ECO 426 (Market Design) - Lecture 10

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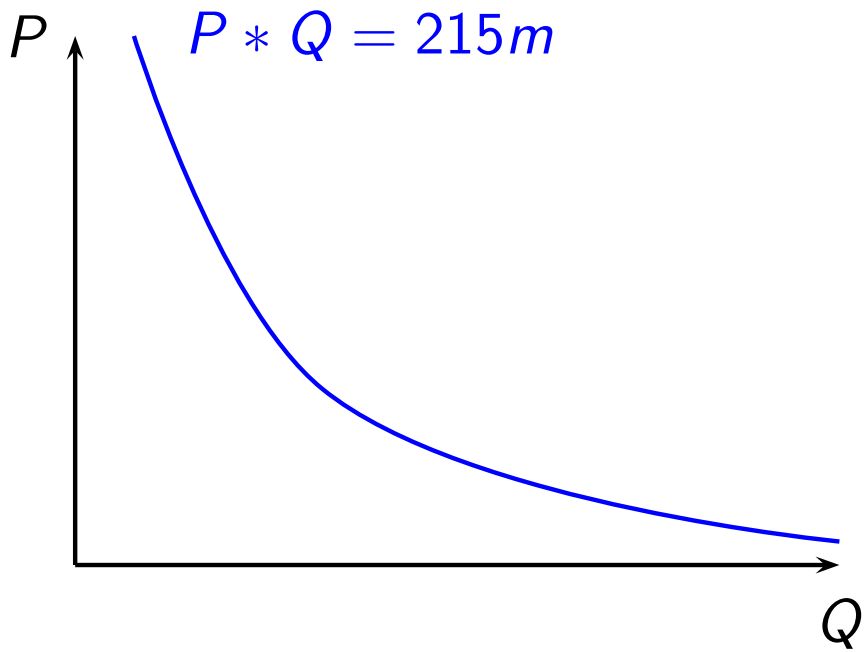

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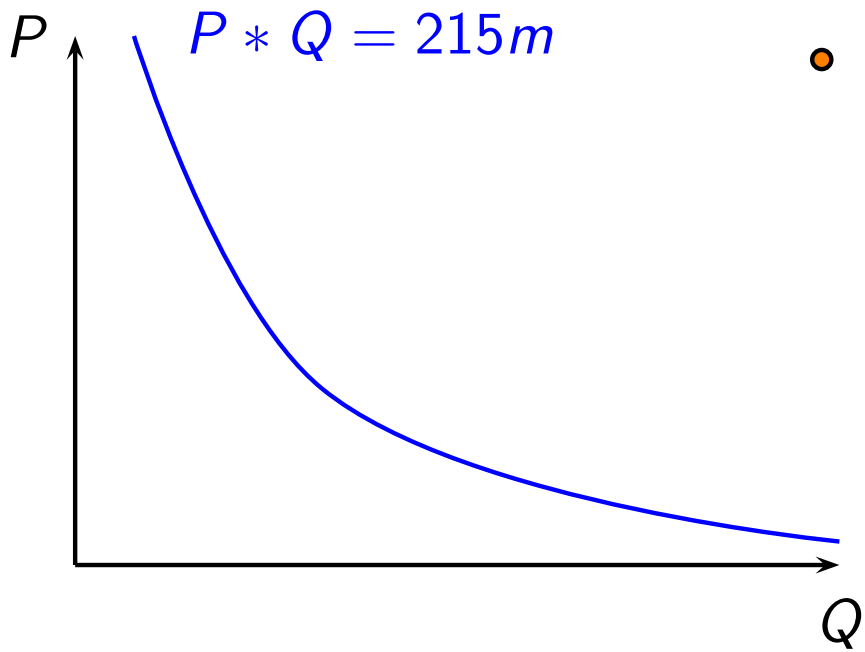

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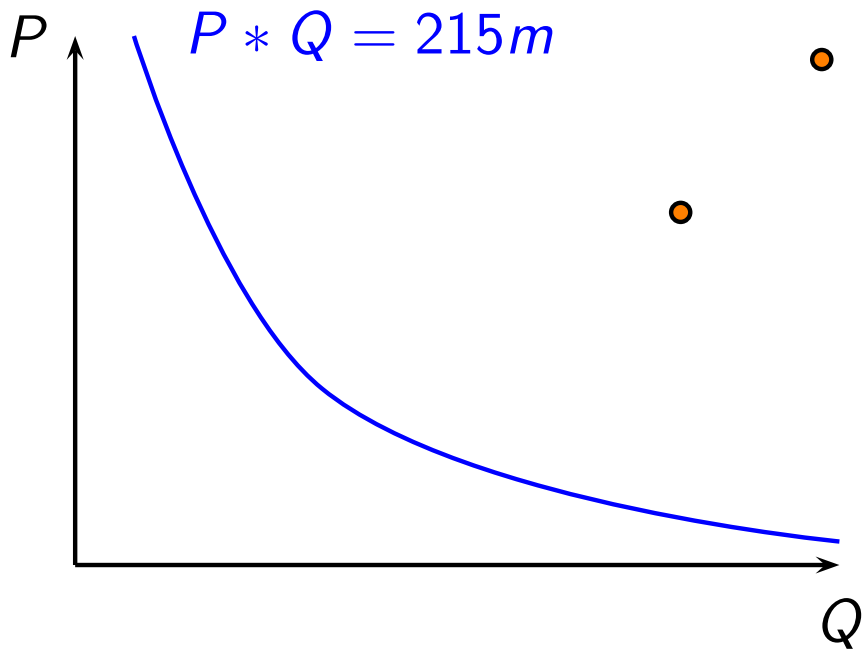

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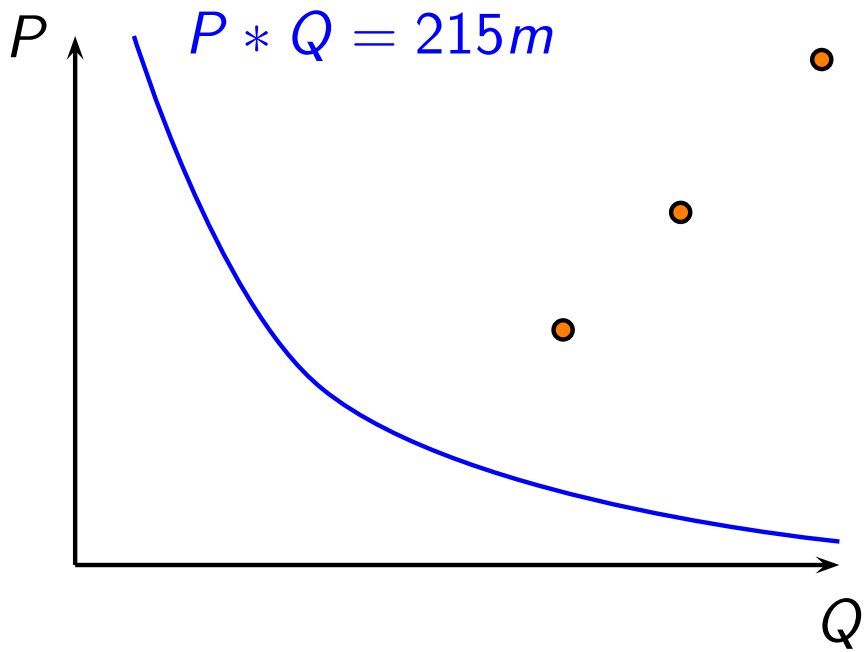

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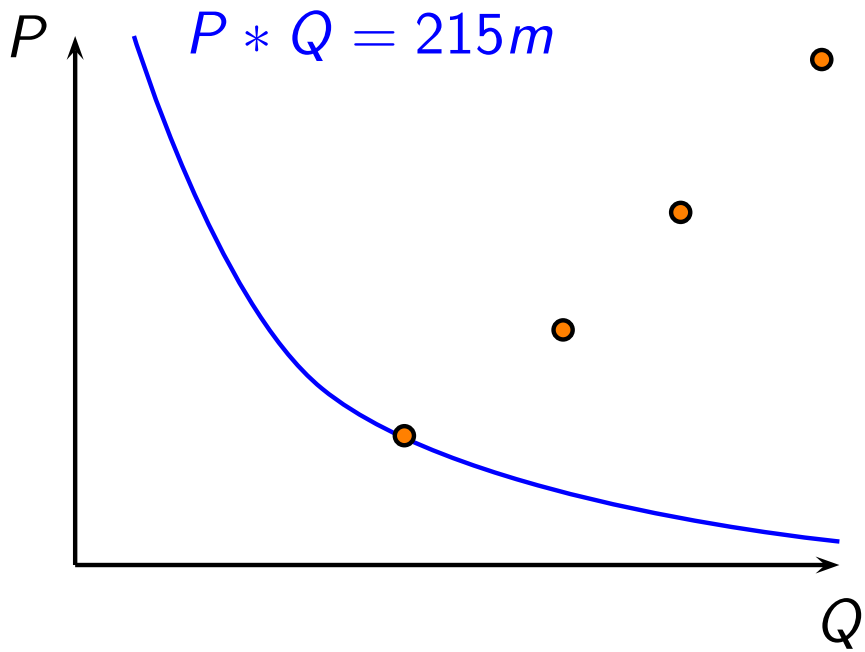

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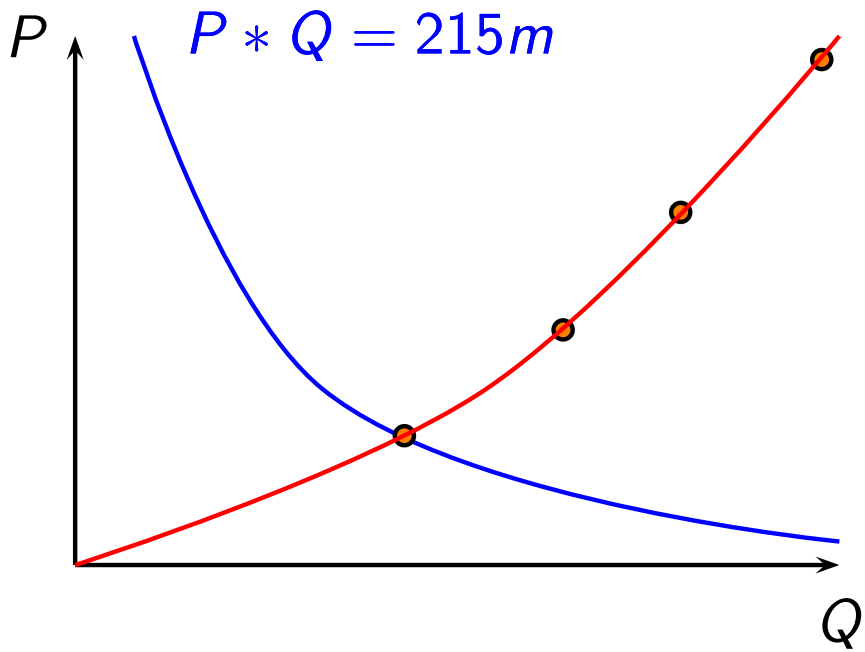

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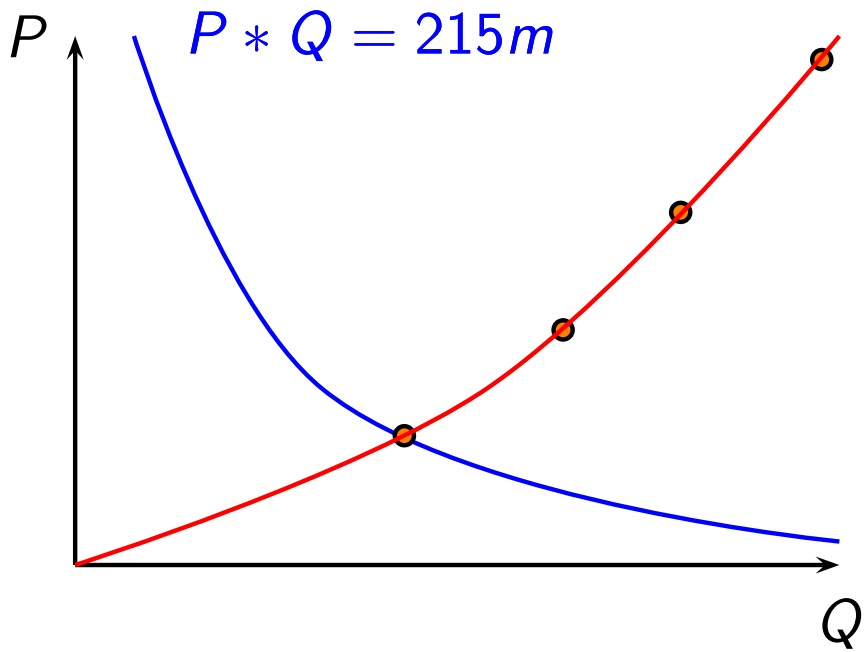

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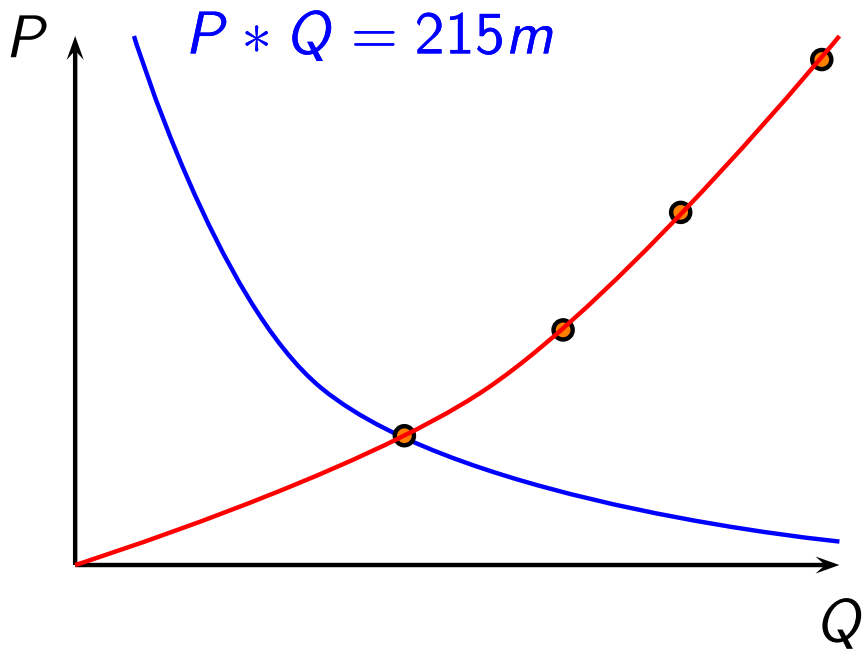

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$$Q \text{ demanded } = Q \text{ supplied}$$

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  - Theorem: It is a dominant strategy, for a bidder with unit demand, to bid her valuation.

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- There are K identical objects to sell (i.e. bidders do not care about the exact object they receive)
- Bidders have unit demand (i.e. they only want to purchase one object)
- There are $N > K$ bidders
- (Independent) private values
- Each bidder submits a sealed bid
- Bids are open
  - K highest bidders each receive one object
  - Winning bidders pay a price equal to $(K+1)^{th}$ highest bid
- Question: What should bidders bid?
  - Theorem: It is a dominant strategy, for a bidder with unit demand, to bid her valuation.
  - Same argument as in second price private value auction

Demand reduction

- Question: What if bidders want more than one unit?

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  - Auction price is 105
  - Bidder 4 profit is 0
- If Bidder 4 reduces his demand to one object bids are: 120, 110, 100, 105
  - Same allocation
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  - Auction price is 105
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- If Bidder 4 reduces his demand to one object bids are: 120, 110, 100, 105
  - Same allocation
  - Auction price is 100
  - Bidder 4 profit is 5

Discriminatory pricing

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- bidders pay different prices (discriminatory pricing)
- the price each bidder pays equals to the negative externality his participation imposes on all other bidders
- bidding truthfully is a dominant strategy

Vickrey auction - example

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  - Removing bidder 1: 2 wins one object and 4 wins two objects
    - Bidder 1 price is 105

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  - Removing bidder 2: 1 wins one object and 4 wins two objects

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    - Bidder 1 price is 105 (displaces one unit of bidder 4)
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  - Removing bidder 3: same allocation

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  - Removing bidder 3: same allocation
    - Bidder 3 price is 0 (displaces nobody)

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  - Removing bidder 2: 1 wins one object and 4 wins two objects
    - Bidder 1 price is 105 (displaces one unit of bidder 4)
  - Removing bidder 3: same allocation
    - Bidder 3 price is 0 (displaces nobody)
  - Removing bidder 4: 1, 2 and 3 win one object each
    - Bidder 4 price is 100 (displaces bidder 3)

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    - Bidder 1 price is 105 (displaces one unit of bidder 4)
  - Removing bidder 2: 1 wins one object and 4 wins two objects
    - Bidder 1 price is 105 (displaces one unit of bidder 4)
  - Removing bidder 3: same allocation
    - Bidder 3 price is 0 (displaces nobody)
  - Removing bidder 4: 1, 2 and 3 win one object each
    - Bidder 4 price is 100 (displaces bidder 3)
- Bidder 4 has no incentive to reduce demand

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  - Bidder 4: wants **two** objects, values **each** \$115

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  - Bidder 2: wants one object, values it \$110
  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$115
- Vickrey pricing

Vickrey auction - example

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  - Bidder 2: wants one object, values it \$110
  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$115
- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)

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  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$115
- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)
  - Bidder 4 pays a total of 210 (displaces bidder 2 and 3)

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- Example: Three (identical) objects for sale, four bidders
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  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$115
- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)
  - Bidder 4 pays a total of 210 (displaces bidder 2 and 3)
  - No incentive to change bids

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- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)
  - Bidder 4 pays a total of 210 (displaces bidder 2 and 3)
  - No incentive to change bids
- Uniform price (4^{th} price auction)

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- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)
  - Bidder 4 pays a total of 210 (displaces bidder 2 and 3)
  - No incentive to change bids
- Uniform price (4^{th} price auction)
  - price is 110

Vickrey auction - example

- Example: Three (identical) objects for sale, four bidders
  - Bidder 1: wants one object, values it \$120
  - Bidder 2: wants one object, values it \$110
  - Bidder 3: wants one object, values it \$100
  - Bidder 4: wants **two** objects, values **each** \$115
- Vickrey pricing
  - Bidder 1 pays 110 (displaces bidder 2)
  - Bidder 4 pays a total of 210 (displaces bidder 2 and 3)
  - No incentive to change bids
- Uniform price (4^{th} price auction)
  - price is 110
  - bidder 4 gains by reducing demand to 1 unit

Multi-item auctions with different goods

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- Multiple objects for sale, K

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- Multiple objects for sale, K
- Objects are different
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- Multiple objects for sale, K
- Objects are different
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- A bidder n
  - has a vector of valuations for each of the objects, $(v_n^1, \ldots, v_n^K)$
  - only wants one of the objects (unit demand)

a simple example of ascending auction

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  - B offers \$2 to good 3

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  - B offers \$0 to good 2
- A gets 3 at \$2, B gets 2 at \$0, C gets 1 at \$1

Ascending auction - preference lists

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  - same as Vickrey prices (strategy proof)