University of Toronto
Faculty of Arts and Sciences

APRIL EXAMINATIONS 2012

ECO316H1S

Duration - 2 hours

No aids are allowed

There are 3 questions in 2 pages. You must give arguments to support your answers.

Question 1. [50 points] Three voters, A, B and C, must elect one of three candidates, a, b or c, into office. Voter A prefers candidate a to candidate b and candidate b to candidate c. Voter B prefers candidate b to candidate c and candidate c to candidate a. Voter C prefers candidate c to candidate a and candidate a to candidate b. Each voter votes for one of the three candidates. A candidate with a majority of votes wins the election, and in the case of a tie the candidate voted by A wins.

For the next three questions assume that voters cast their votes simultaneously.

a) Explain why, regardless of the choices of voters B and C, it is a best response for A to vote for candidate a.

Whenever B and C vote for the same candidate, A's vote does not affect the result (not pivotal). Voting for a or any other candidate gives A the same payoff, hence voting for a is a best response. Whenever B and C vote for different candidates, the candidate A votes for wins the election. Voting for a yields a strictly larger payoff than voting for b or c to A, hence voting for a is a best response to A.

b) Find all Nash equilibria in which voter A votes for candidate a.

Let's characterize the best response function of B and C fixing the action of A to a. When B (C) votes for a any strategy is a best response for C (B) since a is elected regardless. When B votes for b, C's best response is to either vote for a or c, in both cases a will be elected which is preferred to b elected by C. When C votes for b, if B does the same b is elected otherwise a is elected, thus B's best response is to vote for b. If B (C) votes for c, C's (B's) best response it to vote for c. By voting c candidate c is elected, otherwise a wins. Both C and B prefer candidate c to candidate a. The best response functions of B and C have three intersections at the strategy profiles (a, a, a), (a, b, a), and (a, c, c), which are all NE since, by part a), also A is best responding.

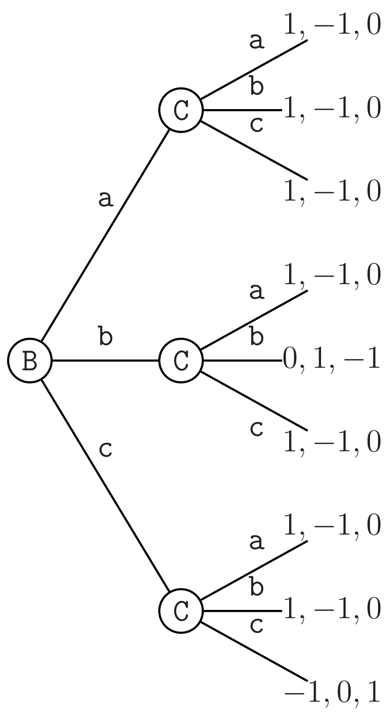
c) For each candidate a, b and c, find a Nash equilibrium in which the candidate is elected (when appropriate you can refer to the Nash equilibria you found in part b).)

The strategy profiles (a,a,a), (b,b,b) and (c,c,c) are Nash equilibria. This is because whenever two players vote for the same candidate it is a best response for the third player to vote for that candidate as well since the third player's vote does not change the outcome of the election.

For the next three questions assume that A votes first, B votes after observing A's choice, and C votes after observing both A and B's votes.

d) Assume that A has voted for candidate a. Using a game tree, model the strategic situation faced by voters B and C and find the backward induction equilibrium.

In the game three below, -1, 0 and 1 are the payoffs associated to the worst, intermediate and best election outcome for each player. The payoffs are ordered as (A,B,C).



In any sub-game perfect Nash equilibrium (backward induction equilibrium) C will vote for c if B voted for c and will not vote for b if B voted for b. Thus, by voting for a or b, B receives a payoff of -1 while he receives a payoff of 1 by voting for c. Any strategy profile such that B votes for c and C votes for an arbitrary candidates after a, does not vote for b afar b and votes for c after c is a sub-game perfect Nash equilibrium. In any equilibrium c wins the election.

e) Repeat the analysis of part d) after assuming that A has voted for candidate b and after assuming that A has voted for candidate c.

A similar analysis to that of part d) shows that: i) when A has voted for B, in any sub-game perfect NE, C votes for a after a, votes for anybody after b and votes for c after c. Voter B votes for b, and thus b wins in any SPNE; ii) when A voted for c, in any sub-game perfect NE C votes differently from B whenever B does not vote for c. In any equilibrium c wins.

f) Use the results for part d) and e) to explain for which candidate A votes in the backward induction equilibrium of the sequential voting game.

The analysis of part d) and e) shows that in any sub game perfect NE c wins whenever A votes for a or for c and b wins otherwise. So in any SPNE A votes for b.

Question 2. [25 points] Consider the two-player zero-sum game described below, where the numbers refer to the payoff of the row player.

	l	r
T	1	2
B	3	0

a) **Using the min-max method** demonstrate that there is no Nash equilibrium in pure strategies.

For the row player, the minimum payoff associated to action T is 1 and the minimum payoff associated to action B is 0. Thus the $\max\min$ value is 1. For the column player, the maximum payoff associated with action l is 3 and the maximum payoff associated with action r is 2. Thus the $\min\max$ value is 2. The game does not have a pure strategy NE because the two values do not coincide.

b) Find the mixed strategy Nash equilibrium of the game.

Let p be the probability that the row player chooses T and q the probability that the column player chooses l. Using the indifference principle, q must equalize the expected payoff from T and B for the row player or

$$1q + 2(1 - q) = 3q.$$

Similarly, p must equalize the expected payoff from l and r for the column player, or

$$1p + 3(1 - p) = 2p.$$

Solving for p and q we have that the strategy profile where the row player plays T with probability $p^* = 3/4$ and the column player plays l with probability $q^* = 1/2$ is the only NE of the game.

c) Re-derive the mixed strategy Nash equilibrium of part b) **using the min-max method**.

A mixed strategy p, achieves the worst (i.e. smallest) payoff to the row player when the column player is best responding to p. Since l is a best response to p when $p \geq 3/4$ and r is a best response when $p < 3/4$, the minimum payoff to the row player associated with strategy p is $1p + 3(1 - p)$ for $p \geq 3/4$ and equals $2p$ for $p < 3/4$. The $\max\min$ value is achieved by $p = 3/4$ and equals $3/2$.

Similarly, a mixed strategy q achieves the worst (i.e. highest) payoff to the column player when the row player is beset responding to q. Since T is a best response for $q \leq 1/2$ and R is a best response for $q > 1/2$, the maximum payoff associated with strategy q is $1q + 2(1-q)$ for $q \leq 1/2$ and $3q$ for $q > 1/2$. The $\min\max$ value is achieved by $q = 1/2$ and equals $3/2$.

Thus the mixed strategy NE of the game is the strategy profile $(p^* = 3/4, q^* = 1/2)$.

Question 3. [25 points] Consider the symmetric two-player game described below.

	a	b	c	d
a	$1,1$	$2,0$	$1,0$	$1,0$
b	$0,2$	$2,2$	$4,1$	$1,1$
c	$0,1$	$1,4$	$4,4$	$1,2$
c	$0,1$	$1,1$	$2,1$	$0,0$

a) Find all Nash equilibria in pure strategies.

The action a is the unique best response to a. The actions a and b are best responses to b. The actions b and c are best responses to c and the actions a, b and c are best responses to d. There are three intersections of the best response functions (i.e three NE), (a, a), (b, b) and (c, c).

b) For each pure strategy explain whether it is evolutionary stable.

The strategy d is not evolutionary stable since (d, d) is not a Nash equilibrium of the game. The strategy a is ES because (a, a) is a strict Nash equilibrium (i.e a is the unique best response to a). The strategy profile (b, b) is a NE, however a is another best response to b. For b to be ES it must be that the payoff to b against a is larger than the payoff to a against itself. Since the former is 0 while the latter is 1, the condition is violated and b is not ES.

Similarly, the strategy profile (c, c) is a NE, however b is another best response to c. For c to be ES it must be that the payoff to c against b is larger than the payoff to b against itself. Since the former is 1 while the latter is 2, the condition is violated and c is not ES.