

Midterm Test  
MONDAY FEBRUARY 24, 2014

**Instructions:** You have 90 minutes to complete this test. There are three questions for a total of 75 points. To obtain credit you **must give an argument** to support each of your answers. No aids allowed.

**Question 1** (25 points)

Consider a kidney exchange program with eight participants. A participant is a patient-donor pair (e.g.  $(t_i, k_i)$ ), and each participant has strict preferences over the set of compatible kidneys and the option of exchanging the donor's kidney for a priority  $w$  on a wait-list for a cadaveric kidney. The preference of the participants are described in the following table

$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$
$k_2$	$k_1$	$k_1$	$k_5$	$k_2$	$k_3$	$k_1$	$k_2$
$k_7$	$k_4$	$k_7$	$k_2$	$k_7$	$k_8$	$w$	$k_4$
$k_1$	$w$	$k_4$	$k_3$	$k_6$	$k_5$	$k_7$	$k_6$
	$k_2$	$k_8$	$k_8$	$k_5$	$w$		$w$
		$k_6$	$k_4$		$k_6$		$k_8$
		$k_3$					

In the table above, the preferences of each patient are described by an ordered list of acceptable exchanges. (That is, an ordered list of the **compatible** kidneys plus the wait-list option  $w$ , if it is acceptable). The list ends with the patient "own" kidney to denote that the participant would prefer remaining with his own kidney (i.e. not exchanging it) to any other alternative. For example, patient  $t_1$  is compatible with two kidneys,  $k_2$  and  $k_7$ , and prefers  $k_2$  to  $k_7$ . Also,  $t_1$  does not find acceptable an exchange to the wait-list. Patient  $t_2$  has two compatible kidneys,  $k_1$  and  $k_4$ , but he is also willing to exchange his donor kidney for a priority  $w$  on a wait-list. The kidney exchange program uses a Top Trading Cycle and Chain mechanism. As chain selection rule, the program uses the longest chain criterion (i.e. the chain with the most number of patients is selected.) Finally, when a chain is selected, the "tail kidney" remains available to the other participants in the program (i.e. in later rounds of the mechanism the remaining participants can receive the tail kidney.)

- 1) Assume that every participant reports his/her preferences honestly and find the outcome of this kidney exchange program. (Describe each step of the mechanism.)
- 2) Does any agent have an incentive to manipulate his/her preferences? If yes provide an example, if no explain why.
- 3) Suppose the rules of the exchange program are changed and, whenever a chain is selected in a round, the tail kidney is removed from the exchange program (i.e. it is no longer available to participants in later rounds.) Is the outcome of the program better or worse for the participants than what you found in part 1)?

**Question 2**(25 points)

Consider the kidney exchange problem of Question 1, and suppose now that only two-way exchanges are allowed. That is, neither exchanges to the wait-list nor exchanges involving more than two patient-donor pairs are allowed.

- 1) Using the preferences table of Question 1, draw a graph that describes all feasible two-way exchanges.
- 2) Suppose the patients' priority ordering is as follows:  $\{t_5, t_8, t_3, t_2, t_7, t_1, t_4, t_6\}$  (i.e.  $t_5$  is the patient with highest priority, and  $t_6$  the patient with the lowest priority.) Using the priority mechanism we studied in class find the outcome of this kidney exchange program.

**Question 3**(25 points)

Consider a school choice problem with five students,  $i_1, i_2, i_3, i_4$  and  $i_5$ , and four schools,  $A, B, C$  and  $D$ . Schools  $A, B$  and  $C$  can admit one student each, and school  $D$  can admit two students. The preferences of each student are described by the following table.

$i_1$	$i_2$	$i_3$	$i_4$	$i_5$
$A$	$B$	$D$	$A$	$B$
$B$	$A$	$A$	$C$	$C$
$C$	$D$	$B$	$B$	$A$
$D$	$C$	$C$	$D$	$D$

Each school orders students with respect to priority for admission. The priority lists are given in the following table

$A$	$i_2$	$i_3$	$i_5$	$i_1$	$i_4$
$B$	$i_3$	$i_1$	$i_2$	$i_5$	$i_4$
$C$	$i_4$	$i_5$	$i_1$	$i_2$	$i_3$
$D$	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$

- 1) Find the outcome of this school choice problem when the DA algorithm with students proposing is used. (Describe each of the steps of the algorithm.)
- 2) Is the outcome of the mechanism in part 1) Pareto efficient?
- 3) Suppose now that  $A, B, C$  and  $D$  are schools in different categories, and the priority list of each school is the ranking of students in the school's category. Find the outcome of this student placement problem when the multi-category serial dictatorship mechanism is used.