ECO316
U of Toronto
Winter 20124
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You have 90 minutes to complete this test. There are four questions for a total of 100 points. To obtain credit you **must give an argument** to support each of your answers.
Note: Question 1 below is quite long. However, with the exception of part i), each of its part can be answered independently.

Question 1. [50 points total] Two individuals, John and Nelly, are trying to lease a rent controlled apartment by bribing the building superintendent (super). John goes to the super first and can give him any amount of money in multiples of one dollar (i.e. 0, 1, 2, 3, ...) Nelly goes to the super after John. The super tells Nelly about how much money John has given him, and explains that she can have the apartment if she is willing to give him a strictly larger bribe (which again must be a multiple of 1 dollar, i.e. 0,1,2,3,...) After Nelly chooses her bribe, the super assigns the apartment and keep **both** bribes.

Both John and Nelly value leasing the rent controlled apartment \$2.5 and leasing a different apartment \$0. They care about the difference between the value of the apartment they lease and the bribe they pay (e.g. if John pays a bribe of \$1, his payoff is $2.5 - 1 = 1.5$ if he leases the rent controlled apartment and $0 - 1 = -1$ if Nelly leases the rent controlled apartment.)

a) (5 points) Explain why offering a bribe strictly larger than \$2 is a strictly dominated strategy for John.

By offering a bribe strictly larger than 2, the best possible payoff for John is strictly negative. This is because even if he gets the apartment he will have paid more that he values it. By offering a bribe of 0, the worst possible payoff for John is 0 (in the case when he does not get the apartment). Thus, regardless of Nelly strategy, 0 always give a payoff strictly larger than any bribe larger than 2 (i.e. 0 strictly dominate any bribe larger than 2.)

b) (5 points) Give and example of a strategy for Nelly.

Offer a bribe of 1 regardless of the bribe offered by John is a strategy for Nelly. (In general a strategy for Nelly is a specification of what bribe Nelly offer: i) after John offers no bribe; **and** after John offers a \$1 bribe; **and** after John offers a \$2 bribe.)

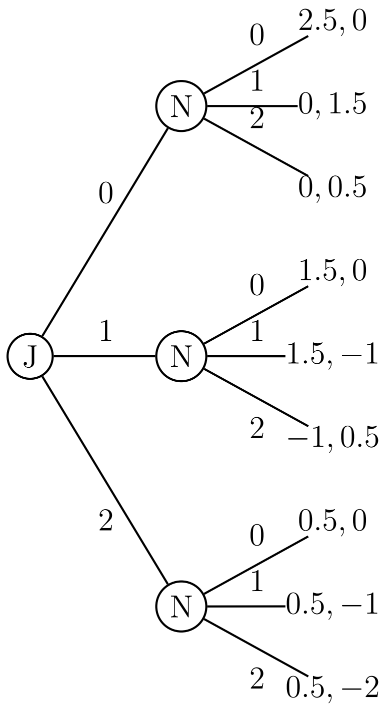
c) (5 points) Give an example of a weakly dominated strategy for Nelly. Any strategy in which Nelly offers a bribe larger and 2 after one John's possible bribes is weakly dominated by the strategy that replaces the bribe larger than 2 dollar with a 0 dollar bribe.

d) (5 points) Give an example of a strictly dominated strategy for Nelly.

The strategy of offering a bribe of \$3 **regardless** of John's bribe is strictly dominated by the strategy of offering a bribe of \$0 **regardless** of John's bribe.

e) (5 points) Consider the case when both John and Nelly can only offer bribes no larger than 2 (i.e. 0,1 and 2) and, using a game tree, model the strategic scenario as an extensive form game with perfect information.

In the game tree below, John's (J) actions is described by the amount of money he gives to the super and Nelly's (N) actions are described by the amount of money she gives. At each terminal node, the first number is John's payoff the second number is Nelly's payoff.



f) (10 points) Find a "backward induction" (i.e. subgame perfect Nash) equilibrium of the game in part e).

If J chooses 0, Nelly's payoff is maximized by choosing 1. If J chooses 1, Nelly's payoff is maximized by choosing 2, and if J chooses 2, Nelly's payoff is maximized by choosing 0. Thus, in any BI equilibrium, Nelly must choose 1 after 0, 2 after 1, and 0 after 2. Given this strategy of Nelly, J's payoff is 0 if he chooses 0, -1 is he chooses 1 and 0.5 if he chooses 2. John's payoff is maximized by 2. The in the (unique) BI equilibrium, J chooses 2, and N's strategy is as described earlier.

Consider a generalization of the strategic scenario described above where John values the rent controlled apartment \$J.5 and Nelly values it \$N.5. Both J and N are integers, (e.g. If J=3 and N=4, John values the rent controlled apartment \$3.5 and Nelly values it \$4.5) and John and Nelly can offer any bribe (i.e. they can offer 0,1,2,3,4,...)

g) (5 points) For what values of J and N will John lease the apartment in the backward induction equilibrium outcome? What bribes will John and Nelly pay in the equilibrium?

h) (5 points) For what values of J and N will Nelly lease the apartment in the backward induction equilibrium outcome? What bribes will John and Nelly pay in the equilibrium?

Solution for both g) and h) In any BI equilibrium, Nelly must choose 0 after any bribe of John b such that $b \geq N$. This is because when John's bribe is greater or equal to N, Nelly can only get the apartment by offering a bribe that is strictly larger than how much she values the apartment, thus she prefers to loose the apartment, in which case it is optimal to offer no bribe. Nelly must also choose $b+1$ whenever $b < N$. This is because a bribe of $b+1$ is the smallest amount that will allow Nelly to get the apartment, and when $b < N$, $b+1$ is still less than how much Nelly values the apartment, thus it is optimal to win the apartment with the smallest possible bribe.

Given Nelly's strategy, the smallest bribe that will allow John to get the apartmen is N. When $N \leq J$, it is optimal for John to offer a bribe of N and obtain the apartment. If $N > J$, it is optimal for John to offer no bribe. Thus, in the (unique) BI equilibrium if $N \leq J$ John gets the apartment, and John and Nelly pay N and 0 respectively; if $N > J$ Nelly gets the apartment and John and Nelly pay 0 and 1 respectively

i) (5 points) Suppose the super knows J and N, and can choose the order in which John and Nelly present their bribes, who should the super choose to go first?

Whenever $N = J$, the amount of money the super receics is the same regardless of the order of the offers. When $N \neq J$, the person with the highest valuation will get the apartment, but the order matter for the amount of money the super receives. For exemple, suppoe $J > N$, if John is first, he pays N to the super, if he goes second he pays 1. So to maximize the super's revenue, he should let the person with the highest valuation go first, unless the peson the other person values teh apartment $0.5, in which case the order should be reversed.

Question 2. [20 points total] Consider the strategic form game described by the following table.

$$\begin{array}{cc|c|c|c|} & & \multicolumn{3}{c}{C} \\ & & l & m & r \\ \hline & T & 0,2 & 3,1 & 2,3 \\ \hline R & M & 1,4 & 2,1 & 4,1 \\ \hline & B & 2,1 & 4,4 & 3,2 \\ \hline \end{array}$$

a) (10 points) Find the set of outcomes that survive iterative deletion of strictly dominated strategies.

Strategy T is strictly dominated by B for the row player. There are no other strictly dominated strategies. After deleting T, the strategic game becomes

	l	m	r
M	$1,4$	$2,1$	$4,1$
B	$2,1$	$4,4$	$3,2$

and there are no more strictly dominated strategies. Thus all the outcomes in table above survive IDSDS.

b) (10 points) Find the set of outcomes that survive iterative deletion of weakly dominated strategies.

In the table above, strategy r is weakly dominated by m for the column player. Eliminating it we obtain the game

	l	m
M	$1,4$	$2,1$
B	$2,1$	$4,4$

M is strictly dominated by B for the row player. Eliminating it

	l	m
B	$2,1$	$4,4$

and now l is strictly dominated by m for the column player. Thus (B, m) is the only outcome that survives IDWDS.

Question 3. [15 points total] Consider the strategic form game described by the following table.

	l	r
T	$1,0$	$0,2$
M	$0,2$	$2,0$
B	$1,1$	$2,1$

Consider a procedure of iterative deletion of weakly dominated strategies where **at most one strategy** is eliminated in each round. Can the set of outcomes that survive the procedure depend on the order in which the strategies are eliminated? If no, explain why. If yes, provide an example.

Yes. In the game above both T and M are weakly dominated by B for the row player. Eliminating T we get

	l	r
M	$0,2$	$2,0$
B	$1,1$	$2,1$

where now r is weakly dominated by l for the column player. Eliminating r

	l
M	$0,2$
B	$1,1$

and now M is strictly dominated by B, thus the only outcome that survives is (B, l). Suppose instead we start the procedure by eliminating M in the first round. We obtain the strategic game below

	l	r
T	$1,0$	$0,2$
B	$1,1$	$2,1$

where l is weakly dominated by r for the column player. Eliminating l we get

	r
M	$0,2$
B	$2,1$

and now M is strictly dominated by B, thus the only outcome that survives the procedur is (B,r) which is different from the outcome we found earlier following a different order of deletion.

Question 4. [15 points total] Given an example of a strategic form game with three (3) players such that **all** of the following properties are satisfied.

1. One player has a strictly dominated strategy.
2. One player has a weakly dominated strategy and no strictly dominated strategy.
3. One player has no dominated strategy.

Consider the following strategic game with three player, A, B and C. Player A has two actions, T and B, Player B has two actions, l and r, and Player C chooses between L and R. We can describe the game through the following two tables. The payoffs are denoted $a_{\dots}$ for A, $b_{\dots}$ for B and $c_{\dots}$ for C.

L

	l	r
T	$a_{TlL}, b_{TlL}, c_{TlL}$	$a_{TrL}, b_{TrL}, c_{TrL}$
B	$a_{BlL}, b_{BlL}, c_{BlL}$	$a_{BrL}, b_{BrL}, c_{BrL}$

R

	l	r
T	$a_{TlR}, b_{TlR}, c_{TlR}$	$a_{TrR}, b_{TrR}, c_{TrR}$
B	$a_{BlR}, b_{BlR}, c_{BlR}$	$a_{BrR}, b_{BrR}, c_{BrR}$

For Player A to have a strictly dominated strategy, say T it msust be the case that

$$a_{BsS} > a_{TsS}$$

for every combination of $s = l, r$ and $S = L, R$. For Player B to have a weakly dominated strategy, say l, but no strictly dominated strategy it must be the case that

$$b_{\sigma rS} \geq b_{\sigma lS}$$

for every combination of $\sigma = T, B$ and $S = L, R$. Further, there must at least one combination of σ and S such that $b_{\sigma rS} > b_{\sigma lS}$ and at least one combination such that $b_{\sigma rS} = b_{\sigma lS}$.
For Player C to have no dominated strategy, it must be the case that for some combinati of $\sigma = T, B$ and $s = l, r$ $c_{\sigma s,R} > c_{\sigma s,L}$ and for some other combination $c_{\sigma s,R} < c_{\sigma s,L}$.