ECO426, Fall 2015
University of Toronto
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Assignment 1

due before 11:59pm on Wednesday October 28, 2015

Instructions: The assignment must be typed and submitted via email to ettore.damiano@utoronto.ca. You should present your arguments both clearly and concisely. I will use the following marking scheme: 30% of the marks are for clarity, 20% of the marks are for conciseness and 50% of the marks are for correctness. You can discuss the problems with other students in the class, however you **must** write your own solution. Also, if when solving a particular problem you received a significant amount of help from one of more student in the class, you should acknowledge it in an appropriately placed footnote.

Question 1

Describe an example of a "priority mechanism" for a two-sided one-to-one matching market. You can come up with your own mechanism and the mechanism needs not have a "real life" interpretation, but the mechanism must be fully specified i.e. it must identify a unique outcome for each set of participants and preferences. Construct an example in which the mechanism you have described yields an outcome that is not stable, or provide a proof that your mechanism is stable.

Question 2

Each child in a group of N children is invited to rank the rest in order of preference as working partners for some assignment. The objective is to find, if possible, a pairing of children that is stable. To that extent consider the following procedure. For each children $A, B, C, \ldots$ we create a "dummy partner" $A', B', C', \ldots$. Each child ranks the dummies putting his own dummy last and ranking each dummy as the corresponding child (e.g. if a child X prefers Y to Z, he will prefer the dummy Y' to the dummy X' to the dummy X'.) The preferences of the dummy X' over children are the same as the preferences of child X.

For example, with a set of five children, A, B, D, E and F, with preferences given by

$$\begin{array}{c|l} A & C \succ B \succ E \succ D \\ B & C \succ D \succ A \succ E \\ C & B \succ D \succ A \succ E \\ D & E \succ A \succ B \succ C \\ E & C \succ B \succ D \succ A \end{array} ,$$

the preferences of children over dummies and dummies over children are

$$\begin{array}{c|l} A & C' \succ B' \succ E' \succ D' \succ A' \\ B & C' \succ D' \succ A' \succ E' \succ B' \\ C & B' \succ D' \succ A' \succ E' \succ C' \\ D & E' \succ A' \succ B' \succ C' \succ D' \\ E & C' \succ B' \succ D' \succ A' \succ E' \end{array} \qquad \begin{array}{c|l} A' & C \succ B \succ E \succ D \succ A \\ B' & C \succ D \succ A \succ E \succ B \\ C' & B \succ D \succ A \succ E \succ C \\ D' & E \succ A \succ B \succ C \succ D \\ E' & C \succ B \succ D \succ A \succ E \end{array} .$$

a) In the example above of a two-sided matching model with students and dummies, find the outcome of the DA algorithm with students proposing.

b) Does the outcome identify a feasible pairing of students?

c) Can you construct an example (e.g. by changing the student preferences) where the outcome of the DA algorithm with student proposing does not yield a feasible pairing of students?

d) Describe the condition that must hold for the outcome of the DA algorithm to identify a feasible pairing of students.

e) Prove that, regardless of the number of students or the student preferences, if the pair student X and dummy Y' is part of the outcome of the DA algorithm with student proposing, then the pair student Y and dummy X' is part of the outcome of the DA algorithm with dummies proposing.

f) What property of the set of stable matching in the two-sided matching model with student and dummies is sufficient for the existence of a stable matching in the children pairing problem?

Question 3

Consider the following mechanism for a house allocation with existing tenants model. First vacant houses are randomly assigned to newcomers in such a way that each newcomer has the same probability of being assigned any of the vacant houses. Given this initial assignment of houses, the TTC mechanism is run to obtain the final allocation of houses to tenants.

a) Is the mechanism described above Pareto Efficient? Is it Strategy Proof?

Compare now the mechanism above with the YRMH-IGYT mechanism for a fixed (and arbitrary) priority ordering.

b) Can an existing tenant receive a worse house with the YRMH-IGYT than she would receive with the TTC after random assignment mechanism? Either prove that it is not possible, or provide an example.

c) Suppose there is only one newcomer. Can she receive a worse house with the YRMH-IGYT than she would receive with the TTC after random assignment mechanism? Either prove that it is not possible, or provide an example.

d) How does the answer to question c) change if there are more than one newcomer?

Question 4

Consider a many to one matching problem with two firms and five workers. The firms, $F = \{f_1, f_2\}$, have a capacity of 3 and 2 respectively, and the workers, $W = \{w_1, w_2, w_3, w_4, w_5\}$, are all acceptable to both firms. All workers find both firms acceptable. Workers w_1 and w_2 prefer f_2 to f_1 and the opposite is true for w_3, w_4 and w_5. Both firms' preferences are responsive. The preferences of f_1 satisfy the following

$$\{w_1\} \succ \{w_2\} \succ \{w_3\} \succ \{w_4\} \succ \{w_5\} \quad \text{and}$$
$$\{w_2\} \succ \{w_4, w_5\} \quad \text{and}$$
$$\{w_3, w_4\} \succ \{w_1\}.$$

The preferences of f_2 satisfy

$$\{w_3\} \succ \{w_4\} \succ \{w_1\} \succ \{w_5\} \succ \{w_2\} \quad \text{and}$$
$$\{w_3\} \succ \{w_1, w_4\}.$$

Firms first declare (simultaneously) a capacity which can be no larger than their true capacity (i.e. firm 1 can declare is has a capacity of 0, 1, 2, or 3). Workers and firms are then matched using the DA algorithm with the workers proposing.

a) Model the scenario above as a strategic game of capacity reporting and find all of its Nash equilibria, or verify that no Nash equilibrium exists.