ECO426, Winter 2015 Ettore Damiano

Midterm Test
Monday February 23, 2015

Instructions: You have 110 minutes to complete this test. There are four (4) questions for a total of 100 points. To obtain credit you **must give an argument** to support each of your answers. No aids allowed.

Question 1 (25 points)
In an house allocation with existing tenants model, consider a variant of the YRMH-IGYT mechanism where, when an agent requests another agent's house they swap their priority position within the mechanism. For example, suppose it is agent a_i's turn and he points to the house of some agent a_j. Then, a_j goes at the beginning of the line, and a_i takes a_j's turn in the queue.

a) Is this mechanism Pareto efficient?

b) Is this mechanism strategy proof?

Question 2 (25 points)
Consider a student placement problem with seven students $\{i_1, i_2, \ldots, i_7\}$ and four schools in two categories, E and M. Schools s_1 and s_2 belong to category E, and have quotas of 1 and 2 respectively. Schools s_3 and s_4 belong to category M and have each a quota of 2. The students rank order in category E is $\{1, 2, 3, 4, 5, 6, 7\}$ (i.e. student i_1 is ranked first, student i_2 second etc.) The students rank order in category M is $\{3, 4, 6, 1, 5, 2, 7\}$. When the multi-category serial dictatorship is run, its outcome (described as the set of students assigned to each of the four schools) is as follows:

Round 1 $(s_1 : i_1), (s_2 : i_2, i_4), (s_3 : i_3, i_6), (s_4 : i_1, i_4)$

Round 2 $(s_1 : i_2), (s_2 : i_4, i_5), (s_3 : i_3, i_6), (s_4 : i_1, i_5)$

Round 3 $(s_1 : i_2), (s_2 : i_4, i_6), (s_3 : i_3, i_6), (s_4 : i_1, i_5)$

Round 4 $(s_1 : i_2), (s_2 : i_4, i_6), (s_3 : i_1, i_3), (s_4 : i_5, i_7)$

Derive **all** the restrictions on the student preferences implied by the outcome of the multi-category serial dictatorship mechanism described above.

Question 3 (25 points)
Consider a housing market problem with eight participants $\{a_1, \ldots, a_8\}$. Each participant owns a house and denote with h_i the house of agent a_i, for $i = 1, \ldots, 8$. The strict preferences of the participants are described in the following table

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
h_3	h_1	h_6	h_5	h_3	h_7	h_3	h_2
h_2	h_6	h_2	h_2	h_1	h_8	h_1	h_4
h_7	h_5	h_4	h_3	h_6	h_5	h_7	h_6
h_1	h_3	h_8	h_8	h_8	h_1		h_3
	h_2	h_5	h_4	h_7	h_6		h_8
		h_3		h_4			
				h_5			

a) Find all Core allocations for this housing market.

Suppose now each house has a price, with p_i denoting the price of house h_i. Given a set of prices, $(p_1, \ldots, p_8)$, each agent can afford to purchase any house that is no more expensive then her own (i.e. agent a_i can afford a house h_j if $p_j \leq p_i$.) Suppose that each agent demands her favorite house among those she can afford.

b) Can you find a set of prices that clear the market (i.e. each house is demanded by exactly one agent)?

c) In general, can an allocation associated with market clearing prices be different from a Core allocation?

Question 4 (25 points) Consider a marriage market with 4 men, $m_1, \ldots, m_4$, and 4 women, $w_1, \ldots, w_4$. Both men and women are strictly ordered with respect to their height, with m_1 (w_1) and m_4 (w_4) being the tallest and shortest man (woman) respectively. Find all stable matchings in each of the following scenarios.

a) All agents strictly prefer taller to shorter partners.

b) All agents with an odd index (i.e. m_1, m_3, w_1, w_3) prefer taller to shorter partners, and the opposite is true for agents with even indexes.

c) The men's preferences are as in part b), but the women's preferences are reversed (i.e. w_1 and w_3 prefer shorter partners while w_2 and w_4 prefer taller partners.)

d) Generalize the model to the case when there are K agents on each side of the market, where K is an even number larger than 4, and find the stable matchings under the three preference scenarios above.