

ECO 426 (Market Design) - Lecture 5

Ettore Damiano

October 19, 2015

Exchanging Kidneys

- Two types of kidney exchanges

Exchanging Kidneys

- Two types of kidney exchanges
 - Pairwise kidney exchange:

Exchanging Kidneys

- Two types of kidney exchanges
 - **Pairwise kidney exchange:** exchange kidney with another patient-donor pair

Exchanging Kidneys

- Two types of kidney exchanges
 - **Pairwise kidney exchange:** exchange kidney with another patient-donor pair

t_1

t_2

k_1

k_2

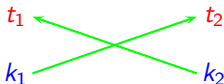
Exchanging Kidneys

- Two types of kidney exchanges
 - **Pairwise kidney exchange:** exchange kidney with another patient-donor pair



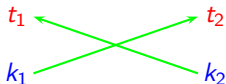
Exchanging Kidneys

- Two types of kidney exchanges
 - **Pairwise kidney exchange:** exchange kidney with another patient-donor pair



Exchanging Kidneys

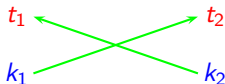
- Two types of kidney exchanges
 - **Pairwise kidney exchange:** exchange kidney with another patient-donor pair



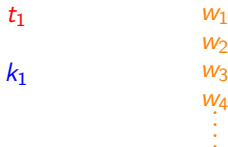
- **Exchange to list:** donate kidney to patient on waiting list in exchange of a better spot on waiting list

Exchanging Kidneys

- Two types of kidney exchanges
 - Pairwise kidney exchange:** exchange kidney with another patient-donor pair

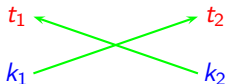


- Exchange to list:** donate kidney to patient on waiting list in exchange of a better spot on waiting list

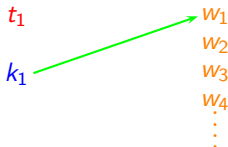


Exchanging Kidneys

- Two types of kidney exchanges
 - Pairwise kidney exchange:** exchange kidney with another patient-donor pair

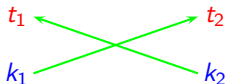


- Exchange to list:** donate kidney to patient on waiting list in exchange of a better spot on waiting list

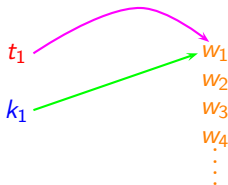


Exchanging Kidneys

- Two types of kidney exchanges
 - Pairwise kidney exchange:** exchange kidney with another patient-donor pair

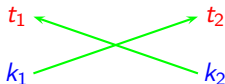


- Exchange to list:** donate kidney to patient on waiting list in exchange of a better spot on waiting list

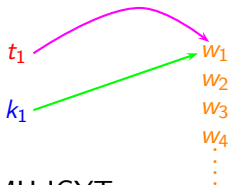


Exchanging Kidneys

- Two types of kidney exchanges
 - Pairwise kidney exchange:** exchange kidney with another patient-donor pair



- Exchange to list:** donate kidney to patient on waiting list in exchange of a better spot on waiting list



- Looks similar to YRMH-IGYT

Kidney exchange problem

- A Kidney exchange problem consists of:

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(\textcolor{red}{t}_1, \textcolor{blue}{k}_1), \dots, (\textcolor{red}{t}_n, \textcolor{blue}{k}_n)\}$

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(\mathbf{t}_1, \mathbf{k}_1), \dots, (\mathbf{t}_n, \mathbf{k}_n)\}$
 - For each patient, \mathbf{t}_i , a set of compatible kidneys $\mathbf{K}_i \subseteq \mathbf{K} = \{\mathbf{k}_1, \dots, \mathbf{k}_n\}$
 - For each patient, \mathbf{t}_i , a (strict) preference ordering over the set of compatible kidneys \mathbf{K}_i

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(\mathbf{t}_1, \mathbf{k}_1), \dots, (\mathbf{t}_n, \mathbf{k}_n)\}$
 - For each patient, \mathbf{t}_i , a set of compatible kidneys $\mathbf{K}_i \subseteq \mathbf{K} = \{\mathbf{k}_1, \dots, \mathbf{k}_n\}$
 - For each patient, \mathbf{t}_i , a (strict) preference ordering over the set of compatible kidneys \mathbf{K}_i **and** the option of exchanging own kidney, \mathbf{k}_i for priority \mathbf{w} on the waiting list

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that
 - The outcome is Pareto efficient,

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that
 - The outcome is Pareto efficient, **it is not possible to improve further the welfare of all**

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that
 - The outcome is Pareto efficient, **it is not possible to improve further the welfare of all**
 - For each patient, the outcome is never worse than not participating in the mechanism,

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that
 - The outcome is Pareto efficient, **it is not possible to improve further the welfare of all**
 - For each patient, the outcome is never worse than not participating in the mechanism, **ensures broad participation, no donor kidney is un-necessarily “wasted”**

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that
 - The outcome is Pareto efficient, **it is not possible to improve further the welfare of all**
 - For each patient, the outcome is never worse than not participating in the mechanism, **ensures broad participation, no donor kidney is un-necessarily “wasted”**
 - The mechanism is strategy proof,

Kidney exchange problem

- A Kidney exchange problem consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
 - For each patient, t_i , a (strict) preference ordering over the set of compatible kidneys K_i **and** the option of exchanging own kidney, k_i for priority w on the waiting list
- **Question:** How do we organize a kidney exchange program such that
 - The outcome is Pareto efficient, **it is not possible to improve further the welfare of all**
 - For each patient, the outcome is never worse than not participating in the mechanism, **ensures broad participation, no donor kidney is un-necessarily "wasted"**
 - The mechanism is strategy proof, **patients have incentive to disclose their preferences honestly**

Kidney exchange - model

- Assumptions:

Kidney exchange - model

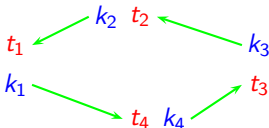
- Assumptions:
 - Multi-way exchanges:

Kidney exchange - model

- Assumptions:
 - Multi-way exchanges: No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

Kidney exchange - model

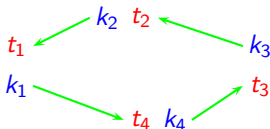
- Assumptions:
 - Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)



Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

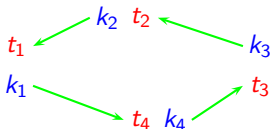


- List exchanges:**

Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

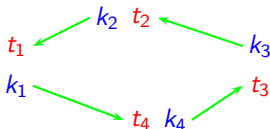


- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)

Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

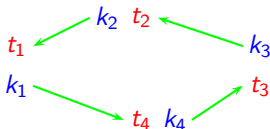


- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)
- Strict preferences:**

Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

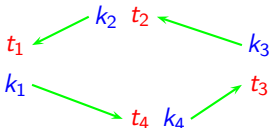


- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)
- Strict preferences:** No patient is indifferent between any two (compatible) alternatives

Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

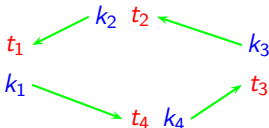


- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)
- Strict preferences:** No patient is indifferent between any two (compatible) alternatives (i.e. strict preferences over: compatible kidneys + trading donor for wait list + remaining with own donor)

Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)

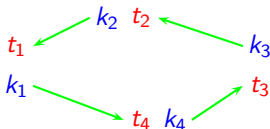


- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)
- Strict preferences:** No patient is indifferent between any two (compatible) alternatives (i.e. strict preferences over: compatible kidneys + trading donor for wait list + remaining with own donor)
- Interpreting live donors' kidneys as “owned” by their respective patients, the problem resembles one of house allocation with existing tenants

Kidney exchange - model

- Assumptions:

- Multi-way exchanges:** No constraint on the number of patient-donor pairs that can participate in an exchange (i.e. multi-way exchanges are allowed)



- List exchanges:** Exchange to list are possible (i.e. exchanging a kidney for a better spot on the wait-list)
- Strict preferences:** No patient is indifferent between any two (compatible) alternatives (i.e. strict preferences over: compatible kidneys + trading donor for wait list + remaining with own donor)
- Interpreting live donors' kidneys as “owned” by their respective patients, the problem resembles one of house allocation with existing tenants
 - Maximizing “supply” of live donors as maximizing participation

TTC(and C)

- TTC mechanism key properties

TTC(and C)

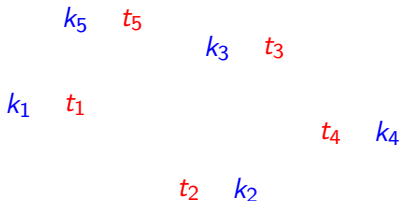
- TTC mechanism key properties
 - Each patient points to favorite kidney or the waiting list

TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney **or the waiting list**
 - Each kidney donor points to his/her patient

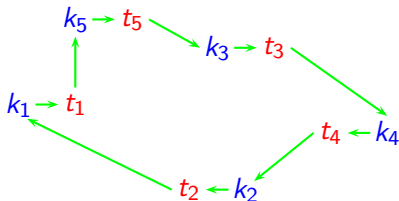
TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney **or the waiting list**
 - Each kidney donor points to his/her patient
- In a given round



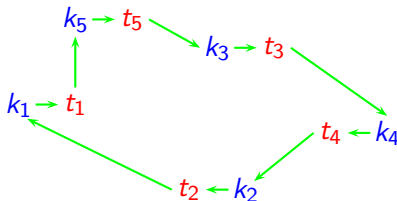
TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney **or the waiting list**
 - Each kidney donor points to his/her patient
- In a given round
 - A cycle might form



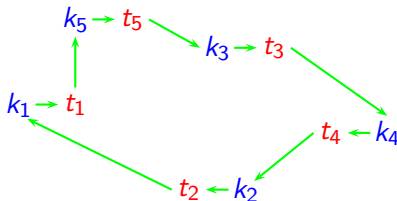
TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney **or the waiting list**
 - Each kidney donor points to his/her patient
- In a given round
 - A cycle might form
 - each patient in the cycle receives the best compatible kidney available



TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney **or the waiting list**
 - Each kidney donor points to his/her patient
- In a given round
 - A cycle might form
 - each patient in the cycle receives the best compatible kidney available
 - A cycle might not form



k_1 t_1

k_2 t_2

w

k_3 t_3

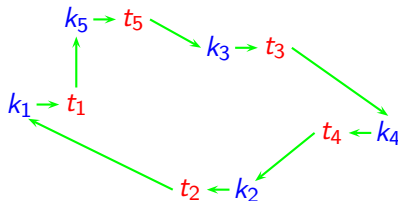
t_n k_n

\dots

t_4 k_4

TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney or the waiting list
 - Each kidney donor points to his/her patient
- In a given round
 - A cycle might form
 - each patient in the cycle receives the best compatible kidney available
 - A cycle might not form
 - some patient point to wait-list



k_1 t_1

k_2 t_2

w
 t_n k_n

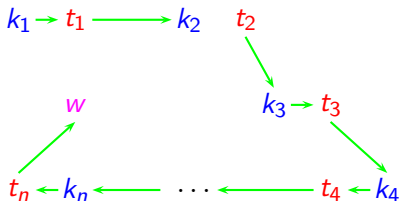
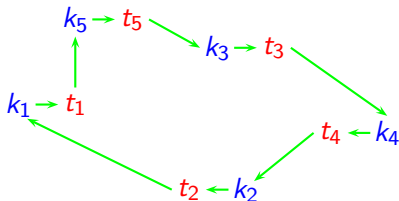
k_3 t_3

...

t_4 k_4

TTC(and C)

- TTC mechanism key properties
 - Each patient points to favorite kidney or the waiting list
 - Each kidney donor points to his/her patient
- In a given round
 - A cycle might form
 - each patient in the cycle receives the best compatible kidney available
 - A cycle might not form
 - some patient point to wait-list
 - If there is no cycle there must be at least a **w-chain**



Top Trading Cycles and Chains mechanism: key ideas

Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:

Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:
 - Carry out exchange

Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:
 - Carry out exchange
 - Remove kidneys and patients in cycle and restart

Top Trading Cycles and Chains mechanism: key ideas

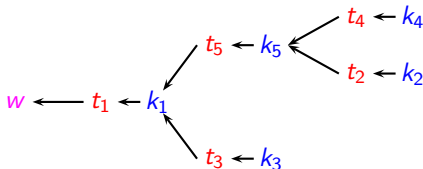
- When a cycle form:
 - Carry out exchange
 - Remove kidneys and patients in cycle and restart
- When no cycle form

Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:
 - Carry out exchange
 - Remove kidneys and patients in cycle and restart
- When no cycle form
 - There can be more than one chain

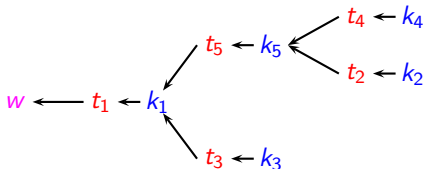
Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:
 - Carry out exchange
 - Remove kidneys and patients in cycle and restart
- When no cycle form
 - There can be more than one chain



Top Trading Cycles and Chains mechanism: key ideas

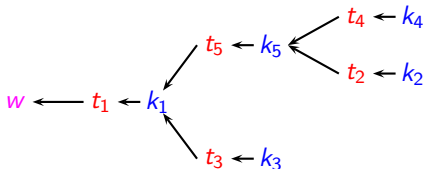
- When a cycle form:
 - Carry out exchange
 - Remove kidneys and patients in cycle and restart
- When no cycle form
 - There can be more than one chain



- Multiple chains can be in “competition” with each other

Top Trading Cycles and Chains mechanism: key ideas

- When a cycle form:
 - Carry out exchange
 - Remove kidneys and patients in cycle and restart
- When no cycle form
 - There can be more than one chain

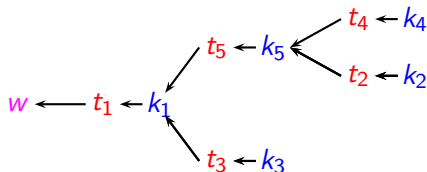


- Multiple chains can be in “competition” with each other
 - Need a **chain selection rule**

- Examples of chain selection rules

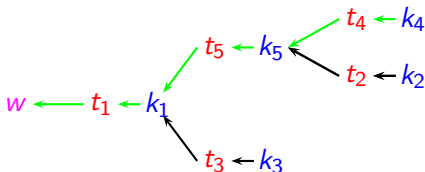
chain selection

- Examples of chain selection rules
- Choose based on length



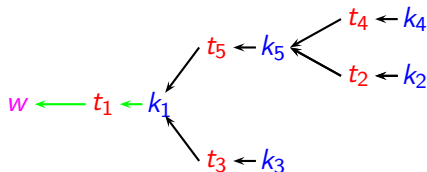
chain selection

- Examples of chain selection rules
- Choose based on length
 - longest chain



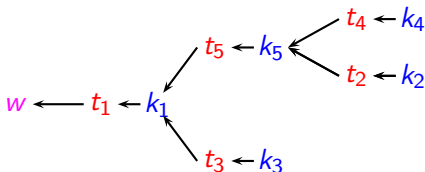
chain selection

- Examples of chain selection rules
- Choose based on length
 - longest chain
 - minimal chain



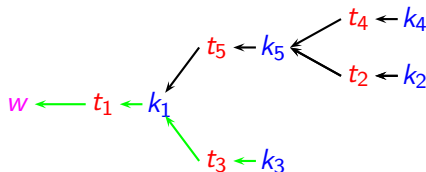
chain selection

- Examples of chain selection rules
- Choose based on length
 - longest chain
 - minimal chain
- Choose based on donor-patient priority



chain selection

- Examples of chain selection rules
- Choose based on length
 - longest chain
 - minimal chain
- Choose based on donor-patient priority
 - Choose chain with the highest priority donor-patient pair (e.g. t_3 , k_3)

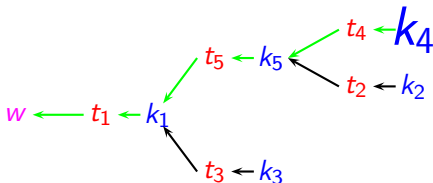


“tail kidney”

- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange

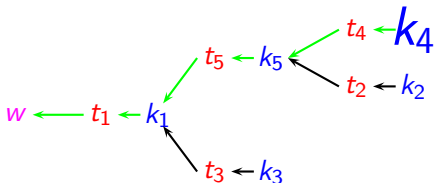
“tail kidney”

- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



“tail kidney”

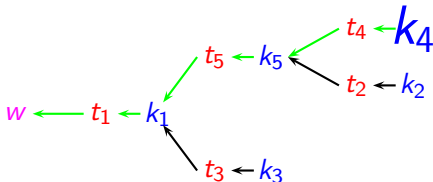
- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



- The tail kidney can be

“tail kidney”

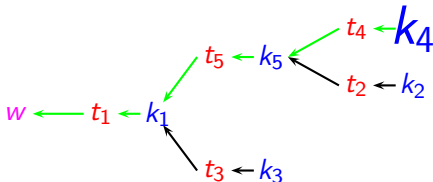
- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



- The tail kidney can be
 - Assigned to some compatible patient on wait list (i.e. list exchange)

“tail kidney”

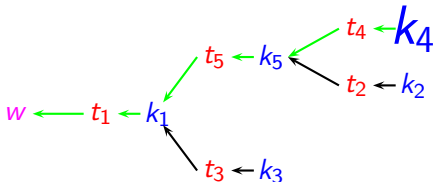
- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



- The tail kidney can be
 - Assigned to some compatible patient on wait list (i.e. list exchange) (might have welfare consequences, Pareto efficiency is not guaranteed)

“tail kidney”

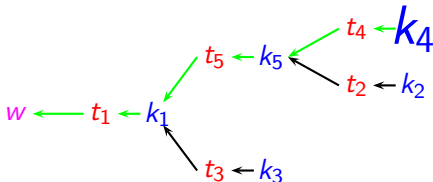
- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



- The tail kidney can be
 - Assigned to some compatible patient on wait list (i.e. list exchange) (might have welfare consequences, Pareto efficiency is not guaranteed)
 - Remain available to remaining patients on the kidney exchange program

“tail kidney”

- The “tail kidney” in a chain (i.e. the kidney of the last patient receiving a transplant in the kidney exchange) is not strictly needed for the exchange



- The tail kidney can be
 - Assigned to some compatible patient on wait list (i.e. list exchange) (might have welfare consequences, Pareto efficiency is not guaranteed)
 - Remain available to remaining patients on the kidney exchange program (guarantees Pareto efficiency)

Combining chain selection and tail kidney options

chain selection

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof,**

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof,**

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof**)

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof,**

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
- 5 Choose chain starting with highest priority patient-donor pair and keep tail kidney

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
- 5 Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof,**

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
- 5 Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof, Pareto efficient**)

Combining chain selection and tail kidney options

- 1 Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- 2 Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- 3 Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- 4 Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
- 5 Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof, Pareto efficient**) - equivalent to YRMH-IGYT

Combining chain selection and tail kidney options

- ① Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
- ② Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
- ③ Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
- ④ Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
- ⑤ Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof, Pareto efficient**) - equivalent to YRMH-IGYT
- Key properties

Combining chain selection and tail kidney options

- ① Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
 - ② Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
 - ③ Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
 - ④ Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
 - ⑤ Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof, Pareto efficient**) - equivalent to YRMH-IGYT
- Key properties
 - Minimal chains for strategy proofness

Combining chain selection and tail kidney options

- ① Choose longest chain and remove tail kidneys (**not strategy proof, not Pareto efficient**)
 - ② Choose longest chain and keep tail kidney (**not strategy proof, Pareto efficient**)
 - ③ Choose minimal chain and keep tail kidney (**strategy proof, Pareto efficient**)
 - ④ Choose chain starting with highest priority patient-donor pair and remove tail kidney (**Strategy proof, not Pareto efficient**)
 - ⑤ Choose chain starting with highest priority patient-donor pair and keep tail kidney (**Strategy proof, Pareto efficient**) - equivalent to YRMH-IGYT
- Key properties
 - Minimal chains for strategy proofness
 - Keep kidney for Pareto efficiency

Constrained kidney exchange

Practical shortcomings:

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict
 - Compatibility is treated as a binary variable (0-1)

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict
 - Compatibility is treated as a binary variable (0-1)
- List exchanges pose a “selection” problem

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict
 - Compatibility is treated as a binary variable (0-1)
- List exchanges pose a “selection” problem
 - Most common blood type is O-type

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict
 - Compatibility is treated as a binary variable (0-1)
- List exchanges pose a “selection” problem
 - Most common blood type is O-type
 - Most likely donor kidney exchanged to wait-list will be O-type incompatible (otherwise the donating patient would have it)

Constrained kidney exchange

Practical shortcomings:

- Multi-way exchanges can be difficult to implement
 - Being illegal to enter a contractual agreement for a “kidney exchange” all surgeries must be performed simultaneously to ensure compliance with the agreed exchange
 - Pairwise kidney exchange requires four “simultaneous” surgeries (two nephrectomies two kidney transplants)
 - Trilateral exchange requires six “simultaneous” surgeries etc.
- Preferences are “in practice” not strict
 - Compatibility is treated as a binary variable (0-1)
- List exchanges pose a “selection” problem
 - Most common blood type is O-type
 - Most likely donor kidney exchanged to wait-list will be O-type incompatible (otherwise the donating patient would have it)
 - List exchanges may harm O-type patient on wait-list

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
- The set of agents N and a compatibility matrix, R , suffice to describe the problem

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
- The set of agents N and a compatibility matrix, R , suffice to describe the problem
 - R is an $N \times N$ matrix

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
- The set of agents N and a compatibility matrix, R , suffice to describe the problem
 - R is an $N \times N$ matrix with

$$r_{i,j} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are compatible} \\ 0 & \text{otherwise} \end{cases}$$

Pairwise Kidney exchange with binary preferences

- A constrained bilateral Kidney exchange problem with binary (compatibility based) preferences consists of:
 - A set of donor-patient pairs $\{(t_1, k_1), \dots, (t_n, k_n)\}$
 - For each patient, t_i , a set of compatible kidneys $K_i \subseteq K = \{k_1, \dots, k_n\}$
- The set of agents N and a compatibility matrix, R , suffice to describe the problem
 - R is an $N \times N$ matrix with

$$r_{i,j} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are compatible} \\ 0 & \text{otherwise} \end{cases}$$

- **Objective:** Find a collection of bilateral kidney exchange among mutually compatible donor-patient pairs

Priority Mechanism

Priority Mechanism

- Order donor-patient pairs according to priorities

Priority Mechanism

- Order donor-patient pairs according to priorities
Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$

Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

Priority Mechanism

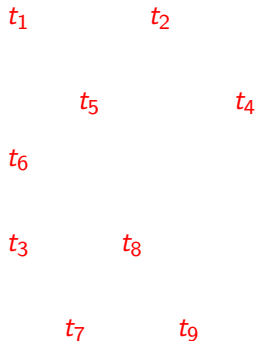
- Order donor-patient pairs according to priorities
Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$
 - medical priority and/or random

Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random

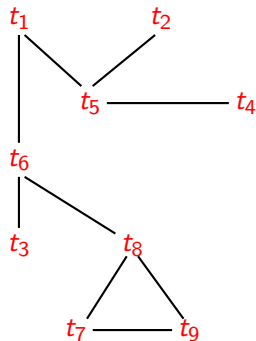


Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random



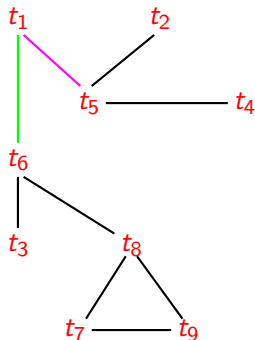
Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random

- match top priority patient if possible
(i.e. if there is a patient-donor pair mutually compatible with the priority 1 patient-donor pair), else skip

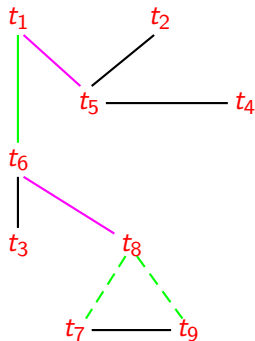


Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random
 - match top priority patient if possible (i.e. if there is a patient-donor pair mutually compatible with the priority 1 patient-donor pair), else skip
 - match priority 2 patient, if possible, in conjunction with priority 1 agent else skip priority 2 agent



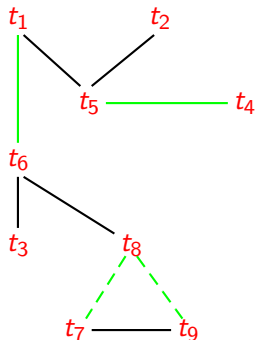
Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random

- match top priority patient if possible (i.e. if there is a patient-donor pair mutually compatible with the priority 1 patient-donor pair), else skip
- match priority 2 patient, if possible, in conjunction with priority 1 agent else skip priority 2 agent
- ...
- match priority n patient, if possible, in conjunction with all earlier priorities, else skip



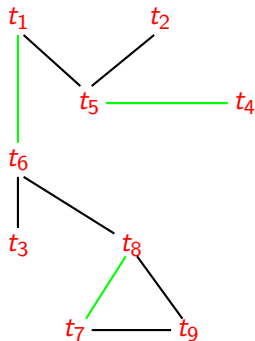
Priority Mechanism

- Order donor-patient pairs according to priorities

Example: Nine patient-donor pairs $\{t_1, t_2, \dots, t_9\}$ priority ordering $\{1, 8, 4, 2, 6, 3, 7, 9, 5\}$

- medical priority and/or random

- match top priority patient if possible (i.e. if there is a patient-donor pair mutually compatible with the priority 1 patient-donor pair), else skip
- match priority 2 patient, if possible, in conjunction with priority 1 agent else skip priority 2 agent
- ...
- match priority n patient, if possible, in conjunction with all earlier priorities, else skip



Priority Mechanism

- The priority mechanism is

Priority Mechanism

- The priority mechanism is
 - Pareto efficient

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B,O-A,A-B,A-B,B-A);

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B, O-A, A-B, A-B, B-A);
HLA incompatible pairs (A-A, A-A, A-A, B-O)

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B,O-A,A-B,A-B,B-A);
HLA incompatible pairs (A-A,A-A,A-A,B-O)
 - Only bilateral exchanges:

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B,O-A,A-B,A-B,B-A);
HLA incompatible pairs (A-A,A-A,A-A,B-O)
 - Only bilateral exchanges: (A-B,B-A) (A-A,A-A) (O-B,B-O)

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B,O-A,A-B,A-B,B-A);
HLA incompatible pairs (A-A,A-A,A-A,B-O)
 - Only bilateral exchanges: (A-B,B-A) (A-A,A-A) (O-B,B-O)
 - Bilateral and trilateral:

Priority Mechanism

- The priority mechanism is
 - Pareto efficient
 - Strategy proof
- Limits:
 - Allowing tri-lateral exchanges can make many more transplants possible
 - Additional benefits from more complex multi-lateral exchanges decline rapidly
- **Example:** Blood incompatible pairs (O-B,O-A,A-B,A-B,B-A);
HLA incompatible pairs (A-A,A-A,A-A,B-O)
 - Only bilateral exchanges: (A-B,B-A) (A-A,A-A) (O-B,B-O)
 - Bilateral and trilateral: (A-B,B-A) (A-A,A-A,A-A)
(B-O,O-A,A-B)

Kidney exchange programs

- New England Program for Kidney Exchange (2004)

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges
 - list exchanges allowed

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges
 - list exchanges allowed
 - altruistic donor exchanges

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges
 - list exchanges allowed
 - **altruistic donor exchanges** (i.e. chains starting from an altruistic live donor, rather than a list exchange)

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges
 - list exchanges allowed
 - **altruistic donor exchanges** (i.e. chains starting from an altruistic live donor, rather than a list exchange)
- Ohio - Living Kidney Donor Program

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges
 - list exchanges allowed
 - **altruistic donor exchanges** (i.e. chains starting from an altruistic live donor, rather than a list exchange)
- Ohio - Living Kidney Donor Program
 - Performed a Six-way paired kidney exchange (September 2011)

Kidney exchange programs

- New England Program for Kidney Exchange (2004)
 - Priority mechanism
 - up to 4-lateral exchanges
 - list exchanges allowed
 - **altruistic donor exchanges** (i.e. chains starting from an altruistic live donor, rather than a list exchange)
- Ohio - Living Kidney Donor Program
 - Performed a Six-way paired kidney exchange (September 2011)
- National program under construction

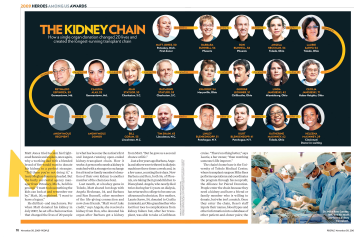
NEAD: Never Ending Altruistic Donor Chain

NEAD: Never Ending Altruistic Donor Chain

- Alliance for Paired Donation

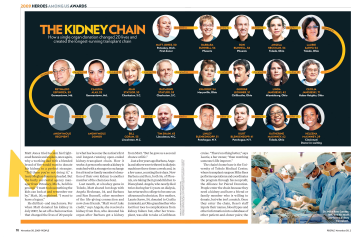
NEAD: Never Ending Altruistic Donor Chain

- Alliance for Paired Donation - 10 kidney transplant chain



NEAD: Never Ending Altruistic Donor Chain

- Alliance for Paired Donation - 10 kidney transplant chain



- National Kidney Registry

NEAD: Never Ending Altruistic Donor Chain

- Alliance for Paired Donation - 10 kidney transplant chain



- National Kidney Registry - 30 kidney transplant chain

