ECO 426 (Market Design) - Lecture 4

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Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:
    - owns a "house," h_a, (H is the set of all houses);
    - has (strict) preferences over the set of houses in the economy
  - the initial allocation might not be efficient (i.e. Pareto efficient)
    - mutually beneficial trades might be possible

Housing Market

- Housing market vs. marriage market
  - one side of the market (houses) has no preferences over matches;
  - agents have an initial endowment (i.e. each agent owns a house)
    - the market starts from a default allocation where each agent is matched to his own house
  - Goal: find a matching that cannot be improved
    - it is not possible to reassign houses making some agent better off and making no agent worse off

Housing Market - CORE

- An allocation is an assignment (matching) of agents to houses such that
  - each agent is assigned exactly one house; and
  - each house is assigned to exactly one agent.
- An allocation in an housing market is described by a **"bijection"** $\mu : A \longrightarrow H$.
- In a housing market, each agent is endowed (owns) one house (e.g. a owns h_a)
- What allocations would we expect to arise if agents can freely dispose of their endowment?

Housing Market - CORE

- Agents in a group $S \subseteq A$ own together (in a "coalition") a subset of the houses in the market H_S
- The agents in a coalition S can "independently" distribute the houses they own, H_S, among themselves.
  - An assignment of the houses in H_S to agents in S, is an allocation in the housing market where the set of agents is S and the set of houses is H_S

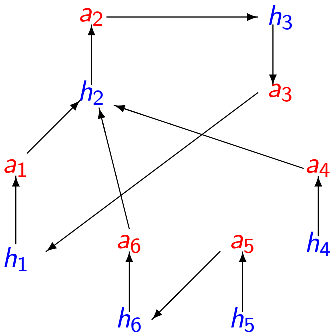
$$\mu_S : S \rightarrow H_S.$$

Housing Market - CORE

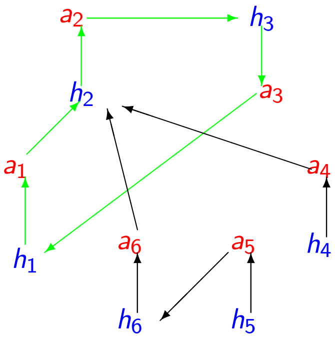
- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that: i) some member of S prefers μ_S to μ; ii) no member of S prefers μ to μ_S.
  - A blocking coalition can find a mutually beneficial trade (i.e. an exchange of houses among members of the coalition that improves all members' welfare with respect to the allocation μ)
- Definition (Core) An allocation is in the core of the housing market if it is not blocked by any coalition.
  - At a core allocation benefits from trade are exhausted
  - In a marriage market, core matchings and stable matchings coincide

Gale's Top trading cycle algorithm

- each agent points to his/her preferred house
- each house points to its owner



Housing Market - TTC

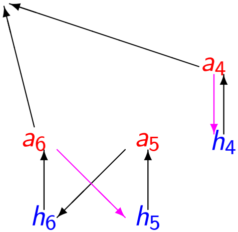


- there is at least one cycle

- remove all cycles assigning houses to agents
  - agents within a cycle exchange houses among each others

Housing Market - TTC

- each remaining agent points to his/her preferred remaining house



- remove all cycles assigning houses to agents
- continue until no agent/house is left

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

- The outcome of the TTC mechanism cannot be blocked
  - cannot make any agent matched in the first round better off (they are getting their favourite house)
  - cannot make any agent matched in the second round better off without making some of the agents matched in the first round worse off
  - cannot make any agent matched in round n better off without making some agents matched in earlier rounds worse off.

TTC and strategic incentives

Theorem The TTC algorithm is a strategy proof mechanism.

- an agent matched in round n cannot, by manipulating his/her preferences, break any of the cycles that form before round n
  - preference manipulation cannot give the agent a house that was assigned earlier than round n.
- getting an house that was assigned in a round later than n does not make the agent better off.

House allocation

- An House allocation problem consists of
  - A collection of N agents, A;
  - A collection of N houses, H;
  - For each agent (strict) a preference ordering over houses.
- House allocation vs. housing market
  - Same preference structure (i.e. only one side has preferences)
  - Same set of possible outcomes (i.e. assignment of a house to each agent)
  - Difference: agents do not "own" houses (i.e. there is no initial allocation of houses to agents.)
- Goal: find an efficient assignment (i.e. one that cannot be improved for all agents)

House allocation - mechanisms

Serial dictatorship

- Agents are given a priority ordering from 1 to N;
- Agents choose houses in order of their priority.

Random serial dictatorship

- Same rules as in serial dictatorship;
- Priority ordering is chosen randomly from the set of all priority orderings of agents.

CORE from random assignment

- Randomly draw an initial assignment.
- Treat the initial assignment as an endowment.
- Use the TTC algorithm to find the unique core allocation given the initial assignment.

House allocation mechanisms - properties

- Serial dictatorship, random serial dictatorship, and core from random endowment are
  - Pareto efficient; and
  - Strategy proof.

House allocation with existing tenants

- Housing market: each agent owns a house;
- Housing allocation: no agents owns a house;
- Housing allocation with existing tenants: some agents "own" houses others do not
  - allocating student housing - with upper year students having the right to keep their current residence (i.e. "own" their house)

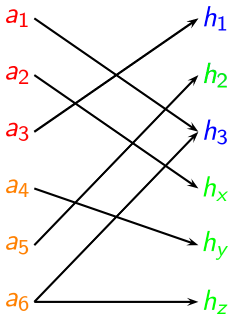
House allocation with existing tenants

- Natural variation of the random serial dictatorship mechanism
  - Students without residence participate in a lottery determining the priority ordering;
  - Students already in residence can:
    - keep their residence; or
    - give up their residence and participate in the lottery.
  - Stock of available houses to choose from is formed by
    - empty houses; and
    - houses of students who choose to participate in the lottery.
- Many US colleges use exactly this mechanism for assigning student residences
- By participating in the lottery, a student in residence might end up with a worse house
  - this might induce some students in residence **not to participate** in the lottery
  - the outcome can be inefficient

YRMH-IGYT mechanism

Example: Existing tenants a_1, a_2, a_3 with houses h_1, h_2, h_3; newcomers agents a_4, a_5, a_6; empty houses h_x, h_y, h_z; randomly chosen priority ordering of agents {4,2,5,6,3,1}

- assign agents their top choice in priority order
- until an agent requests an occupied house
- change the priority ordering placing the existing tenant ahead of requestor {4,2,5,**3**,**6**,1}
- ...repeat each time this happens {4,2,5,**1**,**3**,**6**}



  - YRMH-IGYT stands for: "You request my house - I get your turn"

YRMH-IGYT properties

Theorem The YRMH-IGYT mechanism is Pareto efficient, strategy proof, and makes no existing tenant worse off.

- Solve the participation problem
- Relation to other mechanisms
  - Coincide with Serial dictatorship when no agent has a house
  - Coincide with TTC when all agents are "tenants"
  - IRMH-IGYT is a version of TTC with all unoccupied houses pointing to the agent with the highest priority (among those remaining in the market)

Kidney Exchange

Kidney Exchange

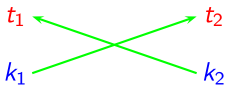
- Shortage of kidneys available for transplantation
  - Yet, the "potential" supply (i.e. the number of not strictly needed functioning kidneys) vastly exceeds the demand
- How do we reduce the shortage?
  - Increase the availability of cadaveric kidneys
  - Increase the supply of live donor kidneys
    - cannot use any "price" mechanism because of legal (and moral) constraints

Kidney donation

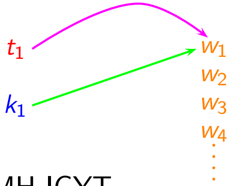
- Live kidney donor must pass two compatibility tests before a transplantation is carried out
  - Blood type match
    - O-type patient can only accept O-type donor
    - A-type patient can accept A and O-type donor
    - B-type patient can accept B and O-type donor
    - AB-type patient can accept all donors
  - Tissue (HLA) compatibility
- Potential inefficiency: when a willing donor does not meet the compatibility tests the kidney is "wasted"
  - most donors are close friends/relatives, unwilling to donate to a stranger
- Possible improvements: donor could be willing to donate to stranger if that improves the chances of their close friend/relative receiving a kidney **i.e a kidney exchange**

Exchanging Kidneys

- Two types of kidney exchanges
  - Pairwise kidney exchange: exchange kidney with another patient-donor pair



  - Exchange to list: donate kidney to patient at top of waiting list in exchange of top spot on waiting list



- Looks similar to YRMH-IGYT

Kidney exchange problem

- A Kidney exchange problem consists of:
  - A set of donor-patient pairs $\{(t_1, k_1), \ldots, (t_n, k_n)\}$
  - For each patient, t_i, a set of compatible kidneys $K_i \subseteq K = \{k_1, \ldots, k_n\}$
  - For each patient, t_i, a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- Question: How do we organize a kidney exchange program such that
  - The outcome is Pareto efficient, it is not possible to improve further the welfare of **all**
  - For each patient, the outcome is never worse than not participating in the mechanism, ensures broad participation, no donor kidney is un-necessarily "wasted"
  - The mechanism is strategy proof, patients have incetive to disclose their preferences honestly