ECO316
U of Toronto
Fall 2012
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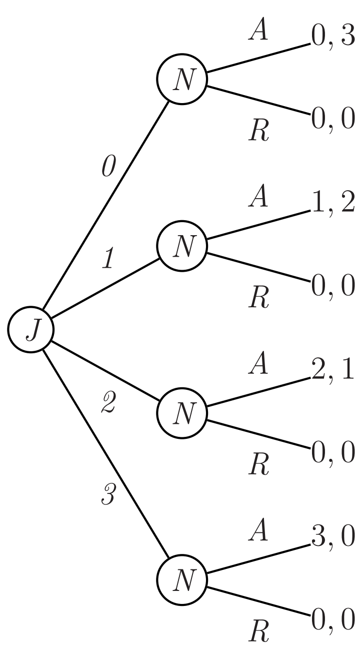
Midterm: October 15, 2012

You have 75 minutes to complete this test. There are three questions for a total of 75 points. To obtain credit you **must give an argument** to support each of your answers.

Question 1. [35 points] John and Nelly are trying to share a 3 dollar prize won by participating in a class experiment. They finally agree to the following simple sharing rule. John will make a proposal about how to divide the sum of money into multiples of one dollar (i.e. John can propose to keep 0 or 1 or 2 or 3 dollars for himself and give the remainder to Nelly). After seeing John's proposed division Nelly can either accept or reject it. An accepted proposal is carried out. If a proposal is rejected the money goes back to the professor, so neither John nor Nelly receives any money. John wants to maximize the amount of money he receives and Nelly wants to maximize the amount of money she receives.

a) (8 points) Model this situation through an appropriate game.

In the game tree below, John's (J) actions is described by the amount of money he proposes to keep for himself. Nelly's (N) actions are labelled with R for rejecting the proposal and A for accepting the proposal. At each terminal node, the first number is John's payoff the second number is Nelly's payoff. John's and Nelly payoff are assumed equal to the amount of money each receive in the game.



b) (5 points) Give an example of a strategy for John and a strategy for Nelly.

Since John only makes a choice at one decision node, a strategy for John is described by the action taken at that decision node. For example proposing to keep 2 dollars is a strategy for John. Nelly makes decisions at 4 decision nodes. A strategy for Nelly must specify an action at each of the four nodes. For example, accepting a proposal if John asks for less than 2 dollars and rejecting a proposal if John asks for 2 or 3 dollars is a strategy for Nelly.

c) (4 points) How many different strategies are there for Nelly?

Nelly has four decision nodes, and at each node she has a choice of one out of two actions. Thus there is a total of $2^4 = 16$ *different strategies available to Nelly.*

d) (10 points) Find all backward induction equilibria (i.e. subgame perfect Nash equilibria).

First notice that Nelly is strictly better off accepting a proposal in which she receives a positive amount of money. Thus in any backward induction (sub game perfect) equilibrium Nelly must accept all proposals where John asks for less than three dollars. Nelly is indifferent between accepting and rejecting the division in which John keeps all the money, so we can construct two backward induction equilibria, one in which Nelly accepts any proposal, and one in which Nelly only rejects if she is offered no money.

Consider the first case. Since Nelly accepts all proposals, proposing to keep 3 dollars maximizes John's payoff. Thus the strategy profile in which John asks for 3 dollars and Nelly accepts any proposal is a backward induction equilibrium.

Consider the second case. Nelly accepts if John proposes to keep 0, 1 or 2 dollars and rejects if John proposes to keep 3 dollars. Now John maximizes his payoff by proposing to keep 2 dollars. Thus the second backward induction equilibrium has John asking for 2 dollars and Nelly accepting all but the proposal in which John asks for 3 dollars.

Consider a slightly more complex sharing rule where, if John's proposal is rejected, before returning the money to the professor John and Nelly try one more time to divide the money using the same method as above but with the roles reversed (i.e. Nelly proposes a division and John decides whether to accept it or not). If Nelly's proposal is also rejected the money goes back to the professor.

e) (8 points) Identify a backward induction equilibrium outcome. (Note: I am not asking that you give the entire equilibrium strategy, just to provide an argument for why a particular outcome is a backward induction equilibrium outcome. Also, for this question, you can assume that whenever a player is indifferent between rejecting and accepting a proposal they will reject it.)

In this game, after John's proposal is rejected, the two players play the same game as in the previous section with the roles reversed. Thus, Nelly anticipates that after rejecting a proposal she will get the payoff that John gets in the equilibrium of the previous game. In the case indifference leads to rejection that payoff is 2 dollars. This implies that Nelly will reject any proposal that gives her less than two dollars. Assuming that she rejects when indifferent she will only accept the proposal in which John gives her 3 dollars. Now consider John's choices at the beginning of the game. If he asks for 0 dollars Nelly will accept and his payoff will be 0. If he asks for any other amount Nelly will reject, in the second round Nelly will offer him 1 dollar and he will accept it. A backward induction equilibrium outcome is one where John first asks for one dollar, Nelly rejects and asks for 2 dollars for herself and John accepts, the payoffs are 1 to John and 2 two Nelly.

Question 2. [20 points] Fill in the missing payoff values in the strategic game described below so that **all** of the following properties hold (partial marks are awarded for a game that satisfies some but not all of the properties.)

1. One of the strategies of the row player is strictly dominated;
2. NO strategy of the column player is dominated (neither strictly nor weakly);
3. only one strategy profile survives the iterative deletion of weakly dominated strategies;
4. multiple strategy profiles survive the iterative deletion of strictly dominated strategies.

	l	r
T	$2,2$	$4,1$
M	$1,3$	$5,z$
B	$1,x$	$y,3$

Only B can be strictly dominated for player 1 and this happens whenever $y < 4$. Since B is strictly dominated for the row player, it never survives neither IDSDS nor IDWDS. For property 3 and 4 to be satisfied in the game below the column player must have one strategy that is weakly but not strictly dominated. This implies that $z = 3$.

	l	r
T	$2,2$	$4,1$
M	$1,3$	$5,z$

Finally, since $z = 3$ In the complete game above x must be strictly smaller than 3 otherwise l weakly dominates r violating property 2.

Question 3. [20 points] The members of a club must decide whether to allow a new member, Julie, to join. The procedure for admitting a new member is simple. Each existing member receives a secret ballot and can: i) vote to admit Julie; ii) vote to NOT admit Julie; or iii) abstain. Julie is admitted as a new member if strictly more than half of the **voting** members voted in favour of her admission (abstaining does not count as voting).

The club has ten existing members: four males and six females. All members only care whether Julie is admitted or not. The male members strictly prefer that Julie be admitted while the female members strictly prefer that Julie not be admitted.

a) (3 points) Model the situation described above as a strategic game (i.e. describe players, strategies and payoffs).

   The players are 10 club members, four males and six females. Each club members has three actions: vote in favour of admitting Julie, vote against admitting Julie and abstaining (i.e. not voting). The payoff to each male (female) club member is 1 (-1) if the number of votes in favour of Julie is strictly larger that the number of votes against and it is -1 (1) otherwise.

b) (9 points) Find all strategies that are weakly dominated.

For a male club member both voting against Julie's admission and abstaining are weakly dominated by voting in favour of admitting Julie. Let denote with x the difference between the votes in favour and against Julie among the other nine club members. If $x < 0$ Julie will not be admitted regardless of the tenth club member, thus his payoff from each of the three strategies is -1. When $x = 0$, Julie is admitted is the tenth club member votes in favor and she is not admitted otherwise, thus the payoff from voting in favour is strictly larger that the payoff from the other two actions. If $x = 1$ voting in favour and abstaining yield a payoff of 1 voting against gives a payoff of -1. And finally, if $x > 1$ the payoff to the tenth club member is 1 regardless of his action. In any scenario is payoff from voting in favor of Julie is at least as large as the payoff from the two other strategies.

Similarly for a female club member voting against Julie weakly dominates voting in favour and abstaining.

c) (4 points) What strategy profiles survive iterative deletion of weakly dominated strategies?

Since for each player all but one strategy are dominated, the only profile of strategies that survives IDWDS is the profile where each male member votes in favour of Julie and each female member votes against her admission.

d) (4 points) What strategy profiles survive iterative deletion of strictly dominated strategies?

No strategy is strictly dominated for any player, thus all strategies survive IDSDS. To see why no strategy is strictly dominated note that if, for example, nine of the 10 club members vote in favour of admission, the payoff choice of the tenth club member does not change the outcome. In this scenario, the payoff to the tenth club member from each of his three strategies is the same. available is the same, thus no strategy is strictly dominated.