[1]ECO316
U of Toronto
Winter 2015
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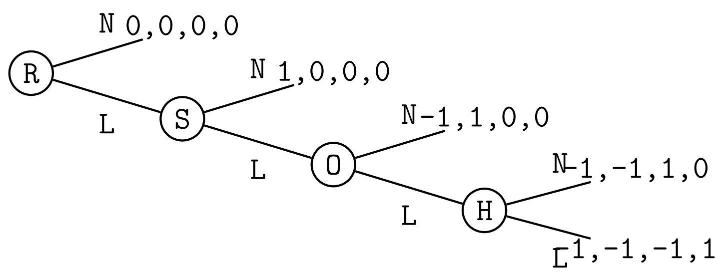
Midterm: February 13, 2015 - No aids allowed

You have 90 minutes to complete this test. There are four questions for a total of 100 points. To obtain credit you **must give an argument** to support each of your answers.

Question 1. [30 points total] It is lunch time. A rabbit is deciding whether to exit its rabbit hole and go eat in the garden patch. If the rabbit exits the hole, a snake will see it and can choose whether or not to exit its snake hole and eat the rabbit. If the snake eats the rabbit, an owl resting on a nearby tree will see it. The owl can fly down the tree and eat the snake, or remain on the tree. If the owl eats the snake, an hawk flying high in the sky will see it. The hawk can continue flying, or dive down to capture and eat the owl. All four animals prefer to have lunch to not having lunch, but they prefer to stay alive with an empty stomach to falling pray to another animal.

a) Model the strategic situation described above with an extensive form game (a game tree.)

See the extensive form game below. For each player the action L stands for having lunch, and the action N for not having lunch. The payoffs are ordered as: Rabbit, Snake, Owl, Hawk.



b) Find the subgame perfect Nash equilibrium (i.e. backward induction equilibrium) of the game.

In any subgame perfect nash equilibrium (SPNE) the Hawk must choose L since, when deciding this action gives a strictly higher payoff (1) than the alternative N (0). Since in any SPNE the hawk chooses L, the payoff to the owl from choosing L is -1 and the owl must choose N in any SPNE. Since the owl chooses N, the payoff to the snake from choosing L is 1, hence the snake must choose L in any SPNE. Finally, since the snake chooses L in any SPNE, the payoff to the rabbit from choosing L is -1 and the rabbit chooses N in any SPNE. Thus, the the strategy profile where the rabbit chooses N, the snake chooses L, the own chooses N and the hawk chooses L, is the only SPNE of the game.

c) For each player, explain whether they have any strictly or weakly dominated strategy.

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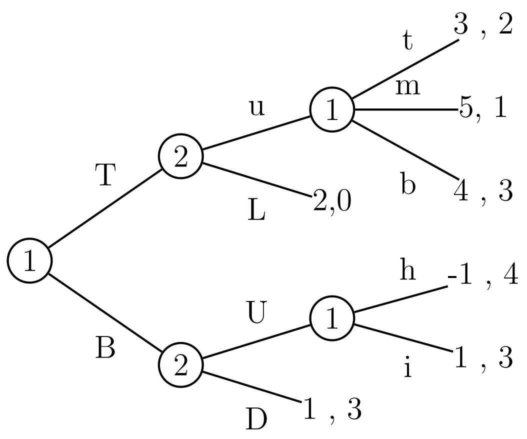
For the hawk the strategy L weakly dominates the strategy N. When all other animals have chosen L, by choosing L the hawk gets a payoff of 1 vs. a payoff of 0 from choosing N. If any of the other animal chooses N, the hawk gets the same payoff of 0 from both strategies. For each of the other players, N does strictly better than L when all previous movers chose L and the next mover chooses L, but L does strictly better than N when all previous movers have chosen L and the next mover chooses N. Thus, neither L nor N is dominated.

d) What strategy profiles survive iterative deletion of strictly dominated strategies? What strategy profiles survive iterative deletion of weakly dominated strategies?

By part c), no player has a strictly dominated strategy, thus all strategy profiles survive IDSDS. By part c), in the first round the strategy N for the hawk is deleted. In the game where the hawk can only choose L, the strategy L is weakly dominated for the owl, so it is erased in the second round. In the game where the own can only choose N, the strategy N is weakly dominated for the snake, so it is deleted in the third round. Finally, in the game where the snake can only choose L, the strategy L is strictly dominated by N for the rabbit. The SPNE is the only strategy profile that survives IDWDS.

Question 2. [30 points] Consider the extensive form game drawn below



a) Describe a strategy for Player 1 and a strategy for Player 2.

A strategy of player 1 is a combination of three actions, one for each of Player 1's decision nodes. For example $\{T, m, h\}$. For Player 2, a strategy is a combinati of two actions, one for each of Player 2's decision node. For example $\{u, U\}$.

b) How many strategies does Player 1 have in this game?

A strategy for Player 1 is a combination of one out of two available action at the initial node, one out of three available actions after 2 chooses u and one out of 2 available actions after 2 chooses U. This gives a total of $2 \times 3 \times 2 = 12$ different strategies for Player 1.

c) Find all subgame perfect Nash equilibria (backward induction equilibria) of this game.

In any subgame perfect Nash equilibrium, Player 1 must choose m and i after player 2 chooses u and U respectively. This implies that the payoff to player 2 from choosing u is 1 vs. a payoff of 0 from choosing L. Thus Player 2 must chose u in any SPNE. SImilarly, the payoff from choosing U is 3 and the payoff from choosing D is also 3, thus Player 2 can choose either U or D in a SPNE. Regardless of that choice, the payoff to Player 1 from choosing B is 1 vs. a payoff of 5 from choosing T. Thus there are two SPNE, where the strategy of Player 1 is $\{T, m, i\}$ and the strategy of Player 2 is either $\{u, U\}$ or $\{u, D\}$. Note that there is a unique SPNE outcome.

d) Can you find a strategy that is weakly dominated for Player 2?

Any strategy where player 2 chooses L is weakly dominated by the strategy that replaces L with u. The new strategy will yield exatly the same payoff when Player 1 chooses B, but a strictly larger payoff when Player 1 chooses T.

e) Can you find a strategy that is strictly dominated for Player 1?

Any strategy of Player 1 where at the initial node Player 1 chooses B is strictly dominated by any strategy where at the initial node Player 1 chooses T. This is because the lowest payoff P1 can get after choosing T (2) is strictly larger than the largest payoff P1 can get after choosing B (1).

Question 3. [25 points total] Consider the strategic form game described by the following table, where x, y, w and z are real numbers.

		C: l	m	r
R	T	x, y	$3, w$	$0, 1$
	M	$1, 0$	$z, 2$	$0, 3$
	B	$0, 1$	$0, 3$	$1, 0$

Find values of x, y, w and z such that **all** of the following properties are satisfied.

a) The row player has a dominated strategy.

Fist note that B does not dominate nor is dominated by either M or T. Since M or T give the same payoff against r, the only possibilities are that M weakly dominates T (when $x < 1$ and $z > 3$) or that T weakly dominates M (when $x > 1$ and $z < 3$.)

b) The column player has no dominated strategy.

This requires that $w < y$ else l is dominated by m.

c) Only one outcome survives the procedure of iterative deletion of weakly dominated strategies.

From the conditions derived in a). If $x < 1$ and $z > 3$ so that T is weakly dominated by M, after deleting T, the strategy m for Player 2 strictly dominates the strategy l. This leaves the game

	m	r
M	$z, 2$	$0, 3$
B	$0, 3$	$1, 0$

where no strategy is dominated since $z > 3 > 0$. If $x > 1$ and $z < 3$ so that M is weakly dominated by T, the only strategy that can be dominated in the remaining game is r which requires that the largest between w and y be at least as large as 1. Since we have already assumed $y > w$, the condition required is $y \geq 1$. After eliminating r T dominates B, and after B is eliminated then l dominates m.

Thus, if $x > 1$, $z < 3$, $y > w$ and $y \geq 1$, the game satisfies all of the properties above.

Question 4. [15 points] Consider the strategic form game described by the following table, where x is a real number (can be either positive or negative).

	A	B
A	$1, -1$	$0, x$
B	$x, 0$	x, x

a) Find all the Nash equilibria, as a function of x (i.e. describe all Nash equilibria as x varies between $-\infty$ and $+\infty$.)

When $x < -1$, A strictly dominate B for both player, thus the unique Nash Equilibrium is the strategy prpofile (A, A). When $-1 \leq x < 0$, A strictly dominate B for the row player so it must be played in any Nash equlibrium. The column player best response to A is B, if $-1 < x$ and is both A and B if $x = -1$. Both (A, A) and (A, B) are NE if $x = -1$ and only (A, B) is a NE if $-1 < x < 0$. When $x > 0$, B strictly dominates A for the column player and the row player best response to B is B. So (B, B) is the only NE. When $x = 0$ both (B, B) and (A, B) are NE, the other two strategy profiles are not since for the row player A is the unique best response to A, and for the column player B is the only best response to A.

b) For what values of x, if any, the game has more than one Nash equilibrium?

From the answer to part a), for $x = -1$ and $x = 0$ there are two NE. There is a unique Nash equlibrium for all other values of x.