ECO316
U of Toronto
Fall 2011
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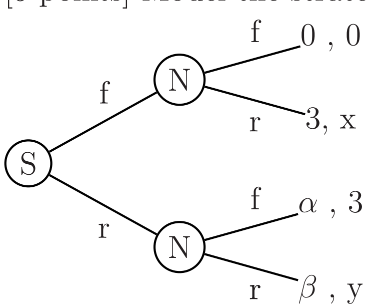
<u>Midterm: October 17, 2011</u>

You have 1 hour to complete this test. There are three questions each worth 20 points. You must explain every answer.

Question 1. [20 points] The Norman army led by William the Conquerer has landed on the Sussex beaches at Hastings in 1066 A.D. and now faces the Saxon army. The Saxon army has the first choice. It can charge the Normans or retreat. The Norman army can respond by either staying on the beach and fighting or by retreating back to Normandy. When both armies fight it is the worse outcome for both the Normans and the Saxons. The best outcome for the Normans is when they fight while the Saxons are retreating. Similarly, the best outcome for the Saxons is when the Normans retreat after being attacked.

a) [5 points] Model the strategic situation described above with a game tree.



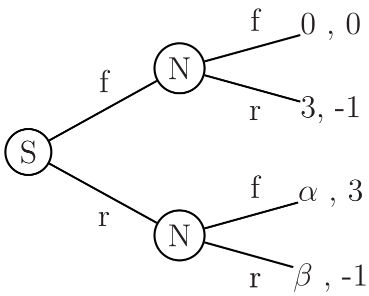
The game above describes the strategic situation faced by the Norman and Saxon armies whenever x, y, α and β are numbers between the payoff from the worst outcome (0) and the payoff from the best outcome (3).

b) [5 points] Find the backward induction solution of the game modelled in part a). Describe both the equilibrium outcome and the equilibrium strategies of the game.

At the decision node after they have been attacked, the Normans strictly prefer retreating to fight, and the opposite is true at the decision node after the Saxons retreat. The Saxon, anticipating that the Normans will retreat if attacked, strictly prefer attacking. Thus the equilibrium outcome is that the Saxon attack and the Norman retreat. The Saxons' equilibrium strategy is to attack, the Normans' equilibrium strategy is to retreat if attacked and attack otherwise.

Now suppose that, before the Saxon army makes its move, William the Conqueror can order that all the Norman's boats be burned. The smoke from the burning boats is clearly visible to the Saxon army, so it will know that the Normans have no boats left. Burning the boats changes the Norman preferences over the outcomes. Now retreating (whether attacked or not) leeds to certain death by drowning, which is the worse possible outcome.

c) [5 points] Change the game in part a) appropriately to reflect the new preferences, and find the backward induction equilibrium outcome.



Now the Normans prefer attacking to retreating at both decision node. The Saxons, anticipating that the Normans will fight back if attacked, prefer retreating to attacking. The backward induction outcome is that the Saxon retreat and the Normans attack.

d) [5 points] If the objective of William the Conqueror is to maximize the payoff of the Norman's army, should he order that the boats be burned?

After burning the boats, the equilibrium outcome is the most preferred outcome by the Normans, and it is strictly preferred to the outcome of the game in part a). Thus, William the Conquerer should order that the boats be burned.

Question 2. [20 points] Consider the two-player strategic game described below.

	l	c	r
T	$2, 2$	$4, 3$	$4, 1$
M	$1, 3$	$3,$	$5,$
B	$1, 4$	$, 3$	$3, 2$

a) [5 points] Fill in the missing payoff values so that: i) the row player ha as a strictly dominated strategy; and ii) the column player has a strategy that is weakly but not strictly dominated.

	l	c	r
T	$2, 2$	$4, 3$	$4, 1$
M	$1, 3$	$3, x$	$5, y = \max\{3, x\}$
B	$1, 4$	$z < 4, 3$	$3, 2$

For example, $x = 2$ $y = 3$ *and* $z = 3$ *(an example suffices).*

b) [5 points] Using the payoff values you chose in part a), solve the game by iterated deletion of dominated strategies.

The strategy B *for the row player is strictly dominated by the strategy* T*, and the strategy* r *for the column player is weakly dominated by either* l *or* c*. After eliminating these two strategies in the resulting game*

	l	c
T	$2,2$	$4,3$
M	$1,3$	$3,x$

the strategy M for the row player is strictly dominated by the strategy T. In the resulting game

	l	c
T	$2,2$	$4,3$

the strategy l for the column player is strictly dominated by the strategy c. Thus iterated deletion of dominated strategies solution of the game is that the row player chooses T and the column player chooses c.

c) [5 points] Is the solution you found in part b) a Nash Equilibrium? Are there other Nash equilibria?

Yes, the strategy profile T, c is a Nash equilibrium. When the column player chooses c the strategy T maximizes the row's player payoff. Thus, T is a best response to c for the row player. Similarly, when the row player chooses T, the strategy c maximizes the column player's payoff. Thus it is a best responses as well. The strategy profile M, r is also a Nash equilibrium. The row player is best responding. For any payoff to the column player from the outcome (M, r), it must be that r is a best response to M, otherwise the strategy r is strictly dominated by either l or c.

d) [5 points] Explain why, as long as the payoffs satisfy the properties stated in part a), the outcome of iterated deletion of dominated strategies does not change and the set of Nash equilibria does not change.

Regardless of the payoff number chosen, the only strategy that can be strictly dominated for the row player is B. The only strategy that can be weakly dominated for the column player is l. Thus these two strategies are eliminated in any game that satisfy the properties stated in a). In the resulting game the strategy M for the row player is strictly dominated, and after that the strategy l is strictly dominated for the column player. These unique outcome is a Nash equilibrium. Also, M, r is always a Nash equilibrium, the argument is already given in the answer to part c).

Question 3. [20 points] John and Nelly share an apartment. The kitchen has become quite dirty and both friends, simultaneously, are deciding whether to clean the kitchen or not. John's most preferred outcome is when Nelly cleans the kitchen by herself. Similarly, Nelly's most preferred outcome is when John cleans by himself. Both roommates prefer to remain with a dirty kitchen to cleaning the kitchen by themselves.

a) [5 points] Model this situation as a strategic form game.

In the strategic game below John is the row player and Nelly is the column player, c *stands for cleaning and* s *stands for not cleaning (shirking)*

	c	s
c	$2, 2$	$1, 4$
s	$4, 1$	x, x

The game satisfy the assumptions in the text whenever x *is in between 1 and 4.*

b) [5 points] Does John have a dominated strategy? What about Nelly?

For John (Nelly), working yields a lower payoff than shirking regardless of what Nelly (John) does. Thus working is dominated by shirking for John (Nelly).

c) [5 points] Find the Nash equilibrium of the game in part a).

The unique Nash equilibrium is that both John and Nelly shirk. This is because regardless of the other player's strategy, both John and Nelly' best response is to shirk.

d) [5 points] Explain, using less than 15 words, under what additional condition on the preferences the game describes a "prisoner's dilemma" situation.

The game above describes a prisoner's dilemma scenario whenever x *is strictly* ~~*larger*~~ smaller *than 2. That is whenever the outcome where both work is preferred by both players to the unique equilibrium outcome where they both shirk.*