ECO 426 (Market Design) - Lecture 7

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November 16, 2015

700 MHz Spectrum Auction

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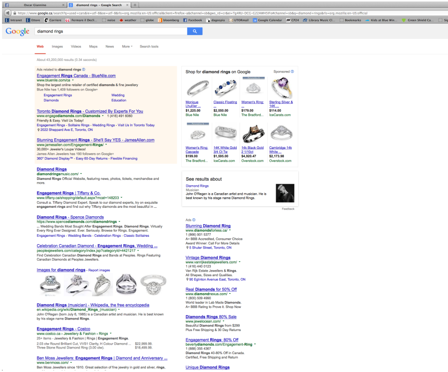

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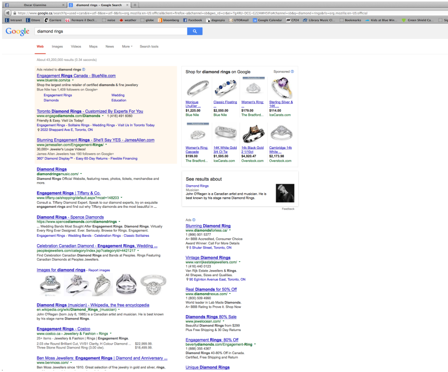

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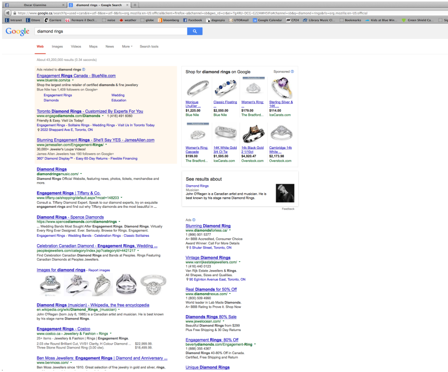

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- Google advertising revenue: USD 42.5bn in 2012

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- Auction design: choose the auction format that best achieve the designer's objective

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- Auction serves as a "price discovery" mechanism
- Look at different auction formats

Independent private values - two bidders example

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- The seller "designs" (i.e. sets the rules) of the auction

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- **Before** observing his valuation, bidder 1 expected profit is
  $$\mathbb{E}\left(\frac{v_1^2}{2}\right) = \frac{1}{6}$$

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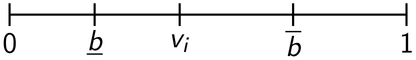

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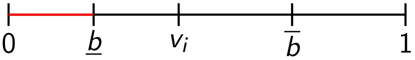

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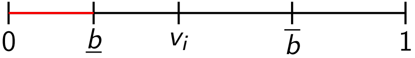

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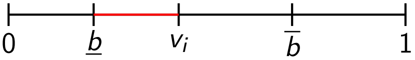

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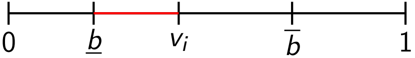

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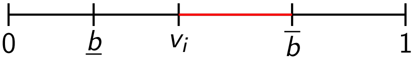

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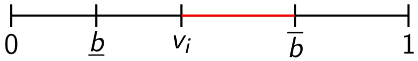

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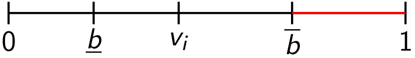

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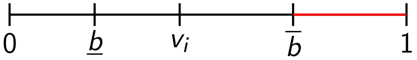

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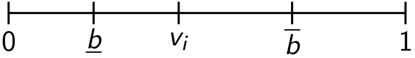

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  - Example: two bidders with valuations 0.4 and 0.6

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