# ECO 426 (Market Design) - Lecture 6

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October 26, 2015

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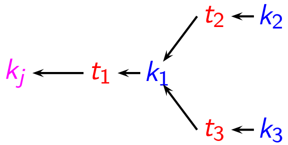


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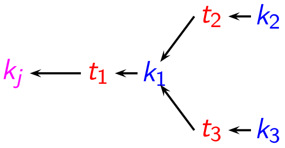


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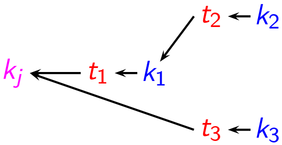


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- Describing "preferences" of a school $s$ by the students' exam scores in the school's category, $c_s$, one obtains an **associated college admissions**

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  - **No waste:** Whenever a student $i$ prefers another school $s$ to the one she is assigned to, school $s$ has no empty slot.
  - **Pareto efficiency:** There is no assignment that makes no **student** worse off and some **student** better off.

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- Proposition. A school placement matching is individually rational and eliminates waste and justified envy, if and only if it is stable in the associated college admission problem.

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- no-waste and individual rationality follow from Pareto efficiency

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| | | slots |
|---|---|---|
| $t_1$ | | |
| | $s_1$ | 1 |
| $t_2$ | | |
| | $s_2$ | 2 |
| $t_3$ | | |
| $t_4$ | $s_3$ | 1 |
| $t_5$ | $s_4$ | 2 |
| $t_6$ | | |

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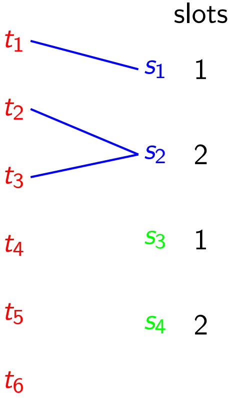

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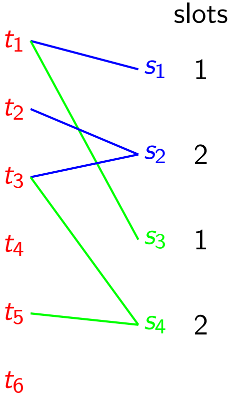

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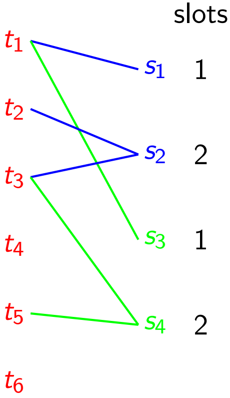

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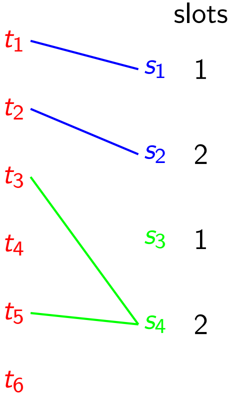

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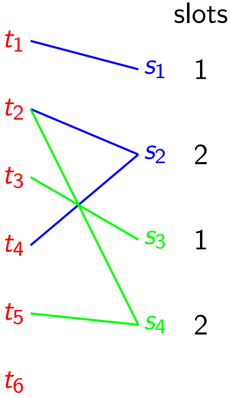

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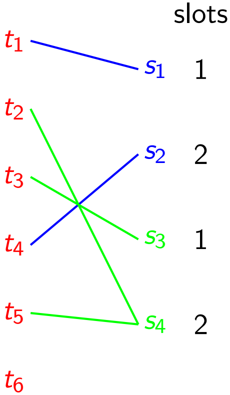

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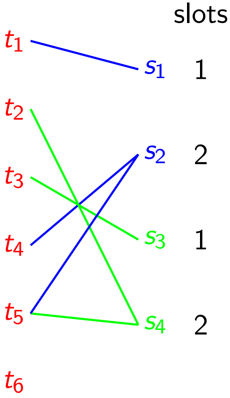

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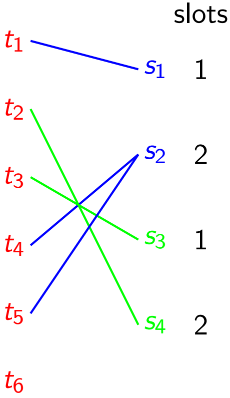

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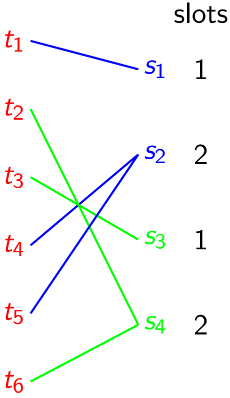

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  - Number of students administratively matched in NYC dropped to 3,000 from 30,000 after change in mechanism

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  - Limits: SIC procedure is not strategy proof.