenants
  - Maximizing "supply" of live donors as maximizing participation

# TTC(and C)

- TTC mechanism key properties
  - Each patient points to favorite kidney or the waiting list
  - Each kidney donor points to his/her patient

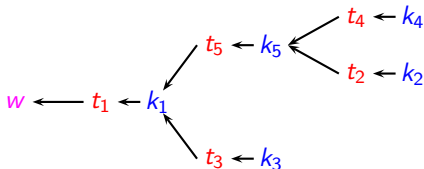
- In a given round

- A cycle might form
  - each patient in the cycle receives the best compatible kidney available
- A cycle might not form
  - some patient point to wait-list
- If there is no cycle there must be at least a **w-chain**



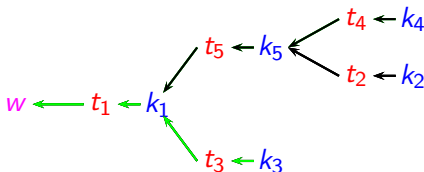
## Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:
  - Carry out exchange
  - Remove kidneys and patients in cycle and restart
- When no cycle form
  - There can be more than one chain



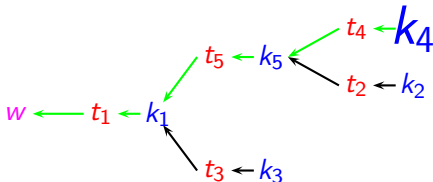
- Multiple chains can be in “competition” with each other
  - Need a **chain selection rule**

- Examples of chain selection rules
- Choose based on length
  - longest chain
  - minimal chain
- Choose based on donor-patient priority
  - Choose chain with the highest priority donor-patient pair (e.g.  $t_3$ ,  $k_3$ )



# “tail kidney”

- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



- The tail kidney can be
  - Assigned to some compatible patient on wait list (i.e. list exchange) (might have welfare consequences, Pareto efficiency is not guaranteed)
  - Remain available to remaining patients on the kidney exchange program (guarantees Pareto efficiency)



## Combining chain selection and tail kidney options

- ❶ Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- ❷ Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- ❸ Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- ❹ Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
- ❺ Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof, Pareto efficient**) - equivalent to YRMH-IGYT
- Key properties
  - Minimal chains for strategy proofness
  - Keep kidney for Pareto efficiency

# Constrained kidney exchange

## Practical shortcomings:

- Multi-way exchanges can be difficult to implement
  - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
    - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
    - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict
  - Compatibility is treated as a binary variable (0-1)
- List exchanges pose a “selection” problem
  - Most common blood type is O-type
  - Most likely donor kidney exchanged to wait-list will be O-type incompatible (otherwise the donating patient would have it)
  - List exchanges may harm O-type patient on wait-list

# Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
  - A set of donor-patient pairs  $\{(t_1, k_1), \dots, (t_n, k_n)\}$
  - For each patient,  $t_i$ , a set of compatible kidneys  $K_i \subseteq K = \{k_1, \dots, k_n\}$
- The set of agents  $N$  and a compatibility matrix,  $R$ , suffice to describe the problem
  - $R$  is an  $N \times N$  matrix with

$$r_{i,j} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are compatible} \\ 0 & \text{otherwise} \end{cases}$$

- **Objective:** Find a collection of bilateral kidney exchange among mutually compatible donor-patient pairs

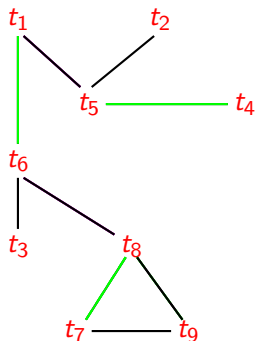
# Priority Mechanism

- Order donor-patient pairs according to priorities

**Example:** Nine patient-donor pairs  $\{t_1, t_2, \dots, t_9\}$  priority ordering  $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random

- match top priority patient if possible (i.e. if there is a patient-donor pair mutually compatible with the priority 1 patient-donor pair), else skip
- match priority 2 patient, if possible, in conjunction with priority 1 agent else skip priority 2 agent
- ...
- match priority  $n$  patient, if possible, in conjunction with all earlier priorities, else skip



# Priority Mechanism

- The priority mechanism is
  - Pareto efficient
  - Strategy proof
- Limits:
  - Allowing tri-lateral exchanges can make many more transplants possible
  - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B,O-A,A-B,A-B,B-A);  
HLA incompatible pairs (A-A,A-A,A-A,B-O)
  - Only bilateral exchanges: (A-B,B-A) (A-A,A-A) (O-B,B-O)
  - Bilateral and trilateral: (A-B,B-A) (A-A,A-A,A-A)  
(B-O,O-A,A-B)

# Kidney exchange programs

- New England Program for Kidney Exchange (2004)
  - Priority mechanism
  - up to 4-lateral exchanges
  - list exchanges allowed
  - **altruistic donor exchanges** (i.e. chains starting from an altruistic live donor, rather than a list exchange)
- Ohio - Living Kidney Donor Program
  - Performed a Six-way paired kidney exchange (September 2011)
- National program under construction

## NEAD: Never Ending Altruistic Donor Chain

- Alliance for Paired Donation - 10 kidney transplant chain



- National Kidney Registry - 30 kidney transplant chain

