

# ECO 426 (Market Design) - Lecture 11

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# Sponsored search auctions

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The screenshot shows a Google search for "diamond rings" with approximately 43,200 results. The page displays several sponsored search results for diamond rings, including:

- Engagement Rings Canada - BlueNite.com**: Shop the largest online retailer of certified diamonds & fine jewelry.
- Toronto Diamond Rings - Customized By Experts For You**: Friendly & Easy Visit Us Today!
- Shining Engagement Rings - Sheryl See YSR - JamesAllen.com**: 30,000+ Jewellers' Choice Award!
- Diamond Rings**: diamondringscanada.com
- Engagement Rings | Tiffany & Co.**: Consult a Tiffany Diamond Expert. Speak to our diamond experts, try on exquisite engagement rings and find out why Tiffany diamonds are the most beautiful in...
- Diamond Rings - Source Diamonds**: Visit: [www.sourcediamonds.com/diamondrings](http://www.sourcediamonds.com/diamondrings)
- Celebration Canadian Diamond - Engagement Rings, Wedding...**: First Celebration Canadian Diamond Rings and Rings at Peoples. Rings Featuring Canadian Diamonds at Peoples Jewellers.
- Icons for diamond rings - Realist images**: Images of various diamond rings.
- Diamond Rings (jewellery) - Wikipedia, the free encyclopedia**: en.wikipedia.org/wiki/Diamond\_Rings\_(jewellery)
- Engagement Rings - Costco**: 20+ Items - Jewellery & Fashion | Rings | Engagement Rings
- Ben Moss Jewellers - Engagement Rings | Diamond and Anniversary...**: Ben Moss Jewellers since 1913. Great selection of the jewelry in gold and silver, rings.
- Shop for diamond rings on Google**: A section titled "Shop for diamond rings on Google" featuring a grid of diamond ring images and prices. Items include: Men's Ring (\$1,235.00), Classic Floating Blue Ring (\$2,880.00), Women's Ring (\$179.00), Sterling Silver & 14K... (\$174.00), Women's Ring (\$188.00), 14K White Gold (\$1,000.00), 14K Black Gold (\$1,000.00), and Celebration 14K (\$1,174.86).
- See results about**: A section titled "See results about" featuring a small image of a diamond ring and the text: "John O'Hagan is a Canadian artist and musician. He is best known by his stage name Diamond Rings."
- Ads: 0**: A section titled "Ads: 0" featuring a small image of a diamond ring and the text: "Shining Diamond Rings... A-1000 Accredited, Consumer Choice Award Winner, Call for More Details 915 584-1001, Toronto, ON."
- Virtual Diamond Rings**: [www.virtualdiamondrings.com](http://www.virtualdiamondrings.com)
- Real Diamonds for 60% Off**: [www.diamondsfor60.com](http://www.diamondsfor60.com)
- Diamonds Rings 50% Sale**: [www.diamondsfor50.com](http://www.diamondsfor50.com)
- Engagement Rings 60% Off**: [diamondsfor60.com/Engagement-Ring](http://diamondsfor60.com/Engagement-Ring)
- Unique Diamond Rings**

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- Google, Yahoo etc.. sell ad spaces linked to keyword searches

The screenshot shows a Google search for "diamond rings". The search results page is divided into two main sections: organic search results on the left and sponsored search results on the right.

**Organic Search Results (Left):**

- Engagement Rings Canada - BlueNite.com**  
Shop the largest online retailer of certified diamonds & fine jewelry  
Blue Nite has 1,495 followers on Google+  
Engagement Rings      Wedding      Information
- Toronto Diamond Rings - Customized By Experts For You**  
www.engagementdiamonds.com/diamonds      1 (416) 610-6100  
Friendly & Easy With Us Today!  
Engagement Rings      Solitaire Rings      Wedding Rings      Visit Us in Toronto Today  
9 2022 Sheppard Ave E, Toronto, ON
- Shining Engagement Rings - She's Say YES - JamesAllen.com**  
www.jamesallen.com/engagement-rings/      800.800. JEWELLERY  
James Allen Jewellers has 101 followers on Display  
2017 Diamond Chapter™      Easy 60-Day Returns      Flexible Financing
- Diamond Rings**  
diamondrings.com/      +  
Diamond Rings Offer Website, featuring news, photos, blogs, merchandise and more.
- Engagement Rings | Tiffany & Co.**  
www.tiffany.com/gifts/engagement-rings/      1-800-748-2373  
Consult a Tiffany Diamond Expert. Speak to our diamond experts, try on exquisite engagement rings and find out why Tiffany diamonds are the most beautiful in...
- Diamond Rings - Source Diamonds**  
www.sourcediamonds.com/diamondrings/      +  
... Wedding Bands Must Bought After Engagement Rings. Diamond Rings. Virtually Every Ring For Diamond. Free Returns. Browse by Rings. Engagement.  
Engagement Rings      Wedding Bands      Celebration Rings      Classic Solitaires
- Celebration Canadian Diamond - Engagement Rings, Weddings**  
www.celebrationcanadiandiamond.com/engagement-rings/      1 (416) 442-1317  
First Celebration Canadian Diamond Rings and Bands at Peoples. Rings Featuring Canadian Diamonds at Peoples Jewellers.
- Icons for diamond rings - Recent images**  
img.icons8.com/diamond-rings/      +  
Diamond Rings (Icons) - Wikipedia, the free encyclopedia  
en.wikipedia.org/wiki/Diamond\_Rings\_(jackets)      +  
John O'Hagan (born July 4, 1981) is a Canadian artist and musician. He is best known by his stage name Diamond Rings.
- Engagement Rings - Costco**  
www.costco.ca/Jewellery/Fashion-Rings/      +  
20+ Items      Jewellery & Fashion Rings | Engagement Rings  
2.02 ct. Lab Created Round Cut, VS1, Color D, H, G Color Diamond - \$22,899.99.  
Three Stone Round Diamond Ring (2.02 ct)      \$10,499.99.
- See More Jewellers: Engagement Rings | Diamond and Anniversary**  
www.diamond.com/      +  
Best Stone Jewellers along 1913. Great selection of the jewelry in gold and silver, rings.

**Sponsored Search Results (Right):**

**Shop for diamond rings on Google**

Men's Ring	Classic Floating	Women's Ring	Stirling Silver & 14K
Men's Ring \$1,238.00 Blue Nite	Classic Floating \$2,889.00 Blue Nite	Women's Ring \$179.00 The Bradford...	Stirling Silver & 14K \$114.00 1stCerts.com

Women's Ring	14K White Gold	14K Black Gold	Cartier Love 14K
Women's Ring \$189.00 The Bradford...	14K White Gold \$2,577.00 1stCerts.com	14K Black Gold \$2,110.00 1stCerts.com	Cartier Love 14K \$2,173.86 Cartier.com

**See results about**

**Diamond Rings**  
Musician  
John O'Hagan is a Canadian artist and musician. He is best known by his stage name Diamond Rings.

**Ads (Sponsored)**

- Shining Diamond Rings**  
www.shiningdiamondrings.com/      +  
1 (800) 807-8077  
A+ BBB Accredited, Consumer Choice Award Winner. Call for More Details  
915 Shuter Street, Toronto, ON
- Virtuoso Diamond Rings**  
www.virtuosodiamondrings.com/      +  
1 (416) 462-9123  
We're No Ordinary Jewellers & Rings  
All Shapes, Sizes and Qualities.  
915 Shuter Street East, Toronto, ON
- Real Diamonds for 50% Off**  
www.diamonds.com/      +  
1 (800) 266-4866  
World leader in Lab-Made Diamonds.  
A+ BBB Rating to Prove It. Shop Now
- Diamonds Rings 50% Sale**  
www.diamonds.com/      +  
Realistic Diamond Rings from \$299  
Plus Free Shipping & 30 Day Returns
- Engagement Rings 60% Off**  
www.diamonds.com/Engagement-Rings/      +  
1 (800) 266-4866  
Engagement Rings 40-60% Off in Canada  
Certified, Free Shipping and Returns
- Unique Diamond Rings**

- Google advertising revenue: USD 42.5bn in 2012

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- An auction is for **one** query of **one** keyword





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  - Firm 3 value is \$2 per-click
- Efficient allocation?
  - Firm 1 gets position *A*
  - Firm 2 gets position *B*
- Total value =  $200 \times \$10 + 100 \times \$4 = \$2,400$

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    - Firm 3 demands nothing
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Finding all competitive equilibrium prices

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# competitive equilibrium prices

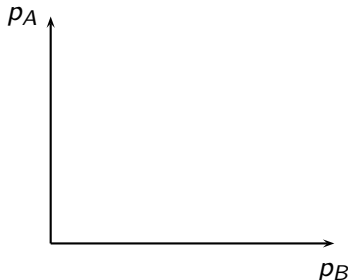
Finding all competitive equilibrium prices

- Competitive equilibrium allocation are efficient
  - Firm 3 must demand nothing
    - $p_A, p_B \geq 2$

# competitive equilibrium prices

Finding all competitive equilibrium prices

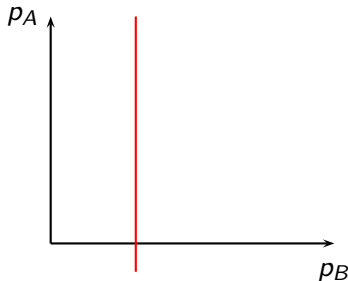
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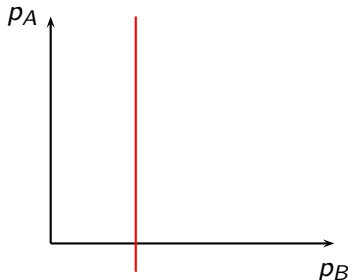




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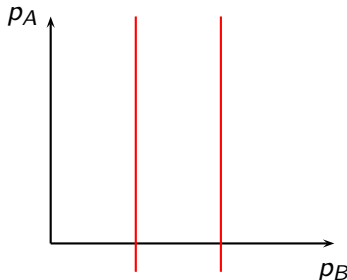
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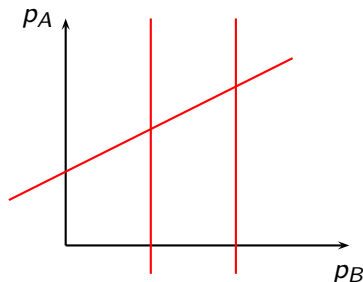
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    - $p_B \leq \$4$



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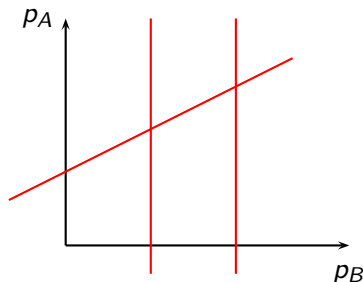
- Competitive equilibrium allocation are efficient
  - Firm 3 must demand nothing
    - $p_A, p_B \geq 2$
  - Firm 2 must demand position  $B$ 
    - $p_B \leq \$4$
    - $(4 - p_B) \times 100 \geq (4 - p_A) \times 200 \Rightarrow p_A \geq 2 + p_B/2$



# competitive equilibrium prices

## Finding all competitive equilibrium prices

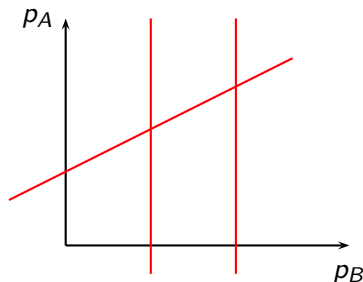
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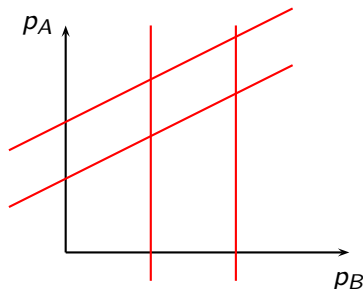
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    - $p_A \leq 10$



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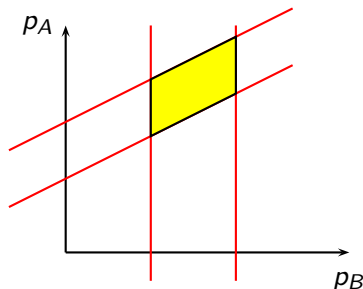
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    - $(4 - p_B) \times 100 \geq (4 - p_A) \times 200 \Rightarrow p_A \geq 2 + p_B/2$
  - Firm 1 must demand position  $A$ 
    - $p_A \leq 10$
    - $(10 - p_A) \times 200 \geq (10 - p_B) \times 100 \Rightarrow p_A \leq 5 + p_B/2$



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    - $(10 - p_A) \times 200 \geq (10 - p_B) \times 100 \Rightarrow p_A \leq 5 + p_B/2$



# pay-your-bid auction

- **Example.** Two positions:  $A$  generates 200 clicks per-day,  $B$  generates 100 clicks per-day. Three advertisers: values \$10, \$4 and \$2 per-click respectively.



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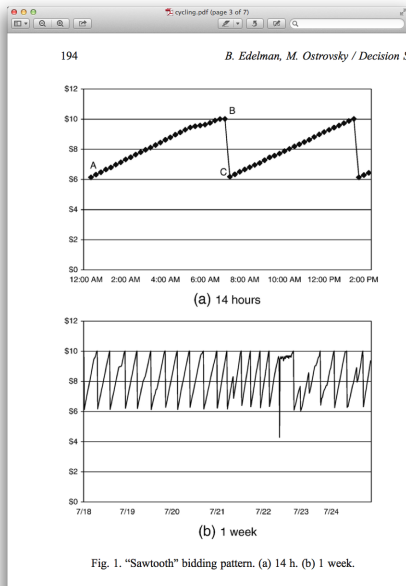
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- pay-your-bid auctions were used in the 1990's (Overture, Yahoo, MSN)

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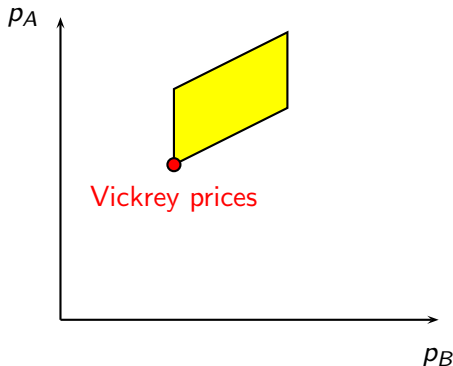
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# General model

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- Efficiency: an allocation is efficient if it is “positive assortative” (i.e. the highest **value** bidder gets top position, and so on...)



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- local envy-free is sufficient for stability

- Stable assignment

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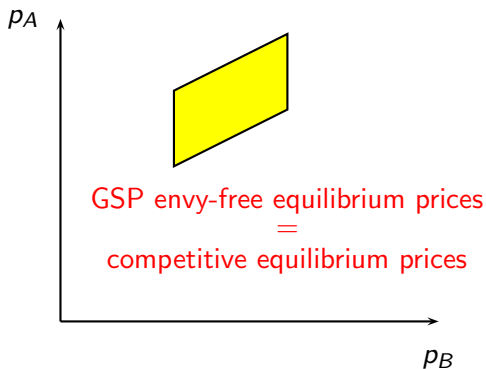
- same as competitive equilibrium
  - efficient allocation
  - demand=supply

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# stability and competitive equilibrium

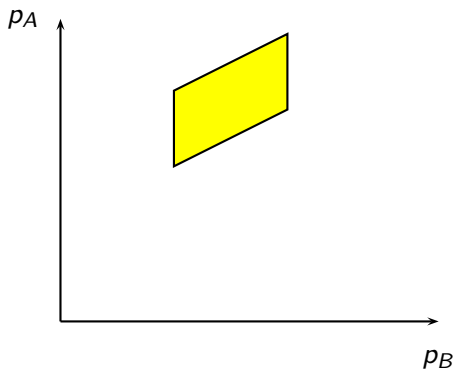
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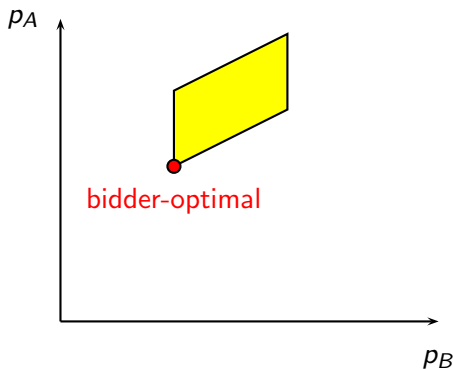
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