University of Toronto
Faculty of Arts and Sciences

APRIL EXAMINATIONS 2014

ECO426H1S

Duration - 2 hours

No aids are allowed

There are 5 questions in 2 pages. **You must give arguments to support your answers.**

Question 1 (20 points)
In a "third price" sealed-bid auction, potential buyers submit bids in sealed envelops. The object is assigned to the participant who submitted the highest bid, and the winner pays a price equal to the third highest submitted bid. (For example, if three participants submit bids of \$3, \$5 and \$2 respectively, the participants who bid \$5 receives the object and pays a price of \$2.)

Suppose there are three (3) bidders, and their private valuations for the object are independent draws from the uniform distribution on the interval $[0, 1]$.

a) Is bidding your own valuation a dominant strategy?

b) Assume that the equilibrium bidding strategies are increasing and linear in the valuations (i.e. they take the form $\beta_(v) = \alpha v$, with $\alpha > 0$.) Using the revenue equivalence theorem, find an equilibrium of the third price auction. (You might find it useful to know that the expected values of the smaller of two independent draws from $U[0, t]$ is equal to $t/3$ for any positive value of t.)

Question 2 (20 points)
There are two identical objects for sale and three potential buyers. Each buyer's valuation can take two possible values, 0 or 1, with equal probability. Buyers' valuations are statistically independent and privately observed. The seller uses a Vickrey auction to sell the two objects. (In case of a tie among two or more bidders the winner/s are determined randomly with equal probability among those bidders.)

a) Find the expected revenue to the seller.

b) Suppose there are $N - 1$ objects and N potential buyers. What happens to the expected revenue when N becomes larger?

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Question 3 (20 points)
Two online advertising spaces are available on a webpage. The top position generates 50 clicks per-day, the second position generates 10 clicks per-day. There are three potential buyers, A,B, and C, with (publicly observed) per-click values of \$2, \$3 and \$7 respectively. The seller is using a generalized second price auction to sell the two positions.

a) Find an inefficient equilibrium of the GSP auction.

b) Find the lowest and the highest possible revenue for the seller in any envy-free equilibrium of the GSP auction.

Question 4 (20 points)
Find the optimal reserve price (i.e. revenue maximizing) in an all-pay auction when there are two bidders with independent private valuations uniformly distributed on the interval [0,1]. (Hint: use the revenue equivalence theorem, following the steps we discussed in class.)

Question 5 (20 points)
Consider the independent private value environment with two bidders whose valuations, v_1 and v_2, are distributed uniformly on the interval [0,1]. The seller is considering whether to sell the object using a first price or a second price auction with no reserve price. The seller utility function is given by

$$u(R) = \sqrt{R},$$

where R is the revenue generated by the auction. The seller wants to choose the auction format that gives him the highest expected utility.

a) Would you advise him to use a first price or a second price auction format? (Hint: suppose the seller learns that the highest of the two valuations is v, would his expected utility be higher in a first price auction or in a second price auction?)

b) Would your answer change when there are more bidders?