ECO426, Fall 2015 Ettore Damiano

Midterm Test
MONDAY NOVEMBER 2, 2015

Instructions: You have 110 minutes to complete this test. There are three (3) questions for a total of 105 points. To obtain credit you **must give an argument** to support each of your answers. No aids allowed.

Question 1 (35 points)
Consider a student pairing problem as in Assignment 1. There are now six (6) children, A, B, C, D, E and F, to be paired for some homework assignment. All partners are acceptable to all children, and the preferences of each children over potential partners are described in the table below

$$\begin{array}{c|c} A & C \succ E \succ D \succ F \succ B \\ B & F \succ C \succ D \succ A \succ E \\ C & E \succ D \succ F \succ B \succ A \\ D & A \succ F \succ B \succ C \succ E \\ E & B \succ D \succ C \succ F \succ A \\ F & C \succ A \succ B \succ D \succ E \end{array}.$$

For each child create a "dummy partner" and describe preferences of children over dummies and dummies over children as in Assignment 1.

a) For the two-sided matching model with students and dummies, find the outcome of the DA algorithm with student proposing.

b) Is the matching of students and dummies that you found in part a) feasible? (i.e. does it correspond to a possible pairing of students for their homework assignment?)

c) For each student, find the best and the worst dummy partner among all stable matchings of the two-sided matching model with students and dummies.

d) Find all stable matchings of the two-sided model of student and dummies.

e) Prove that, for any set of students and preferences, the number of stable matchings in the two-sided model with students and dummies is always at least as large as the number of stable matchings in the one-sided student model.

Question 2 (35 points)
Consider a kidney exchange program with nine participants. A participant is a patient-donor pair (e.g. (t_i, k_i)), and each participant has strict preferences over the set of compatible kidneys, and the option, denoted w, of exchanging the donor's kidney for a high priority position on a wait-list for cadaveric kidneys. The preference of the participants are described in the following table

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
k_3	k_8	k_2	k_7	k_4	k_4	k_2	k_1	k_3
k_8	k_7	k_1	k_5	w		k_4	k_2	w
w	k_3	k_9	k_6				w	
		w						

In the table above, the preferences of each patient are described by an ordered list of acceptable exchanges. (That is, an ordered list of the **compatible** kidneys plus the wait-list option w, if it is acceptable). For example, patient t_1 is compatible with two kidneys, k_3 and k_8, and prefers k_3 to k_8. Also, t_1 finds acceptable an exchange to the wait-list.
The kidney exchange program uses a Top Trading Cycle and Chain mechanism, and a chain selection rule, to determined the live donor kidney exchanges and the list exchanges that will be performed.

a) Do the details of the chain selection rule (e.g. minimal chain, longest chain, etc.) matter for the final outcome of the exchange program?

Now suppose that only two-way exchanges are possible. That is, neither exchanges to the wait-list nor exchanges involving more that two patient-donor pairs are allowed. Further, the patients are priority ordered with t_1 having the highest priority, t_2 the second highest, t_3 the third etc. up to t_9 with the lowest priority.

b) Using the priority mechanism we studied in class, find the outcome of the kidney exchange program.

c) For each patient, compare the welfare across the outcomes of the mechanism in part a) and b).

d) How can the outcome of the exchange program vary when the patients' priority ranking changes?

Question 3 (35 points)
Consider a school placement problem with eight students, $\{i_1, \ldots, i_8\}$, and four schools, $\{s_1, \ldots, s_4\}$, each with a quota of two students. The student strict preferences over schools are described in the following table

i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
s_1	s_2	s_3	s_2	s_1	s_2	s_3	s_2
s_3	s_1	s_1	s_3	s_2	s_1	s_2	s_4
s_2	s_3	s_2	s_1	s_3	s_3	s_1	s_1
s_4	s_4	s_4	s_4	s_4	s_4	s_4	s_3

There are two school categories, E and M, with s_1 and s_2 in category E and s_3 and s_4 in category M. The order of students with respect to their category E test scores is $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (i.e. student i_1 has the highest and student i_9 the lowest test score). The ordering in category M is instead $\{2, 5, 1, 7, 6, 8, 3, 4\}$ (i.e. student i_2 has the highest score and student i_4 the lowest.)

a) Assign student to schools using the multi-category serial dictatorship mechanism (illustrates each steps).

After each student has been assigned to a school from part a), a market opens where they can freely trade their assigned spot. The "market prices" for a spot in each school are determined by the following algorithm.

At the start, all prices are zero (i.e. there are four prices $p_1 = p_2 = p_3 = p_4 = 0$ for a spot each school s_1,...,s_4, respectively). In each round, given the current prices, each student can "afford" a spot at any school that is no more expensive then the school he/she has been assigned to after the multi-category serial dictatorship mechanism. For example, if the prices are such that $p_1 < p_2 < p_3 < p_4$, a student who is assigned to school s_3, can demand a spot at school s_1, s_2 or s_3, but not s_4. Each agent demands his/her favorite school among those he/she can afford. The price of any school who is demanded by more than two students increases by one unit, and a new round of the algorithm starts. The algorithm ends when no school is demanded by more than two students, (i.e. each school is demanded by exactly two students,) and each student is assigned to the school he/she demands.

b) What are the final "market prices" and allocation of students? (describe each step)

c) Is there a different mechanism that would have achieved the same allocation of students to schools?