ECO 426 (Market Design) - Lecture 3

Ettore Damiano

September 28, 2014

one-sided strategy proofness

- A stable mechanism can be strategy proof for one-side of the market

- A stable mechanism can be strategy proof for one-side of the market

  Theorem: The men (women) proposing deferred acceptance algorithm is strategy-proof for the men (women).

one-sided strategy proofness

- A stable mechanism can be strategy proof for one-side of the market

  Theorem: The men (women) proposing deferred acceptance algorithm is strategy-proof for the men (women).

- If the true preferences are such that there is only one stable matching, no agent can benefit from misreporting their preferences

- A stable mechanism can be strategy proof for one-side of the market

  Theorem: The men (women) proposing deferred acceptance algorithm is strategy-proof for the men (women).

- If the true preferences are such that there is only one stable matching, no agent can benefit from misreporting their preferences
  - the unique stable matching is the outcome of both the DA men and DA women proposing algorithm

one-sided strategy proofness

- A stable mechanism can be strategy proof for one-side of the market

  Theorem: The men (women) proposing deferred acceptance algorithm is strategy-proof for the men (women).
- If the true preferences are such that there is only one stable matching, no agent can benefit from misreporting their preferences
  - the unique stable matching is the outcome of both the DA men and DA women proposing algorithm
- When there are multiple stable matchings, how much can a woman gain by manipulating her preferences in the DA men proposing mechanism?

benefits from preference manipulation

benefits from preference manipulation

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

$$m_1 \mid w_1 \quad w_2 \quad w_4 \quad w_3 \quad w_6$$

benefits from preference manipulation

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

m_1	w_1	w_2	w_4	w_3	w_6
m_1	w_1	w_2	w_4		

benefits from preference manipulation

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

m_1	w_1	w_2	w_4	w_3	w_6
m_1	w_1	w_2	w_4		

- Theorem Provided all other participants are truthful, in the DA men proposing mechanism, a woman can achieve her best possible match by truncating her preference list and stopping with the man who is the best achievable in any stable matching.

benefits from preference manipulation

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

$$\begin{array}{c|ccccc} m_1 & w_1 & w_2 & w_4 & w_3 & w_6 \\ m_1 & w_1 & w_2 & w_4 & & \end{array}$$

- Theorem Provided all other participants are truthful, in the DA men proposing mechanism, a woman can achieve her best possible match by truncating her preference list and stopping with the man who is the best achievable in any stable matching.
    - Limits to preference manipulation: can yield at most the best partner across stable matchings

benefits from preference manipulation

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

$$\begin{array}{c|ccccc} m_1 & w_1 & w_2 & w_4 & w_3 & w_6 \\ m_1 & w_1 & w_2 & w_4 & & \end{array}$$

- Theorem Provided all other participants are truthful, in the DA men proposing mechanism, a woman can achieve her best possible match by truncating her preference list and stopping with the man who is the best achievable in any stable matching.
  - Limits to preference manipulation: can yield at most the best partner across stable matchings
  - Bound is tight: there exists a "simple" manipulation strategy that achieves the best possible match

benefits from preference manipulation

- An agent can truncate his/her preference list by reporting as unacceptable one or more acceptable partner, starting from the least desirable

m_1	w_1	w_2	w_4	w_3	w_6
m_1	w_1	w_2	w_4		

- Theorem Provided all other participants are truthful, in the DA men proposing mechanism, a woman can achieve her best possible match by truncating her preference list and stopping with the man who is the best achievable in any stable matching.
  - Limits to preference manipulation: can yield at most the best partner across stable matchings
  - Bound is tight: there exists a "simple" manipulation strategy that achieves the best possible match
  - The manipulation strategy is informationally demanding

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)
   - Thus the manipulating woman is matched in every stable matching under the new preferences

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)
   - Thus the manipulating woman is matched in every stable matching under the new preferences
3. Any matching that gives the manipulating woman an even better partner is blocked under the true preferences.

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)
    - Thus the manipulating woman is matched in every stable matching under the new preferences
3. Any matching that gives the manipulating woman an even better partner is blocked under the true preferences. The pair that blocked under the true preferences still blocks after the manipulation

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)
   - Thus the manipulating woman is matched in every stable matching under the new preferences
3. Any matching that gives the manipulating woman an even better partner is blocked under the true preferences. The pair that blocked under the true preferences still blocks after the manipulation
4. Therefore, the manipulating woman is getting her best possible match after the manipulation.

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)
   - Thus the manipulating woman is matched in every stable matching under the new preferences
3. Any matching that gives the manipulating woman an even better partner is blocked under the true preferences. The pair that blocked under the true preferences still blocks after the manipulation
4. Therefore, the manipulating woman is getting her best possible match after the manipulation.

- Agents who are unmatched in a stable matching, are unmatched in all stable matchings (rural hospital theorem),

benefits from preference manipulation - sketch of proof

1. The woman optimal matching is still a stable matching after the manipulation
2. The set of matched agents is the same under any stable matching (rural hospital theorem)
   - Thus the manipulating woman is matched in every stable matching under the new preferences
3. Any matching that gives the manipulating woman an even better partner is blocked under the true preferences. The pair that blocked under the true preferences still blocks after the manipulation
4. Therefore, the manipulating woman is getting her best possible match after the manipulation.

- Agents who are unmatched in a stable matching, are unmatched in all stable matchings (rural hospital theorem), hence they cannot gain from preference manipulation.

exernalities

- We implicitly assumed no externalities

exernalities

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only

exernalities

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist

exernalities

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.

exernalities

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple,

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students.

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.

exernalities

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s

- We implicitly assumed no externalities
    - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
    - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
    - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
    - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)
  - h_2 prefers s to m or w

exernalities

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)
  - h_2 prefers s to m or w
  - s prefers h_1 to h_2

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)
  - h_2 prefers s to m or w
  - s prefers h_1 to h_2
  - No stable matching:

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)
  - h_2 prefers s to m or w
  - s prefers h_1 to h_2
  - No stable matching:
    - if m and w are employed, s and h_2 block the matching

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)
  - h_2 prefers s to m or w
  - s prefers h_1 to h_2
  - No stable matching:
    - if m and w are employed, s and h_2 block the matching
    - if s is employed by h_2, s and h_1 block the matching

- We implicitly assumed no externalities
  - Preferences of each agents are defined over own partners only
- If agents care about others' matches a stable matching might not exist
- Example: **Couples** might care about joint location when looking for jobs.
  - m and w are two medical students in a couple, s is a single medical students. h_1 and h_2 are two hospitals in the same area, each with one opening.
  - Both m and w find unacceptable a job at one hospital if the partner is not hired by the other hospital
  - h_1 prefers m or w to s (i.e. $m \succ w \succ s$ or $w \succ m \succ s$)
  - h_2 prefers s to m or w
  - s prefers h_1 to h_2
  - No stable matching:
    - if m and w are employed, s and h_2 block the matching
    - if s is employed by h_2, s and h_1 block the matching
    - if s is employed by h_1 the couple and the two hospitals "block"

large markets

- Incentive to manipulate preferences are “small” in large markets

- Incentive to manipulate preferences are “small” in large markets
  - The proportion of woman who can benefit from manipulation shrinks to zero in the DA men proposing mechanism as the number of agents grows

- Incentive to manipulate preferences are "small" in large markets
  - The proportion of woman who can benefit from manipulation shrinks to zero in the DA men proposing mechanism as the number of agents grows (and agents preferences are independent uniform draws over all possible rankings)

large markets

- Incentive to manipulate preferences are "small" in large markets
  - The proportion of woman who can benefit from manipulation shrinks to zero in the DA men proposing mechanism as the number of agents grows (and agents preferences are independent uniform draws over all possible rankings)
  - The loss from switching from DA men proposal to DA women proposal does not make a big difference (1998 change in the NRMP)

- Incentive to manipulate preferences are “small” in large markets
  - The proportion of woman who can benefit from manipulation shrinks to zero in the DA men proposing mechanism as the number of agents grows (and agents preferences are independent uniform draws over all possible rankings)
  - The loss from switching from DA men proposal to DA women proposal does not make a big difference (1998 change in the NRMP)
- Probability that a stable matching exists with a fixed number of couples converges to one as the number of agents grows.

large markets

- Incentive to manipulate preferences are “small” in large markets
  - The proportion of woman who can benefit from manipulation shrinks to zero in the DA men proposing mechanism as the number of agents grows (and agents preferences are independent uniform draws over all possible rankings)
  - The loss from switching from DA men proposal to DA women proposal does not make a big difference (1998 change in the NRMP)
- Probability that a stable matching exists with a fixed number of couples converges to one as the number of agents grows.
  - Consistent with practice - NRMP has always been able to find a stable matching

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
    - Each firm f has a quota q of jobs to fill

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
  - Each firm f has a quota q of jobs to fill
  - Each firm f (strictly) ranks workers

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
  - Each firm f has a quota q of jobs to fill
  - Each firm f (strictly) ranks workers
  - Replacing a worker with a higher ranked worker (or a vacancy with an acceptable worker) makes f better off.

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
  - Each firm f has a quota q of jobs to fill
  - Each firm f (strictly) ranks workers
  - Replacing a worker with a higher ranked worker (or a vacancy with an acceptable worker) makes f better off.
- Stable matching definition changes:

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
  - Each firm f has a quota q of jobs to fill
  - Each firm f (strictly) ranks workers
  - Replacing a worker with a higher ranked worker (or a vacancy with an acceptable worker) makes f better off.
- Stable matching definition changes:
  - each firm does not exceed its quota;

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
  - Each firm f has a quota q of jobs to fill
  - Each firm f (strictly) ranks workers
  - Replacing a worker with a higher ranked worker (or a vacancy with an acceptable worker) makes f better off.
- Stable matching definition changes:
  - each firm does not exceed its quota;
  - there is not a worker and a firm pair such that: i) the worker prefers the firm to his current match;

many-to-one matching

- Firms often have multiple openings to fill (while workers are still looking for one job)
- Matching is a pairing of a firm to (possibly) many workers
- Need to define preferences of firms over multiple workers
- Simplest possible extension (responsive preferences):
  - Each firm f has a quota q of jobs to fill
  - Each firm f (strictly) ranks workers
  - Replacing a worker with a higher ranked worker (or a vacancy with an acceptable worker) makes f better off.
- Stable matching definition changes:
  - each firm does not exceed its quota;
  - there is not a worker and a firm pair such that: i) the worker prefers the firm to his current match; and ii) the firm prefers the worker to one of its current workers (or vacancy).

- In the DA algorithm can treat each firm as “multiple” firms, one for each vacancy, with identical preferences over workers.

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold

many-to-one matching

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)

many-to-one matching

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms

many-to-one matching

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms
  - All firms fill the same number of position and the same workers find jobs, across all stable matchings

many-to-one matching

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms
  - All firms fill the same number of position and the same workers find jobs, across all stable matchings
    - Vacancy rate in each hospital is constant across all stable mechanism

many-to-one matching

- In the DA algorithm can treat each firm as “multiple” firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms
  - All firms fill the same number of position and the same workers find jobs, across all stable matchings
    - Vacancy rate in each hospital is constant across all stable mechanism
    - Cannot change the vacancy rate in rural hospital if sticking to stable mechanism

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms
  - All firms fill the same number of position and the same workers find jobs, across all stable matchings Rural hospital theorem
    - Vacancy rate in each hospital is constant across all stable mechanism
    - Cannot change the vacancy rate in rural hospital if sticking to stable mechanism

many-to-one matching

- In the DA algorithm can treat each firm as “multiple” firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms
  - All firms fill the same number of position and the same workers find jobs, across all stable matchings Rural hospital theorem
    - Vacancy rate in each hospital is constant across all stable mechanism
    - Cannot change the vacancy rate in rural hospital if sticking to stable mechanism
- Some results do not hold

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
  - The DA algorithm yields a stable matching (a stable matching exists)
  - Firms proposing DA results in best stable matching for firms
  - All firms fill the same number of position and the same workers find jobs, across all stable matchings Rural hospital theorem
    - Vacancy rate in each hospital is constant across all stable mechanism
    - Cannot change the vacancy rate in rural hospital if sticking to stable mechanism
- Some results do not hold
  - No stable mechanism is strategy proof for the hospital

many-to-one matching

- In the DA algorithm can treat each firm as "multiple" firms, one for each vacancy, with identical preferences over workers.
- Some results still hold
    - The DA algorithm yields a stable matching (a stable matching exists)
    - Firms proposing DA results in best stable matching for firms
    - All firms fill the same number of position and the same workers find jobs, across all stable matchings Rural hospital theorem
        - Vacancy rate in each hospital is constant across all stable mechanism
        - Cannot change the vacancy rate in rural hospital if sticking to stable mechanism
- Some results do not hold
    - No stable mechanism is strategy proof for the hospital (no stable mechanism is collusion proof)

more general preferences

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)

more general preferences

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)
- Preferences of a firm are described by an ordered list of subsets of workers

more general preferences

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)
- Preferences of a firm are described by an ordered list of subsets of workers

Example $F = \{f_1, f_2\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)
- Preferences of a firm are described by an ordered list of subsets of workers

Example $F = \{f_1, f_2\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has a quota of 2 and "responsive" preferences (w_1, w_2, w_3)

more general preferences

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)
- Preferences of a firm are described by an ordered list of subsets of workers

Example $F = \{f_1, f_2\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has a quota of 2 and "responsive" preferences (w_1, w_2, w_3)

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

more general preferences

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)
- Preferences of a firm are described by an ordered list of subsets of workers

Example $F = \{f_1, f_2\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has a quota of 2 and "responsive" preferences (w_1, w_2, w_3)

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 2 has arbitrary preferences

- More generally, firms might care about the composition of their workforce (e.g. an hospital might not want to hire two neurosurgeons)
- Preferences of a firm are described by an ordered list of subsets of workers

Example $F = \{f_1, f_2\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has a quota of 2 and "responsive" preferences (w_1, w_2, w_3)

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 2 has arbitrary preferences

$$f_2 \mid \{w_1, w_3, w_5\} \quad \{w_2, w_4\} \quad \{w_1, w_2, w_3\} \quad \{w_1\} \quad \{w_1 w_2\} \quad \emptyset$$

substitutable preferences

- With arbitrary preferences a stable matching might not exists

substitutable preferences

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"
- Given a set of workers A, the set of workers rejected by firm f if it were able to choose freely is denoted $R_f(A)$

substitutable preferences

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"
- Given a set of workers A, the set of workers rejected by firm f if it were able to choose freely is denoted $R_f(A)$

  Definition: A firm f has substitutes preferences if,

$$A' \subset A \quad \Rightarrow \quad R_f(A') \subseteq R_f(A)$$

substitutable preferences

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"
- Given a set of workers A, the set of workers rejected by firm f if it were able to choose freely is denoted $R_f(A)$

  Definition: A firm f has substitutes preferences if,

$$A' \subset A \quad \Rightarrow \quad R_f(A') \subseteq R_f(A)$$

  - the set of workers rejected does not shrink when the set of workers available for choosing expands

substitutable preferences

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"
- Given a set of workers A, the set of workers rejected by firm f if it were able to choose freely is denoted $R_f(A)$

  Definition: A firm f has substitutes preferences if,

$$A' \subset A \quad \Rightarrow \quad R_f(A') \subseteq R_f(A)$$

  - the set of workers rejected does not shrink when the set of workers available for choosing expands
  - rules out complementarities among workers

substitutable preferences

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"
- Given a set of workers A, the set of workers rejected by firm f if it were able to choose freely is denoted $R_f(A)$

  Definition: A firm f has substitutes preferences if,

$$A' \subset A \quad \Rightarrow \quad R_f(A') \subseteq R_f(A)$$

  - the set of workers rejected does not shrink when the set of workers available for choosing expands
  - rules out complementarities among workers
  - responsive preferences are always substitutes,

substitutable preferences

- With arbitrary preferences a stable matching might not exists
  - cf. non existence of stable matching with externalities
- Restrict to "substitutable preferences"
- Given a set of workers A, the set of workers rejected by firm f if it were able to choose freely is denoted $R_f(A)$

  Definition: A firm f has substitutes preferences if,

$$A' \subset A \quad \Rightarrow \quad R_f(A') \subseteq R_f(A)$$

  - the set of workers rejected does not shrink when the set of workers available for choosing expands
  - rules out complementarities among workers
  - responsive preferences are always substitutes, the reverse is not true

substitutable preferences

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

substitutable preferences

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has "responsive" as well as "substitutes" preferences

$$f_1 \;\Big|\; \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

substitutable preferences

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has "responsive" as well as "substitutes" preferences

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 3's preferences are "substitutes" but not "responsive"

$$f_3 \mid \{w_2, w_3\} \quad \{w_1, w_3\} \quad \{w_1, w_2\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has "responsive" as well as "substitutes" preferences

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 3's preferences are "substitutes" but not "responsive"

$$f_3 \mid \{w_2, w_3\} \quad \{w_1, w_3\} \quad \{w_1, w_2\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- never reject w_2 and w_3; reject w_1 only if both w_2 and w_3 are available

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has "responsive" as well as "substitutes" preferences

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 3's preferences are "substitutes" but not "responsive"

$$f_3 \mid \{w_2, w_3\} \quad \{w_1, w_3\} \quad \{w_1, w_2\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- never reject w_2 and w_3; reject w_1 only if both w_2 and w_3 are available

- Firm 2 has arbitrary preferences

$$f_2 \mid \{w_1, w_3, w_5\} \quad \{w_2, w_4\} \quad \{w_1, w_2, w_3\} \quad \{w_1\} \quad \{w_1 w_2\} \quad \emptyset$$

substitutable preferences

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has "responsive" as well as "substitutes" preferences

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 3's preferences are "substitutes" but not "responsive"

$$f_3 \mid \{w_2, w_3\} \quad \{w_1, w_3\} \quad \{w_1, w_2\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

  - never reject w_2 and w_3; reject w_1 only if both w_2 and w_3 are available

- Firm 2 has arbitrary preferences

$$f_2 \mid \{w_1, w_3, w_5\} \quad \{w_2, w_4\} \quad \{w_1, w_2, w_3\} \quad \{w_1\} \quad \{w_1 w_2\} \quad \emptyset$$

  - w_1 is rejected if all workers but w_3 are available,

Example $F = \{f_1, f_2, f_3\}$ and $W = \{w_1, w_2, w_3, w_4, w_5\}$

- Firm 1 has "responsive" as well as "substitutes" preferences

$$f_1 \mid \{w_1, w_2\} \quad \{w_1, w_3\} \quad \{w_2, w_3\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- Firm 3's preferences are "substitutes" but not"responsive"

$$f_3 \mid \{w_2, w_3\} \quad \{w_1, w_3\} \quad \{w_1, w_2\} \quad \{w_1\} \quad \{w_2\} \quad \{w_3\} \quad \emptyset$$

- never reject w_2 and w_3; reject w_1 only if both w_2 and w_3 are available

- Firm 2 has arbitrary preferences

$$f_2 \mid \{w_1, w_3, w_5\} \quad \{w_2, w_4\} \quad \{w_1, w_2, w_3\} \quad \{w_1\} \quad \{w_1 w_2\} \quad \emptyset$$

- w_1 is rejected if all workers but w_3 are available, and is not rejected when all workers are available.

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.
- DA algorithm with workers proposing

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.
- DA algorithm with workers proposing
  - In each round a firm **"holds"** the **favorite set** of workers among those proposing and those held from previous round

substitutable preferences - existence of stable matching

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.
- DA algorithm with workers proposing
    - In each round a firm **"holds"** the **favorite set** of workers among those proposing and those held from previous round and rejects the remaining

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.
- DA algorithm with workers proposing
    - In each round a firm **"holds"** the **favorite set** of workers among those proposing and those held from previous round and rejects the remaining
    - If a worker is rejected by a firm in a given round, a new offer by the same worker to the same hospital would be rejected in any later round

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.
- DA algorithm with workers proposing
  - In each round a firm **"holds"** the **favorite set** of workers among those proposing and those held from previous round and rejects the remaining
  - If a worker is rejected by a firm in a given round, a new offer by the same worker to the same hospital would be rejected in any later round
  - When the algorithm ends the outcome is stable (no workers offer to an hospital that he prefers would be accepted)

substitutable preferences - existence of stable matching

Theorem Suppose firms have substitutes preferences. Then the DA algorithm yields a stable matching.

- A stable matching exists.
- DA algorithm with workers proposing
  - In each round a firm **"holds"** the **favorite set** of workers among those proposing and those held from previous round and rejects the remaining
  - If a worker is rejected by a firm in a given round, a new offer by the same worker to the same hospital would be rejected in any later round
  - When the algorithm ends the outcome is stable (no workers offer to an hospital that he prefers would be accepted)
- A firm never "regrets" making a rejection.

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:
    - owns a "house," h_a,

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:
    - owns a "house," h_a, (H is the set of all houses);

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:
    - owns a “house,” h_a, (H is the set of all houses);
    - has (strict) preferences over the set of houses in the economy

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:
    - owns a "house," h_a, (H is the set of all houses);
    - has (strict) preferences over the set of houses in the economy
  - the initial allocation might not be efficient (i.e. Pareto efficient)

Matching when only one side has preferences

Some allocation problems can be modelled as two-sided matching markets but with one side not having any preferences over the possible allocations

- Housing market (allocating houses to individuals)
  - A collection of individuals, A (agents)
  - each agent $a \in A$:
    - owns a "house," h_a, (H is the set of all houses);
    - has (strict) preferences over the set of houses in the economy
  - the initial allocation might not be efficient (i.e. Pareto efficient)
    - mutually beneficial trades might be possible

Housing Market

- Housing market vs. marriage market

- Housing market vs. marriage market
  - one side of the market (houses) has no preferences over matches;

Housing Market

- Housing market vs. marriage market
  - one side of the market (houses) has no preferences over matches;
  - agents have an initial endowment (i.e. each agent owns a house)

Housing Market

- Housing market vs. marriage market
  - one side of the market (houses) has no preferences over matches;
  - agents have an initial endowment (i.e. each agent owns a house)
    - the market starts from a default allocation where each agent is matched to his own house

- Housing market vs. marriage market
  - one side of the market (houses) has no preferences over matches;
  - agents have an initial endowment (i.e. each agent owns a house)
    - the market starts from a default allocation where each agent is matched to his own house
  - Goal: find a matching that cannot be improved

Housing Market

- Housing market vs. marriage market
  - one side of the market (houses) has no preferences over matches;
  - agents have an initial endowment (i.e. each agent owns a house)
    - the market starts from a default allocation where each agent is matched to his own house
  - Goal: find a matching that cannot be improved
    - it is not possible to reassign houses making some agent better off and making no agent worse off

- An allocation is an assignment (matching) of agents to houses such that

Housing Market - CORE

- An allocation is an assignment (matching) of agents to houses such that
  - each agent is assigned exactly one house; and

- An allocation is an assignment (matching) of agents to houses such that
  - each agent is assigned exactly one house; and
  - each house is assigned to exactly one agent.

Housing Market - CORE

- An allocation is an assignment (matching) of agents to houses such that
  - each agent is assigned exactly one house; and
  - each house is assigned to exactly one agent.
- An allocation in an housing market is described by a **"bijection"** $\mu : A \rightarrow H$.

Housing Market - CORE

- An allocation is an assignment (matching) of agents to houses such that
  - each agent is assigned exactly one house; and
  - each house is assigned to exactly one agent.
- An allocation in an housing market is described by a **"bijection"** $\mu : A \longrightarrow H$.
- In a housing market, each agent is endowed (owns) one house (e.g. a owns h_a)

Housing Market - CORE

- An allocation is an assignment (matching) of agents to houses such that
  - each agent is assigned exactly one house; and
  - each house is assigned to exactly one agent.
- An allocation in an housing market is described by a **"bijection"** $\mu : A \longrightarrow H$.
- In a housing market, each agent is endowed (owns) one house (e.g. a owns h_a)
- What allocations would we expect to arise if agents can freely dispose of their endowment?

Housing Market - CORE

- Agents in a group $S \subseteq A$ own together (in a "coalition") a subset of the houses in the market H_S

Housing Market - CORE

- Agents in a group $S \subseteq A$ own together (in a "coalition") a subset of the houses in the market H_S
- The agents in a coalition S can "independently" distribute the houses they own, H_S, among themselves.

Housing Market - CORE

- Agents in a group $S \subseteq A$ own together (in a "coalition") a subset of the houses in the market H_S
- The agents in a coalition S can "independently" distribute the houses they own, H_S, among themselves.
  - An assignment of the houses in H_S to agents in S, is an allocation in the housing market where the set of agents is S and the set of houses is H_S

Housing Market - CORE

- Agents in a group $S \subseteq A$ own together (in a "coalition") a subset of the houses in the market H_S
- The agents in a coalition S can "independently" distribute the houses they own, H_S, among themselves.
  - An assignment of the houses in H_S to agents in S, is an allocation in the housing market where the set of agents is S and the set of houses is H_S

$$\mu_S : S \rightarrow H_S.$$

Housing Market - CORE

- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that:

Housing Market - CORE

- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that: i) some member of S prefers μ_S to μ;

- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that: i) some member of S prefers μ_S to μ; ii) no member of S prefers μ to μ_S.

- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that: i) some member of S prefers μ_S to μ; ii) no member of S prefers μ to μ_S.
    - A blocking coalition can find a mutually beneficial trade (i.e. an exchange of houses among members of the coalition that improves all members' welfare with respect to the allocation μ)

Housing Market - CORE

- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that: i) some member of S prefers μ_S to μ; ii) no member of S prefers μ to μ_S.
  - A blocking coalition can find a mutually beneficial trade (i.e. an exchange of houses among members of the coalition that improves all members' welfare with respect to the allocation μ)
- Definition (Core) An allocation is in the core of the housing market if it is not blocked by any coalition.
  - At a core allocation benefits from trade are exhausted

Housing Market - CORE

- Definition (Blocking) A coalition of agents S **blocks** an allocation μ, if there is an assignment μ_S of the houses owned by the coalition to the members of the coalition S, such that: i) some member of S prefers μ_S to μ; ii) no member of S prefers μ to μ_S.
  - A blocking coalition can find a mutually beneficial trade (i.e. an exchange of houses among members of the coalition that improves all members' welfare with respect to the allocation μ)
- Definition (Core) An allocation is in the core of the housing market if it is not blocked by any coalition.
  - At a core allocation benefits from trade are exhausted
  - In a marriage market, core matchings and stable matchings coincide

Gale's Top trading cycle algorithm

Gale's Top trading cycle algorithm

- each agent points to his/her preferred house

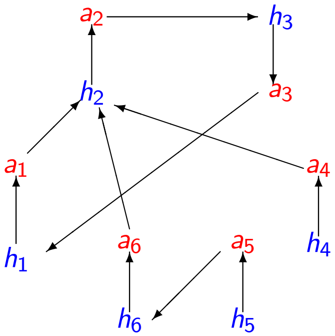
Gale's Top trading cycle algorithm

- each agent points to his/her preferred house
- each house points to its owner

Housing Market - TTC

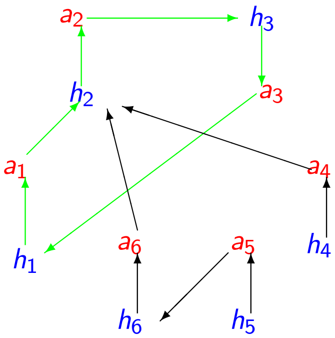
Gale's Top trading cycle algorithm

- each agent points to his/her preferred house
- each house points to its owner



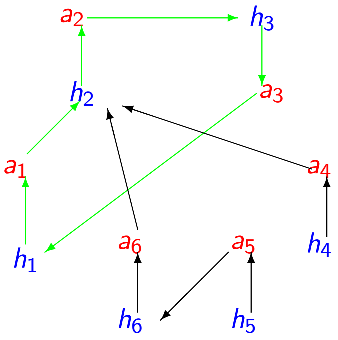
- there is at least one cycle

- there is at least one cycle



Housing Market - TTC

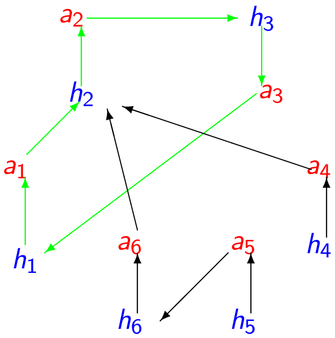
- there is at least one cycle



- remove all cycles assigning houses to agents

Housing Market - TTC

- there is at least one cycle



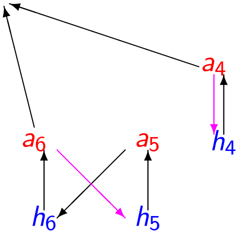
- remove all cycles assigning houses to agents
  - agents within a cycle exchange houses among each others

Housing Market - TTC

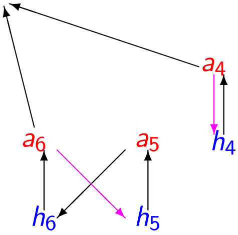
- each remaining agent points to his/her preferred remaining house

Housing Market - TTC

- each remaining agent points to his/her preferred remaining house



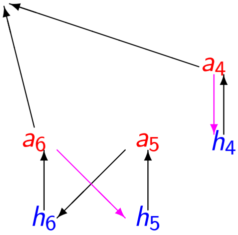
- each remaining agent points to his/her preferred remaining house



- remove all cycles assigning houses to agents

Housing Market - TTC

- each remaining agent points to his/her preferred remaining house



- remove all cycles assigning houses to agents
- continue until no agent/house is left

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

- The outcome of the TTC mechanism cannot be blocked

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

- The outcome of the TTC mechanism cannot be blocked
  - cannot make any agent matched in the first round better off

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

- The outcome of the TTC mechanism cannot be blocked
  - cannot make any agent matched in the first round better off (they are getting their favourite house)

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

- The outcome of the TTC mechanism cannot be blocked
  - cannot make any agent matched in the first round better off (they are getting their favourite house)
  - cannot make any agent matched in the second round better off without making some of the agents matched in the first round worse off

TTC and core

Theorem The outcome of the TTC mechanism is the unique core allocation of the housing market.

- The outcome of the TTC mechanism cannot be blocked
  - cannot make any agent matched in the first round better off (they are getting their favourite house)
  - cannot make any agent matched in the second round better off without making some of the agents matched in the first round worse off
  - cannot make any agent matched in round n better off without making some agents matched in earlier rounds worse off.

TTC and strategic incentives

Theorem The TTC algorithm is a strategy proof mechanism.

TTC and strategic incentives

Theorem The TTC algorithm is a strategy proof mechanism.

- an agent matched in round n cannot, by manipulating his/her preferences, break any of the cycles that form before round n

TTC and strategic incentives

Theorem The TTC algorithm is a strategy proof mechanism.

- an agent matched in round n cannot, by manipulating his/her preferences, break any of the cycles that form before round n
  - preference manipulation cannot give the agent a house that was assigned earlier than round n.

Theorem The TTC algorithm is a strategy proof mechanism.

- an agent matched in round n cannot, by manipulating his/her preferences, break any of the cycles that form before round n
  - preference manipulation cannot give the agent a house that was assigned earlier than round n.
- getting an house that was assigned in a round later than n does not make the agent better off.

preview of next lecture

- Housing allocation

- Housing allocation - agents have no claim on the set of houses

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools
- House allocation with existing tenants

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools
- House allocation with existing tenants - some agents have a claim on some houses others do not

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools
- House allocation with existing tenants - some agents have a claim on some houses others do not
  - how do we ensure wide participation?

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools
- House allocation with existing tenants - some agents have a claim on some houses others do not
  - how do we ensure wide participation?
- Applications in market design:

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools
- House allocation with existing tenants - some agents have a claim on some houses others do not
  - how do we ensure wide participation?
- Applications in market design:
  - Kidney exchange

preview of next lecture

- Housing allocation - agents have no claim on the set of houses
  - allocating students to dorms
  - allocating students to schools
- House allocation with existing tenants - some agents have a claim on some houses others do not
  - how do we ensure wide participation?
- Applications in market design:
  - Kidney exchange
  - School assignment