ECO316, Winter 2014
University of Toronto
Ettore Damiano

Assignment

Question 1. Consider the two players private values model discussed in class, where the two players valuations are v_L and v_H, with $v_L < v_H$.

a) Find all equilibria of the second price auction (including those in which players use dominated strategies).

b) Suppose there are N player, with valuations $v_1 < v_2 < \ldots < v_N$. For each player find an equilibrium of the second price auction where that player wins.

Question 2. (from the December 2011 final exam) Two bidders compete for a single object in an all-pay auction. They simultaneously submit one of two possible bids p_H, p_L with $p_H > p_L > 0$. The object is worth $\$v$ to both and it is assigned to the highest bidder. Ties are broken by a fair coin flip. Each bidder pays a price equal to his submitted bid whether he receives the object or not. The payoff to each bidder is equal to the value of the object times the probability of receiving it minus the price paid in the auction.

a) Model this situation as a strategic form game.

	p_l	p_h
p_l	$v/2 - p_l,\ v/2 - p_l$	$-p_l,\ v - p_h$
p_h	$v - p_h,\ -p_l$	$v/2 - p_h,\ v/2 - p_h$

b) Find the range of values of v for which one action in the game is dominated.

c) For every value of v find the Nash equilibria of the game.

Hoping to generate higher bids and hence more revenue, the auctioneer decides to add a consolation prize worth $r < v$ to the auction. In this modified auction the highest bidder (or the winner of the coin flip in case of equal bids) receives the object worth v and the other bidder receives the object worth r.

d) For a fixed r and every value of v larger than r study what happens to the revenue generated by the auction after the consolation prize is added.

e) How would the equilibrium change if the two players choose their bids sequentially rather than simultaneously?

Question 3. (from Osborne 86.1) An action affects each of two people. The right to choose the action is sold in a second-price auction. That is, the two people simultaneously submit bids, and the one who submits the higher bid chooses her favorite action and pays (to a third party) the amount bid by the other person, who pays nothing. (Assume that if the bids are the same, person 1 is the winner.) For $i = 1, 2$, the payoff of person i when the action is a and person i pays m is $u_i(a) - m$. In the game that models this situation, find for each player a bid that weakly dominates all the player's other bids (and thus find a Nash equilibrium in which each player's equilibrium action weakly dominates all her other actions.)

Question 4. (from Osborne 88.1) Consider a *third*-price sealed bid auction, which differs from a first- and a second-price auction only in that the winner (the person who submits the highest bid) pays the third highest price. (Assume that there are at least three bidders.)

a) Show that for any player i the bid of v_i weakly dominates any lower bid but does not weakly dominate any higher bid. (To show the latter, for any bid $b_i > v_i$, find bids for the other players such that player i is better off bidding b_i than bidding v_i.

b) Show that the action profile in which each player bids her valuation is not a Nash equilibrium.

c) Find a Nash equilibrium. (There are ones in which every player submits the same bid.

Question 5. In many auctions, the auctioneer sets a reserve pricea price below which she will not sell the object. Consider a second-price sealed-bid auction with two bidders, each of whose valuations can take one of three values 0,1/2 and 1 with equal probability. Denote by r the reserve price. If both bids are greater than r, then the player who submits the higher bid wins and pays a price equal to the lower bid. If one bid is greater than r and the other is less than r, then the player who submits the bid greater than r wins and pays r. If both bids are less than r, then the object is not sold. Find the value of r that maximizes the expected revenue of the auctioneer.