University of Toronto
Faculty of Arts and Sciences

APRIL EXAMINATIONS 2015

ECO426H1S

Duration - 2 hours

No aids are allowed

There are 4 questions in 2 pages. **You must give arguments to support your answers.**

Question 1 (25 points)
In an "average price" sealed-bid auction, potential buyers submit bids in sealed envelops. The object is assigned to the participant who submitted the highest bid, and the winner pays a price equal to the average submitted bid. (For example, if three participants submit bids of \$3, \$4 and \$2 respectively, the participant who bids \$4 receives the object and pays a price of $\$3 = (3 + 4 + 2)/3$.)

Suppose there are $N > 1$ bidders, and their private valuations for the object are independent draws from the uniform distribution on the interval $[0, 1]$.

a) Is bidding your own valuation a dominant strategy?

b) Assume that the equilibrium bidding strategies are increasing and linear in the valuations (i.e. they take the form $\beta(v) = \alpha v$, with $\alpha > 0$.) Using the revenue equivalence theorem, find an equilibrium of the average price auction. (I am asking you to provide the equilibrium bidding strategy as a function of the number of participants i.e. N. For partial marks you can find the equilibrium for an example with a fixed N e.g $N = 2$ or $N = 3$ etc.)

Question 2 (30 points)
There are two identical objects for sale and two potential buyers. A buyer whose value for a single object is v, values the pair of objects at $(1 + \beta)v$, where β is a number between 0 and 1. (The buyer "discounts" owning a second unit of the good by the factor β.) The buyers' valuations, v_1 and v_2, are between 0 and 1, drawn from some joint distribution F and are privately observed. (The actual distribution F does not matter for the answer to the questions below, but if you find it useful you can assume that v_1 and v_2 are independent draws from the uniform distribution.) The seller uses a Vickrey auction to sell the two objects. (In case of a tie among two or more bidders the winner/s are determined randomly with equal probability among those bidders.)

a) Describes the revenue to the seller as a function of v_1, v_2 and β.

b) Show that the expected revenue to the seller is increasing in β.

c) Would the expected revenue to the seller increase or decrease if the seller auctions off only one unit of the good? (Explain how the comparison varies with the value of β.)

Suppose now the seller auctions off the two objects sequentially (i.e. one unit at a time) using a second price auction for both objects. Assume that, in the second auction, each bidder bids her valuation (i.e. v_i if bidder i lost the first auction and βv_i if bidder i won the first auction.)

d) Show that each bidder i bidding βv_i in the first auction is a (Bayesian) Nash equilibrium. (Hint: for bidder i, compare the value of winning the first auction to the value of losing it as a function of the other bidder value v_j.)

Question 3 (25 points)
Two online advertising spaces are available on a webpage. The top position generates 100 clicks per-day, the second position generates 40 clicks per-day. There are four potential buyers, A,B, C, and D, with (publicly observed) per-click values of \$10, \$1, \$5 and \$8 respectively. The seller is using a generalized second price auction to sell the two positions.

a) Find an inefficient equilibrium of the GSP auction.

b) Find the lowest and the highest possible revenue for the seller in any envy-free equilibrium of the GSP auction.

Suppose now the seller can make one more advertising space available on the webpage. This position would generate 20 clicks per-day and would not affect the number of clicks generated by the other two positions.

c) Find the lowest possible revenue for the seller in any envy-free equilibrium of the GSP auction, and compare it to the revenue you found in part b).

Question 4 (20 points)
A single object is being sold through a second price auction. Consider a private value environment with two bidders whose valuations, v_1 and v_2, are distributed uniformly on the interval [0,1], but are not necessarily independent. Precisely, the two valuations are given by

$$v_1 = \alpha\omega_1 + (1-\alpha)\omega_2 \quad and \quad v_2 = \alpha\omega_2 + (1-\alpha)\omega_1.$$

In the above α is a number between $\frac{1}{2}$ and 1, and ω_1, ω_2 are two independent draws from the uniform distribution on the interval [0,1].

a) Describe the revenue to the seller as a function of α, ω_1 and ω_2

b) Calculate the seller expected revenue and study how it changes with α.