

ECO 426 (Market Design) - Lecture 2

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 - exploding offers (i.e. offers with a very short deadline)

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 - The algorithm was changed in the late 90s from hospital proposing to students proposing

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- Example: priority equal to the product of mutual rankings (i.e. couples that rank each other first have highest priority, couples with mutual ranking 1-2 have second highest priority etc. etc.)

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- Is stability of the matching outcome important for the success of a centralized matching program?

TABLE I
STABLE AND UNSTABLE (CENTRALIZED) MECHANISMS

Market	Stable	Still in use (halted unraveling)
American medical markets		
NRMP	yes	yes (new design in '98)
Medical Specialties	yes	yes (about 30 markets)
British Regional Medical Markets		
Edinburgh ('69)	yes	yes
Cardiff	yes	yes
Birmingham	no	no
Edinburgh ('67)	no	no
Newcastle	no	no
Sheffield	no	no
Cambridge	no	yes
London Hospital	no	yes
Other healthcare markets		
Dental Residencies	yes	yes
Osteopaths (<'94)	no	no
Osteopaths (\geq '94)	yes	yes
Pharmacists	yes	yes
Other markets and matching processes		
Canadian Lawyers	yes	yes (except in British Columbia since 1996)
Sororities	yes (at equilibrium)	yes

From: Roth, 2002, "The economist as engineer: game theory, experimentation, and computation as tools for economic design;" *Econometrica*, 1341--1378

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- The triplet: i) $M \cup W$; ii) \mathcal{H} ; and iii) P ; define a strategic game with ordinal preferences.

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- **Definition (Best response)** A strategy q_x is a best response to a strategy profile q_{-x} for player x if the matching $\mathcal{H}(q_x, q_{-x})$ is (weakly) preferred by x to the matching $\mathcal{H}(q'_x, q_{-x})$ for any other strategy q'_x available to x .

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- Whether a player has a dominant strategy depends on the matching mechanism (\mathcal{H}), as well as on the agent's true preferences.
- If for a mechanism \mathcal{H} , P_x is a dominant strategy for player x , then player x has no reason (i.e. he/she never gains) to misreport her preferences within that mechanism.

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 - But the outcome can be inefficient i.e. every agent could be made better off by some other matching

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 - But the outcome can be **not stable**

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 - Is there any strategy proof **and** stable mechanism? **No**

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 - The example also shows that there is no Nash equilibrium where all players report their true preferences as long as the mechanism is stable
 - Relaxing the requirement that reporting the true preferences is a dominant strategy, and only requiring that everybody reporting honestly is a Nash equilibrium does not help