ECO426, Winter 2014
University of Toronto
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Assignment 1

DUE BEFORE 1PM ON MONDAY FEBRUARY 10, 2014

Instructions: The assignment must be typed and submitted via email to ettore.damiano@utoronto.ca. You should present your arguments both clearly and concisely. I will use the following marking scheme: 30% of the marks are for clarity, 20% of the marks are for conciseness and 50% of the marks are for correctness. You can discuss the problems with other students in the class, however you **must** write your own solution. Also, if when solving a particular problem you received a significant amount of help from one of more student in the class, you should acknowledge it in an appropriately placed footnote.

Problem 1

Consider the following example of a marriage market with strict preferences. There are four men, $M = \{m_1, m_2, m_3, m_4\}$, and five women $W = \{w_1, w_2, w_3, w_4, w_5\}$. The preferences of each man and woman are described (in the form of ordered lists of acceptable mates) in the following two tables.

m_1	w_2	w_3	w_1	w_5	
m_2	w_3	w_2	w_5	w_1	w_4
m_3	w_1	w_2	w_3	w_4	w_5
m_4	w_5	w_3	w_2	w_1	w_4

w_1	m_2	m_1	m_3	m_4
w_2	m_4	m_3	m_2	m_1
w_3	m_3	m_4	m_1	
w_4	m_2	m_3	m_4	m_1
w_5	m_1	m_2	m_4	m_3

1) Using the Deferred Acceptance algorithm, find the men-optimal and the women -optimal stable matchings. (Describe each of the steps of the algorithm)

2) Find a stable matching different from those you found in 1). Compare the preferences of the agents across the three stable matchings you found.

3) The "Rural Hospital Theorem" states that the set of unmatched agents is the same across all stable matchings. Only using the fact that a men-optimal and a women-optimal stable matching exist, (i.e. among the stable matchings, there is one that is favored by all men, and one that is favored by all women,) demonstrate that the set of unmatched agents must be the same in the men- and women- optimal stable matchings.

Problem 2

Consider the following formal definition of "responsive" preferences.

Definition (*responsive preferences*) *The preferences of a firm f over sets of workers are* **responsive** *if there is an ordered list of acceptable workers, P, such that*

a) If w is preferred to w' according to P, and C is a set of workers that includes w' but not w, then the set of workers $C \setminus w' \cup w$ (i.e. the set of workers obtained by replacing w' with w in C) is not worse than C for f; and

b) If w is an unacceptable worker according to P and C is a set of workers that includes w, then the set of workers $C \setminus w$ (i.e. the set of workers obtained by removing w from C) is not worse than C for f.

Suppose there are four workers $W = \{w_1, w_2, w_3, w_4\}$, and the preferences of a firm are described by an ordered list of **acceptable sets** of workers.

1) For each of the example of firm preferences below explain if they are: i) responsive; ii) substitutes; or iii) neither responsive nor substitutes.

i)	$\{w_1, w_2, w_3\}$	$\{w_1, w_3\}$	$\{w_2, w_3\}$	$\{w_1, w_2\}$	$\{w_3\}$	$\{w_1\}$	$\{w_2\}$	
ii)	$\{w_1, w_3, w_4\}$	$\{w_1, w_3\}$	$\{w_2, w_3\}$	$\{w_1, w_2\}$	$\{w_3\}$	$\{w_1\}$	$\{w_2\}$	
iii)	$\{w_1, w_3\}$	$\{w_1, w_4\}$	$\{w_2, w_3\}$	$\{w_1, w_2\}$	$\{w_3\}$	$\{w_1\}$	$\{w_4\}$	$\{w_2\}$
iv)	$\{w_4, w_3\}$	$\{w_1, w_3\}$	$\{w_1, w_2\}$	$\{w_3\}$	$\{w_1\}$	$\{w_4\}$	$\{w_2\}$	

Suppose now there are two firms, $F = \{f_1, f_2\}$, with a capacity of 3 and 2 respectively, and five workers, $W = \{w_1, w_2, w_3, w_4, w_5\}$, all acceptable to both firms. All workers find both firms acceptable. Workers w_1 and w_2 prefer f_2 to f_1 and the opposite is true for w_3, w_4 and w_5. Both firms' preferences are responsive. The preferences of f_1 satisfy the following

$$\{w_1\} \succ \{w_2\} \succ \{w_3\} \succ \{w_4\} \succ \{w_5\} \quad \text{and}$$

$$\{w_2\} \succ \{w_4, w_5\} \quad \text{and}$$

$$\{w_3, w_4\} \succ \{w_1\}.$$

The preferences of f_2 satisfy

$$\{w_3\} \succ \{w_4\} \succ \{w_1\} \succ \{w_5\} \succ \{w_2\} \quad \text{and}$$

$$\{w_3\} \succ \{w_1, w_4\}.$$

Firms first declare (simultaneously) a capacity which can be no larger than their true capacity (i.e. firm 1 can declare is has a capacity of 0, 1, 2, or 3). Workers and firms are then matched using the DA algorithm with the workers proposing.

2) Model the scenario above as a strategic game of capacity reporting and find all of its Nash equilibria, or verify that no Nash equilibrium exists.

Problem 3

Consider a housing market problem with six agents $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ and their respective houses $H = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$. The preference of agents over houses are described in the following table

a_1	a_2	a_3	a_4	a_5	a_6	a_7
h_2	h_6	h_4	h_5	h_2	h_1	h_1
h_3	h_3	h_1	h_6	h_1	h_4	h_5
h_4	h_2	h_7	h_2	h_4	h_5	h_4
h_6	h_1	h_2	h_4	h_3	h_6	h_3
h_5	h_4	h_3	h_1	h_7	h_2	h_7
h_1	h_5	h_5	h_7	h_5	h_3	h_2
h_7	h_7	h_6	h_3	h_6	h_7	h_6

1) Using the Top Trading Cycle mechanism, find the core assignment. (Describe each step of the mechanism.)

2) Suppose that one of the agents can exclude one or more of the others (and their houses) from the market (i.e. can prevent other agents from participating in the market). For each agent, find the the best outcome he/she can achieve in the TTC mechanism by excluding other agents, and find the smallest group of agents he/she needs to exclude from the market to achieve it.

Problem 4

Consider a housing allocation with existing tenants problem with four existing tenants $\{a_1, a_2, a_3, a_4\}$ with occupied houses $\{h_1, h_2, h_3, h_4\}$ respectively, three newcomers $\{a_5, a_6, a_7\}$ and three empty houses $\{h_5, h_6, h_7\}$. The preference of agents over houses are described in the following table

a_1	a_2	a_3	a_4	a_5	a_6	a_7
h_2	h_2	h_1	h_2	h_7	h_3	h_1
h_3	$\vdots$	h_6	h_5	h_2	h_5	h_2
h_1		h_7	h_6	h_1	h_2	h_3
$\vdots$		h_3	h_3	$\vdots$	h_1	h_7
		$\vdots$	h_4		$\vdots$	h_5
			$\vdots$			$\vdots$

1) For the priority ordering $\{a_6, a_2, a_5, a_3, a_7, a_1, a_4\}$, find the outcome of the YRMH-IGYT mechanism. (Describe each step of the mechanism.)

2) For each agent, find his/her best possible outcome in the YRMH-IGYT across all possible priority orderings.