ECO 426 (Market Design) - Lecture 11

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Sponsored search auctions

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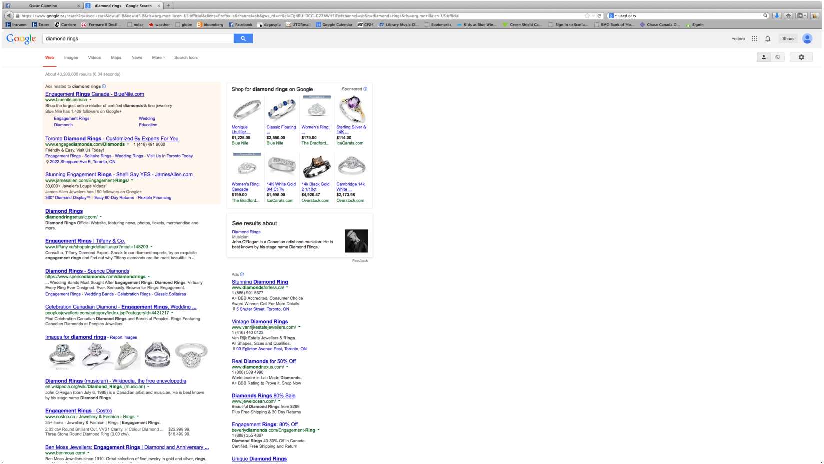

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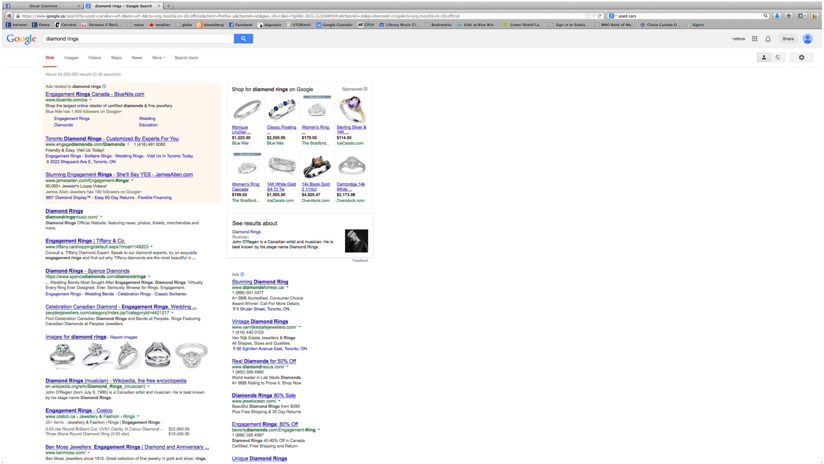

- Google advertising revenue: USD 42.5bn in 2012

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  - Bids order determine the ad position within the web-page
- An auction is for **one** query of **one** keyword

efficient allocation

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Example:

- Two positions on a web-page:

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- Efficient allocation?
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- Total value $= 200 \times \$10 + 100 \times \$4 = \$2,400$

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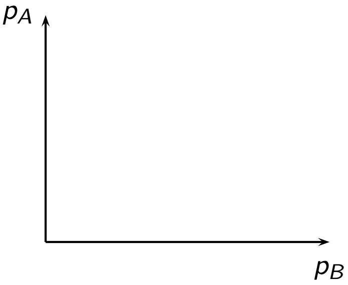

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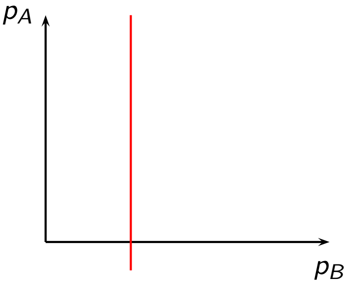

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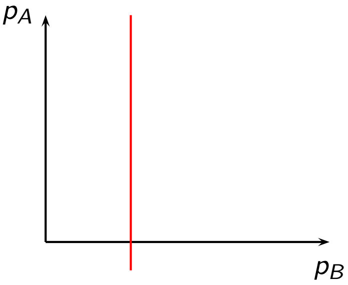

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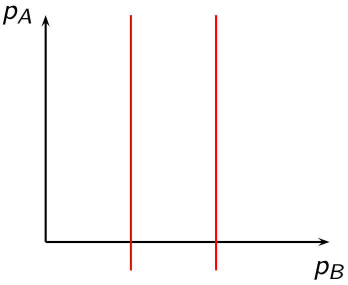

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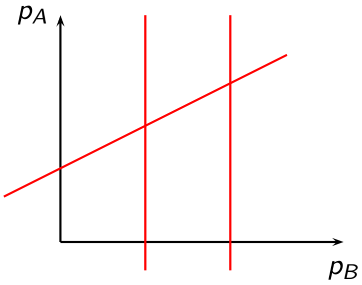

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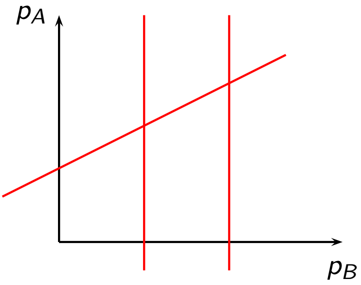

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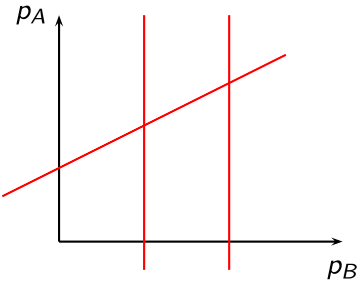

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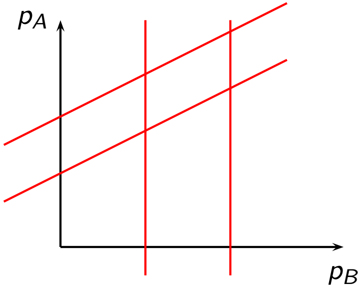

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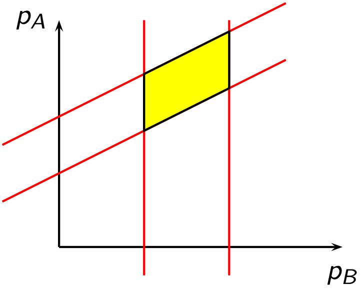

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  ⋮

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- pay-your-bid auctions were used in the 1990's (Overture, Yahoo, MSN)

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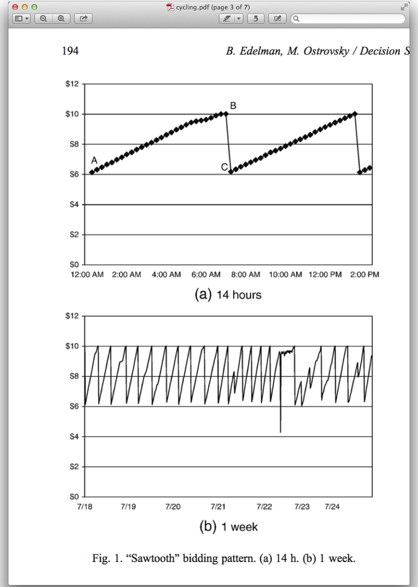

Fig. 1. "Sawtooth" bidding pattern. (a) 14 h. (b) 1 week.

Vickrey auction

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    - Firm 2 displaces firm 3 for 100 clicks $\Rightarrow$ pays $\$2 \times 100 = \$200$

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    - Firm 1 displaces firm 3 for 100 clicks and firm 2 for 100 clicks

Vickrey auction

- Example. Two positions: A generates 200 clicks per-day, B generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.
- In a Vickrey auction it is a dominant strategy to bid own valuation. (i.e. Firm 1 bids \$10, firm 2 bids \$4 and firm 3 bids \$2)
  - Allocation: Firm 1 gets A firm 2 gets B, firm 3 gets nothing (efficient)
  - Prices:
    - Firm 3 pays nothing
    - Firm 2 displaces firm 3 for 100 clicks $\Rightarrow$ pays $\$2 \times 100 = \$200$
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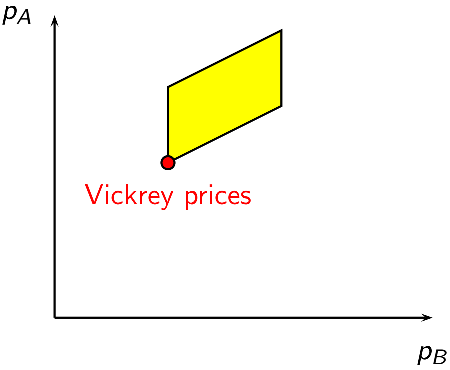

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- same as competitive equilibrium
  - efficient allocation
  - demand=supply

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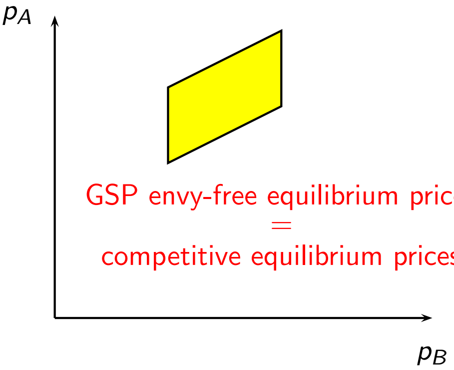

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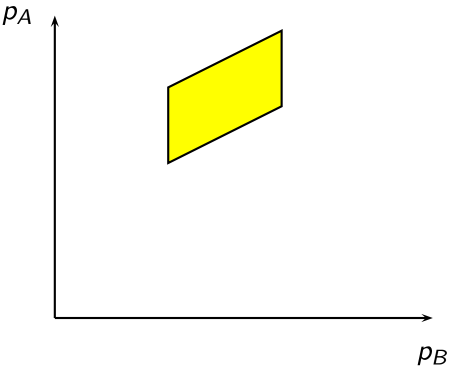

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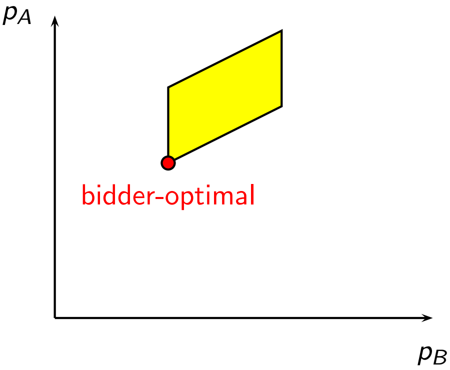

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