First in Village or Second in Rome?

Ettore Damiano
University of Toronto
Li, Hao
University of Toronto
Wing Suen
The University of Hong Kong

March 7, 2008

Abstract: Though individuals prefer to join groups with high quality peers, there are also advantages from being high up in the pecking order within a group. We show that sorting of agents in this environment results in an overlapping interval structure in the type space. Segregation and mixing coexist in a stable equilibrium. When transfers are possible our stable equilibrium corresponds to a competitive equilibrium in which agents bid for relative positions. The equilibrium entails too little segregation compared to the efficient allocation. A greater degree of egalitarianism within organizations leads to greater segregation across organizations, but can potentially improve the efficiency of equilibrium allocation. Since competition is most intense for agents with intermediate talent, effective personnel policies to attract talent differ systematically between high-quality and low-quality organizations.

JEL Classification: C78, D83, M50

Corresponding Author: Ettore Damiano. Address: Department of Economics, University of Toronto, 150 St. George Street, Toronto, Ontario M5S3G7, Canada. Phone: (416) 946-5821. Fax: (416) 978-6713. E-mail: ettore.damiano@utoronto.ca.

Acknowledgments: We thank Luis Garicano, John Hartwick, Tony Latter, Rich Romano, Aloysius Siow, Zhigang Tao, and Bruce Weinberg for helpful comments, and Priscilla Man and Ron Chan for able research assistance. We have also benefited from many comments and suggestions by the Editor and two anonymous referees.

"I had rather be the first in this village than second in Rome."

—Attributed to Julius Caesar by Plutarch, *Life of Caesar*.

1. Introduction

Segregation and mixing typically coexist in the distribution of talents among organizations. For example, productive economists are disproportionately represented in top research departments but at the same time second-tier departments often have economists who are more productive than some economists in a first-tier institution. A plausible explanation is that economists who prefer to have great colleagues also recognize that the more productive researchers within individual departments have greater access to departmental resources. This paper builds on this idea and develops an equilibrium sorting model to explain the pattern of talent distribution across organizations. The premise of our model is that interactions among individuals in an organization typically involve both cooperation and competition. In making mutually exclusive choices of which jobs to take, which schools to attend, or which social clubs to join, economic agents are concerned with both the "peer effect" and the "pecking order effect."

The peer effect is well-documented in the education literature (e.g., Coleman et al., 1966; Summers and Wolfe, 1977; Sacerdote 2001). Lazear (2001) has a model of educational technology in which the peer effect arises because learning is reduced for all other students when one student disrupts the class. More generally, the peer effect is seen as a consequence of the complementarity between characteristics of agents in a match. Naturally, people desire to join organizations with high-quality members if being in the company of high-quality colleagues raises their own utility or productivity. But the motive to join the company of high-quality peers can also be present without complementarity. For example, potential employers' imperfect information about individual student quality leads to their use of the school average to improve on their estimate, as described in the statistical discrimination literature (e.g., Aigner and Cain, 1977). High-quality peers can therefore confer an informational externality on one another.

The pecking order effect is related to people's concern for their relative status. It can arise from the effects on self-esteem developed through interactions with other agents in an organization. The idea that an individual's utility depends not only on the level of his consumption, but also on how that level compares with that of his reference group, has a long history in economics (e.g., Duesenberry, 1949; Frank, 1985; Solnick and Hemenway, 1998). In educational psychology, researchers speak of a "big fish–small pond" effect (Marsh and Parker, 1984). A study of academically talented students in Israeli primary schools finds that those who participate in special homogeneous classes for the gifted have lower academic self-concept and greater test anxiety than do those who participate in regular mixed-ability classes (Zeidner and Schleyer, 1998).[1] The pecking order effect can also arise when some resources within a group or an organization are allocated through non-market means. Cole et al. (1992) describe a model in which the competition for mates leads to a concern for relative ranking. As Postlewaite (1998) points out, such kind of non-market allocation is ubiquitous. Resources within an organization such as corner offices or decision making power in recruitment are often allocated according to the relative status of the incumbents. Furthermore, people higher up in the pecking order have a greater chance of success in internal promotion tournaments. In school choices, the pecking order effect is present because grades are relative and depend on underlying ability of fellow students. It is more important when grades have more serious implications, as demonstrated by the so-called top-ten-percent law in the state of Texas, which guarantees that students who finish in the top ten percent of their graduating class earn automatic admission to the Texas public university of their choice.[2]

Even men as great as Julius Caesar at some point might not be able to have it both ways. The rest of us are constantly reminded by the trade-off between the peer effect and the pecking order effect. Because the groups or organizations we are trying to join have limited capacities, individuals who are most desirable are expected to make any choice as

[1] In a longitudinal study of secondary school students in Hong Kong, a city which has a highly achievement-segregated school system, Marsh et al. (2000) find that school-average ability has a negative effect on a student's academic self-concept, and that lower academic self-concept in turn adversely affects the student's subsequent academic achievement as reflected in standardized test scores. These researchers also find that higher perceived school status has a counterbalancing positive effect on self-concept. This is consistent with the informational externality we discuss in connection with the peer effect.

[2] For empirical evidence that the adoption of the top-ten-percent law in Texas influenced high school students' enrollment decisions, see Cullen, Long and Reback (2005). The relation of their empirical findings to the theoretical results of the present paper is discussed in Section 5.1.

they wish, while those who are least desirable have no real choice. It may thus appear that the sorting choices can be determined sequentially, with the more desirable agents choosing before the less desirable ones. But generally those ranked at the bottom of the first-tier organization will want to switch to the top of the second-tier organization. Sorting choices cannot be determined independently because for each agent both the peer effect and the pecking order effect in an organization depend on the choices made by other agents.

This paper presents a model where agents with heterogeneous, one-dimensional type face the trade-off between the peer effect and the pecking order effect in choosing between two organizations of fixed sizes. We model the peer effect by assuming that individual agents care about the average type in the organization they join, and we model the pecking order effect through the concern for their ranking according to type within the organization. While highly stylized, the model is meant to capture the essential features of sorting of talents in the presence of the peer and pecking order effects. We define a "sorting equilibrium" as an allocation of types among the organizations such that no agent whose type is higher than the lowest type in the other organization wishes to move. Sorting of agents is shown to result in an overlapping interval structure in the type space: the set of types that end up in each organization is an interval, and both the highest type and the lowest type are higher in the organization with a higher average type. Thus, in equilibrium segregation occurs for the top and the bottom types while the intermediate types mix between the two organizations. Further, the equilibrium overlapping-interval pattern of segregation and mixing is locally stable.

In a transferable utility model, we interpret a rank in an organization as a relative position and assume that complementarities exist between the agent type and the productivity of the relative position he holds in the organization. When agents bid competitively for different relative positions in the organizations, the complementarities allow higher type agents to outbid lower types for higher-ranked positions. The resulting competitive equilibrium yields the same overlapping interval allocation of types in the two organizations and a positive assortative matching between types and relative positions within each organization, as the stable sorting equilibrium without transfers. The transferable utility model thus provides a foundation for the notion of sorting equilibrium without transfers, and a natural framework to evaluate the welfare properties of the sorting equilibrium. The equilibrium allocation is inefficient; it does not maximize the aggregate utility because agents fail to internalize the peer effect on other agents when moving across organizations to take advantage of the pecking order effect. The result is an equilibrium allocation that exhibits too little segregation. Under a linearity assumption on the form of complementarities between the agent type and the concerns for relative position and average ability, we show that maximizing aggregate utility requires sorting types into overlapping intervals as in the sorting equilibrium or the competitive equilibrium, but with less overlapping.

In our model the degree of segregation, measured in terms of differences in average ability across organizations, increases as the concern for relative ranking becomes less important.[3] In the context of school choice, this result suggests that a greater degree of egalitarianism within schools can lead to greater segregation by ability across schools. Similarly, for fixed salary structures in a sports league, a lower team salary cap reduces the relative importance of the pecking order effect, and may result in a greater team quality difference, which is counter-productive in restoring the competitive balance in the league. Further, since the sorting equilibrium exhibits too little segregation compared to the efficient allocation, egalitarian policies may increase the aggregate welfare. Our model also sheds light on how organizational policies affect the sorting of talents. In a sorting equilibrium with an overlapping interval structure, competition is most intense for agents with intermediate ability, because they are mobile across organizations. This suggests that a high-quality organization benefits from policies that cater more to its low-status members than to its high-status members, while the low-quality organization benefits from policies that cater more to its high-status members.

In the remainder of this section we review the existing literature that is most germane to the present study. In Section 2, we present the setup where agents with identical trade-off between average ability and relative ranking sort into two ex ante identical, equal-sized organizations. Section 3 deals with the case of no transfers. We show that all equilibria take

[3] While it is known that the peer effect alone leads to complete segregation of agents by type (Becker, 1973; Benabou, 1993; Kremer, 1993), in our model complete segregation occurs in equilibrium if the pecking order effect is sufficiently weak, not necessarily absent.

the form of overlapping intervals. We define stable equilibrium and show that a unique stable equilibrium outcome exists for any trade-off between average ability and relative ranking. In Section 4 we introduce transfers and demonstrate that the sorting equilibrium we have identified can be implemented as a competitive equilibrium. Welfare analysis of the equilibrium shows that there is too little segregation in equilibrium compared to the efficient allocation. In Section 5, we offer comparative statics results regarding system-wide and organization-specific factors that affect agents' concern for relative ranking. We also examine the effect on equilibrium sorting of uniform and idiosyncratic preference biases for organizations due to attributes independent of the trade-off between average ability and relative ranking. Section 6 concludes with a brief summary and a short discussion of some limitations of the present paper.

1.1. Related literature

There is a rich literature on the impact of peer effect in education. Streaming students by their ability has long been a controversial issue in education policy (e.g., Slavin 1987; Gamoran et al., 1995). One controversy involves the conflict between efficiency and egalitarianism. Education researchers typically attribute the efficiency of ability streaming to the fact that teachers can tailor their instruction to a more homogeneous group of students (Lou et al., 2000). Economic theory suggests that positive assortative matching maximizes the sum of outputs if the production function is supermodular (Becker 1973). Lazear (1991) provides a micro-foundation for the existence of the peer effect in the classroom, and discusses the implications of the peer effect for streaming, class size, and other issues. Arnott and Rowse (1987) use a reduced-form education production function with peer effects to derive the optimal allocation of students and educational expenditure across classes. In the latter model, the extent of segregation by ability is limited by diminishing returns to educational expenditure.

Peer effect also features extensively in the literature on locational choices. De Bartolome (1990) develops a community choice model where families care about the provision of a public service (schooling) in the community. Families have high or low ability children. High ability children benefit more from expenditures on education (input effect) and all children benefit from having more high ability children in the school (peer effect).

Segregation occurs with only input effect or peer effect, and when both effects are present mixed schools may obtain in equilibrium. The focus of that paper is on efficiency of school financing. Fernandez and Rogerson (1996) study a similar model but without the peer effect, and focus on welfare improving policies. They find that policies that increase the fraction of relatively wealthy individuals in the poorest neighborhood make everybody better off. Becker and Murphy (2000, chap. 5) discuss the implications of the peer effect for residential segregation. They use a two-type model to illustrate how multiple equilibria, tipping, and inefficiency may arise in a competitive land market when people prefer to have "good" neighbors.

In comparison, pecking order effect has received less attention. Kremer and Maskin (1996) study a one-sided matching model with a production function in which the more able worker within a two-person firm gets to perform the more productive task.[4] Garicano and Rossi-Hansberg (2004) study how specialization and communication affect the equilibrium allocation of knowledge workers. By making the allocation of job positions endogenous, these two models capture some aspect of the pecking order effect. These models are particularly useful for analyzing earnings inequality, but they do not yield easily interpretable results concerning the degree of segregation by ability across organizations. In our paper, we use a reduced-form approach to study the pecking order effect, without committing to any specific channel in which relative ranks matter—be it due to social psychological factors such as self-esteem, to non-market means of distributing resources within organizations, or to particular forms of production technologies. This approach allows us to develop a flexible model that yields unambiguous results concerning the systematic differences between high-quality and low-quality organizations.

The result of coexistence of segregation and mixing in the sorting of ability in the present paper is derived in a many-to-one matching framework, where ability is one-dimensional but enters the utility function of an agent through both the peer effect and

[4] Legros and Newman (2002) consider more general models and derive conditions for pairwise matching to be positive assortative. Hartwick and Kanemoto (1984) analyze the formation of a hierarchy of groups where each agent's utility from joining a group depends on a variable associated with the group and a value associated with the agent's rank within the group. There is no mixing in their equilibrium because the variable associated with the group depends only on a boundary type.

the pecking order effect. Legros and Newman (2007) consider a pairwise matching model with limited transferability, and derive necessary and sufficient conditions for positive assortative matching of types. The many-to-one framework, in which each organization is matched with a continuum of agents, distinguishes our paper from theirs. In our model, with or without transfers, positive assortative matching between agent type and relative ranking obtains within each organization, but it does not obtain across organizations in the sense that there is mixing or overlapping of types instead of a complete segregation. In another many-to-one framework, in which many students enroll in each school, Epple and Romano (1998) derive a competitive equilibrium with some overlapping of student types across schools.[5] Unlike our paper, they assume that students differ by both their family income and academic aptitude, so that the overlapping occurs in the sense that the best, but poor, students in a school with a lower average student aptitude have higher aptitude than the worst, but rich, students in a school with a higher average. In contrast, type is one-dimensional in our model and mixing occurs in a stronger form, in that a proportion of the same type agents join different organizations in trading off the peer effect against the pecking order effect.

2. The Setup

There is a mass of 2 of agents to be allocated into two organizations, A and B. For simplicity, suppose that the organizations have the same capacity of 1. Agents are characterized by their one-dimensional type θ, also referred to as ability. The distribution of θ is given by the distribution function F on the support $[\underline{\theta}, \overline{\theta}]$, with the corresponding density function f. We assume that f is continuous.

Preferences of agents over the two organizations depend on the average ability of agents in that organization, and on their individual ranking of ability within that organization. Although peer effects can work through many other channels—for example, agents may prefer organizations with a less dispersed type distribution or those with a high average

[5] Epple and Romano (1998) assume that the type of agents is observable. Damiano and Li (forthcoming) consider a random pairwise matching model with one-dimensional types and characterize price competition when type information is private.

in the first quartile due to the benefits of role models—in this paper we focus only on the simplest formulation of the peer effect through the average type of the organization. At the same time, we represent the concern for the pecking order effect by assuming that preferences of the agents are continuous in their ranking within an organization: this is justified even if there is a discontinuous payoff from being at the top of an organization, because any uncertainty regarding who will eventually end up at the top would render the preferences continuous in the ex ante ranking of ability. Formally, for each $i = A, B$, let m_i be the average ability of agents in organization i, also referred to as the quality of i, and let $r_i(\theta)$ be the quantile rank of an agent of type θ in i. We assume that the utility from joining organization i is given by

$$V_i(\theta) = \alpha r_i(\theta) + m_i, \tag{1}$$

where α is a positive weight that agents put on ranking relative to quality in the organization. The payoff is zero if an agent does not join either organization.

Equation (1) embeds three assumptions about the individual utility function. First, the concern for ranking and the concern for organization quality are additively separable; second, the marginal rate of substitution between relative rank and average ability is type-independent; and third, the marginal rate of substitution is constant. These assumptions allow us to characterize equilibrium pattern of sorting in a convenient way. We will comment on the role played by these assumptions in the analysis below.

Equation (1) is the simplest functional form that satisfies the property that all agents face identical, constant trade-off between relative ranking and average ability. A more general form that retains this property allows interactions between types and the concerns for ranking and average ability, so that the utility function takes the multiplicatively separable form,

$$V_i(\theta) = l(\theta)\left(\alpha r_i + m_i\right), \tag{2}$$

for some positive-valued, strictly increasing, and differentiable function l. When there are no side transfers, equilibrium sorting is identical under equation (1) or (2); without loss of generality we carry out the analysis under equation (1). When utility is transferable, the form of l matters. By assuming that l is an increasing function, we are positing in (2) that there are complementarities between agent type and relative ranking, and between type and average ability. In Section 4 we provide a more detailed discussion of equation (2) and use it to characterize competitive and efficient allocations.

3. Sorting without Transfers

When transfers are not allowed, we can focus on the interaction in the individual utility function between concerns for relative ranking and concerns for peer quality. Most of our results in this section apply to a more general case than equation (1) in which relative rank and average ability are not perfect substitutes. This case is represented by

$$V_i(\theta) = v(r_i(\theta), m_i), \tag{3}$$

where v is increasing in both arguments and differentiable.

3.1. Overlapping intervals

A *feasible allocation* in this model is a pair (H_A, H_B) of cumulative type distributions in organizations A and B such that $H_A(\theta) + H_B(\theta) = F(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. For each $i, j = A, B$, $i \neq j$, when $H_i(\theta)$ is constant in a neighborhood of some type θ, types around θ are all allocated to organization j only. If both H_A and H_B are strictly increasing in a neighborhood of θ, then types around θ are split between A and B. Let T_i be the *support set* of organization i, or the set of types that do not exclusively choose organization j, defined as the closure of the set of types at which H_i is strictly increasing.

We assume that an agent can join organization i, either when the capacity of the organization is not yet filled, or when the capacity is filled but the agent's type is higher than the lowest type of that organization. Given this assumption, a natural equilibrium concept requires that at an equilibrium allocation no agent with the option of changing organization should strictly prefer to do so. Since the payoff from not joining an organization is zero, the capacity of each organization will always be filled. For the following definition, note that given a feasible allocation (H_A, H_B), the rank $r_i(\theta)$ of type θ in each organization i is well-defined, and is equal to $H_i(\theta)$.

Definition 1. *A sorting equilibrium is a feasible allocation* (H_A, H_B) *such that for each* $i, j = A, B$, $i \neq j$, *if* $\theta \in T_i$ *and* $\theta > \inf T_j$, *then* $V_i(\theta) \geq V_j(\theta)$.

Our equilibrium concept is a natural notion of pairwise stability in a two-sided, many-to-one matching model with a continuum of agents on one side and two organizations on the other side. The preference of the agents is defined by equation (1). Each organization strictly prefers to add more agents of any type so long as the capacity constraint is not met, and for sets of agents that meet the capacity constraints, it prefers one with a higher average type to one with a lower average.[6] Alternatively, Definition 1 corresponds to a subgame perfect equilibrium outcome of the following extensive form game between the agents and the two organizations. In the first stage of the game, all agents simultaneously choose to apply to either A or B or both and pay a small cost per application; in the second stage, after observing the pool of applicants each organization admits a subset of the pool of size at most equal to its capacity. In this game, the payoff of a type θ agent is given by equation (1) if admitted by organization i, and is zero if not admitted by either organization, minus any application cost incurred. The payoff to each organization is the average type of the admitted agents. It is straightforward to see that in any subgame perfect equilibrium outcome all agents apply to and are admitted by one of the two organizations and each organization has a size of 1. The condition in Definition 1 then corresponds to the no deviation condition for type θ agent in a subgame perfect equilibrium.

Definition 1 is a simplification of reality. If agents have multi-dimensional attributes, it is not straightforward to define higher types and lower types. For example, an organization may hire workers who are less directly productive but who provide role models to improve its human capital stock (Athey et al., 2000). Even when agents can be unambiguously ranked on a single ability dimension, it is not always the case that incumbents in an organization necessarily welcome newcomers who have high ability. Our model is more

[6] For an excellent analysis of two-sided matching models with finite agents, see Roth and Sotomayor (1990). Due to the payoff externalities introduced by the peer effect and the pecking order effect, the results surveyed by Roth and Sotomayor do not apply to our model. Further, in the model with finite agents, the payoff externalities cause difficulties in interpreting the notion of pairwise stability as an equilibrium concept. Such difficulties disappear in our model with a continuum of agents, because a deviation by a pair of an agent and an organization has a negligible impact on the payoffs of other agents.

likely to apply when entry into an organization is controlled by an outside decision maker who is primarily interested in maximizing the average ability of the organization. In Section 4 we allow side transfers and show that any equilibrium allocation according to Definition 1 can be supported as a competitive equilibrium where agents bid for positions of different ranking in organizations. Now we introduce a particularly simple form of allocation.

DEFINITION 2. *A feasible allocation is of an overlapping interval form if there exist types x, y, with $x \geq y$, such that the two support sets are $[y, \overline{\theta}]$ and $[\underline{\theta}, x]$.*

In an overlapping interval form, the support set T_i of each organization i is an interval. There are two critical types x and y, with $x \geq y$. All agents with type greater than x go to one organization, say A. All agents with type lower than y go to the other organization. Agents with type in the range $[y, x]$ are present in both organizations. The following lemma shows that all sorting equilibrium allocations take the form of overlapping intervals.

LEMMA 1. *Any sorting equilibrium allocation takes the overlapping interval form.*

PROOF. First we show that the support sets T_A and T_B are ordered in any sorting equilibrium: if $m_A \geq m_B$, then $\sup T_A \geq \sup T_B$ and $\inf T_A \geq \inf T_B$. Suppose $\sup T_A < \sup T_B$. Then there is a type $\theta \in T_B$ such that $r_B(\theta) < 1$ and $r_A(\theta) = 1$. Since $m_A \geq m_B$, agents of type θ strictly prefer A. This is a contradiction since type θ has the option of moving to A. A similar argument establishes that $\inf T_A \geq \inf T_B$.

Next we show that in any sorting equilibrium the support set T_i of each organization i, $i = A, B$, is an interval in the type space. Suppose not, and, without loss of generality, suppose that T_A is not an interval. Then there exist two types θ and $\tilde{\theta}$, with $\theta < \tilde{\theta}$, such that all types on the interval $(\theta, \tilde{\theta})$ choose organization B exclusively while types in small neighborhoods below θ and above $\tilde{\theta}$ are in the support set of organization A. Then, $H_A(\theta) = H_A(\tilde{\theta})$ and $H_B(\theta) < H_B(\tilde{\theta})$. Since types θ and $\tilde{\theta}$ have the option of switching between A and B, both types must be indifferent between the two organizations, or

$$\alpha r_A(\theta) + m_A = \alpha r_B(\theta) + m_B;$$

$$\alpha r_A(\tilde{\theta}) + m_A = \alpha r_B(\tilde{\theta}) + m_B.$$

The above two contradict each other (recall that $r_i(t) = H_i(t)$ in each i.) *Q.E.D.*

Lemma 1 contains two results: the support sets of the two organizations are ordered, and there are no gaps in the two support sets. The first result depends only on the assumption that the individual utility function $V_i(\theta)$ increases with $r_i(\theta)$ and m_i. The second result of no gaps in the support sets depends on the assumption that the marginal rate of substitution between relative ranking and average ability is type-independent in the individual utility function. However, even if relative ranking and average ability are imperfect substitutes as in equation (3), the contradiction argument for the no-gap result in the proof of Lemma 1 goes through. Thus, neither additive separability of the pecking order effect and the peer effect nor the constancy of the marginal rate of substitution is necessary for the result of overlapping intervals.[7]

While it has been recognized in the literature that positive complementarities lead to assortative matching and negative complementarities tend to produce mixing, there are few general results in the literature concerning the equilibrium allocation when both elements are present. In principle, "mixing" can take a variety of forms and this can render the analysis unwieldy.[8] In our model, support sets contain no holes, and mixing takes the special form of overlapping intervals, drastically simplifying the subsequent analysis.

3.2. Stable sorting equilibrium

Fix an overlapping interval allocation. Without loss of generality, we assume that $T_A = [y, \overline{\theta}]$ and $T_B = [\underline{\theta}, x]$. The restrictions on H_A and H_B imposed by the overlapping interval form are: (i) $H_A(\theta) = 0$ and $H_B(\theta) = F(\theta)$ for $\theta \in [\underline{\theta}, y]$; (ii) $H_A(\theta) = F(\theta) - 1$ and $H_B(\theta) = 1$ for $\theta \in [x, \overline{\theta}]$; and (iii) $H_A(\theta)$ and $H_B(\theta)$ are strictly increasing for $\theta \in (y, x)$. See

[7] When the marginal rate of substitution depends on the type, there may be gaps in the support sets. For example, if the parameter α in equation (1) is replaced by a decreasing function of θ, so that higher types are less concerned with the pecking order effect than lower types are, then the support set of the higher quality organization may have gaps (the other support set continues to be an interval).

[8] In Kremer and Maskin (1996), workers with one-dimensional, heterogeneous types form pairwise matches to produce. There are two tasks, "manager" and "assistant," in each matched pair, and the output is $\theta^2\tilde{\theta}$, where θ is the manager's type and $\tilde{\theta}$ is the assistant's type. When types are distributed uniformly over a relatively narrow range, total output is maximized by dividing the mass of workers into two equal-size overlapping intervals of types that are managers and of types that are assistants, and matching them positive assortatively. However, when the type distribution has a sufficiently large support, the set of types that are managers and the set of types that are assistants are no longer intervals. This feature makes the analysis of the extent of segregation in Kremer and Maskin (1996) quite intractable.

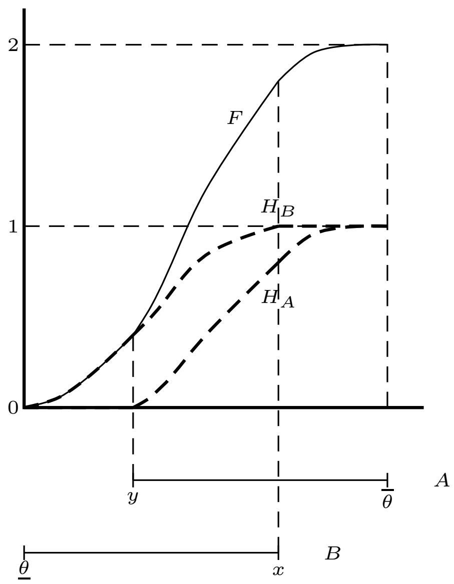

Figure 1

Figure 1 for an illustration. Note that feasibility of the allocation implies that $x \geq \theta_e \geq y$, where θ_e denotes the median type in the population. There are two extreme cases of overlapping interval allocations: if $x = y = \theta_e$, we have a perfectly segregated allocation; if $x = \overline{\theta}$ and $y = \underline{\theta}$, and $H_A(\theta) = H_B(\theta) = \frac{1}{2}F(\theta)$, we have a perfectly mixed allocation.

If $x > y$ in an overlapping interval allocation, all agents of type $\theta \in [y, x]$ must be indifferent between organization A and organization B. Recall that $r_i(\theta) = H_i(\theta)$ for each $i = A, B$. The indifference conditions for the threshold types x and y are

$$\begin{aligned} \alpha(2 - F(x)) &= m_A - m_B; \\ \alpha F(y) &= m_A - m_B. \end{aligned} \tag{4}$$

These two indifference conditions imply

$$2 - F(x) = F(y). \tag{5}$$

The above equation defines a one-to-one relation between y and x, with $x = \overline{\theta}$ implying $y = \underline{\theta}$, and $x = \theta_e$ implying $y = \theta_e$. For all θ between y and x, a similar indifference condition holds:

$$\alpha(H_B(\theta) - H_A(\theta)) = m_A - m_B. \tag{6}$$

Thus, the difference in ranking for any type θ between A and B is constant on the interval $[y, x]$. This implies that types between y and x are evenly split between A and B. Thus,

$$H_A(\theta) = \begin{cases} F(\theta) - 1, & \text{if } x < \theta \leq \overline{\theta} \\ \frac{1}{2}\left(F(\theta) - F(y)\right), & \text{if } y < \theta \leq x \\ 0, & \text{if } \theta \leq y \end{cases} \tag{7}$$

and $H_B(\theta) = F(\theta) - H_A(\theta)$.

With the above results, an overlapping interval allocation can be characterized by a single variable. It is more convenient to define an equilibrium in terms of the difference in average types, $m_A - m_B$. Denote $z = m_A - m_B$. To be consistent with our assumption that $T_A = [y, \overline{\theta}]$ and $T_B = [\underline{\theta}, x]$, we have $z \geq 0$. The largest possible value of z is $\overline{\mu}_e - \underline{\mu}_e$, where $\overline{\mu}_e$ and $\underline{\mu}_e$ are the conditional mean above and below θ_e, respectively. Given any $z \in [0, \overline{\mu}_e - \underline{\mu}_e]$, we use equations (4) to define the threshold types x and y. Note that since $x \geq \theta_e \geq y$, equations (4) cannot be satisfied if $z > \alpha$ (this occurs only if $\alpha < \overline{\mu}_e - \underline{\mu}_e$); in this case we let $x = y = \theta_e$, as all types prefer A to B but only types above θ_e have the option of moving to A. Then, we can obtain m_A as a function of z:

$$m_A(z) = \int_{\underline{\theta}}^{\overline{\theta}} t \, \mathrm{d}H_A(t),$$

where $H_A(t)$ is given by equation (7). From the identity $m_A + m_B = 2\mu$, where μ is the unconditional mean of the type distribution F, we get a necessary and sufficient condition for a sorting equilibrium with quality difference z:

$$D(z) \equiv 2(m_A(z) - \mu) = z. \tag{8}$$

If there exists $z \in (0, \alpha)$ such that (8) is satisfied, then we have a sorting equilibrium with partial overlapping, where the threshold types x and y satisfy $\underline{\theta} < y < \theta_e < x < \overline{\theta}_e$ by equations (4). If equation (8) is satisfied by $z = \overline{\mu}_e - \underline{\mu}_e$, then we have a perfectly segregated equilibrium allocation, with $x = y = \theta_e$. Finally, since $z = 0$ implies $x = \overline{\theta}$ and $y = \underline{\theta}$ by equations (4), and hence equation (8) is satisfied by $z = 0$, a perfectly mixed allocation is always an equilibrium.

The equilibrium condition $D(z) = z$ may admit more than one solution in z in the range $[0, \overline{\mu}_e - \underline{\mu}_e]$. However, some of these solutions may be "unstable" with respect to small perturbations in the equilibrium allocation. Following the standard stability concept, we say that a sorting equilibrium z is *stable* if $D'(z) < 1$.[9] We have the following characterization result for the basic model.

PROPOSITION 1. *There exists a unique stable sorting equilibrium outcome for any α, and it is (a) perfect segregation if $\alpha \leq \overline{\mu}_e - \underline{\mu}_e$, (b) partial overlapping if $\overline{\mu}_e - \underline{\mu}_e < \alpha < \overline{\theta} - \underline{\theta}$, and (c) perfect mixing if $\alpha \geq \overline{\theta} - \underline{\theta}$.*

PROOF. We already know that $D(0) = 0$ for any α. Further, using equations (4) and (8), we can show that the derivative of $D(z)$ is given by

$$D'(z) = \frac{1}{\alpha}(x - y).$$

From equations (4), we know that x decreases and y increases with z, and thus $D(z)$ is a concave function. There are three cases; see Figure 2 for an illustration. (a) If $\alpha \leq \overline{\mu}_e - \underline{\mu}_e$, then $D'(0) > 1$ and $x = y = \theta_e$ for all $z \in [\alpha, \overline{\mu}_e - \underline{\mu}_e]$. We have $D'(\overline{\mu}_e - \underline{\mu}_e) = 0$, and $z = \overline{\mu}_e - \underline{\mu}_e$ is the only stable solution. (b) If $\overline{\mu}_e - \underline{\mu}_e < \alpha < \overline{\theta} - \underline{\theta}$, then $D'(0) > 1$, and $D(\overline{\mu}_e - \underline{\mu}_e) < \overline{\mu}_e - \underline{\mu}_e$ because $x > \theta_e > y$ when $z = \overline{\mu}_e - \underline{\mu}_e$ from equation (4). There is a unique interior stable solution to $D(z) = z$. (c) If $\alpha \geq \overline{\theta} - \underline{\theta}$, then $D'(0) \leq 1$ and therefore $D(z) < z$ for all $z > 0$. The only solution to $D(z) = z$ is $z = 0$, which is stable. *Q.E.D.*

The above proposition shows that segregation and mixing generally coexist in equilibrium allocations of types across organizations. Since there is no inherent difference between organization A and organization B, for any equilibrium in which $m_A > m_B$, there is another equilibrium involving the same allocation but with the identities of A and B reversed. Nevertheless, each equilibrium is locally stable in the sense that small disturbances to the allocation do not spread.

Proposition 1 establishes the existence of a unique stable sorting equilibrium in our model for any utility parameter α and type distribution F. The uniqueness part of the result is due to the assumption of linear individual utility function (equation 1), but the

[9] We adopt the convention that $D'(0)$ and $D'(\overline{\mu}_e - \underline{\mu}_e)$ are defined by continuity, from above and from below respectively. Also, we say that $z = 0$ is stable if $D'(0) = 1$ and $D(z) < z$ for z just above 0.

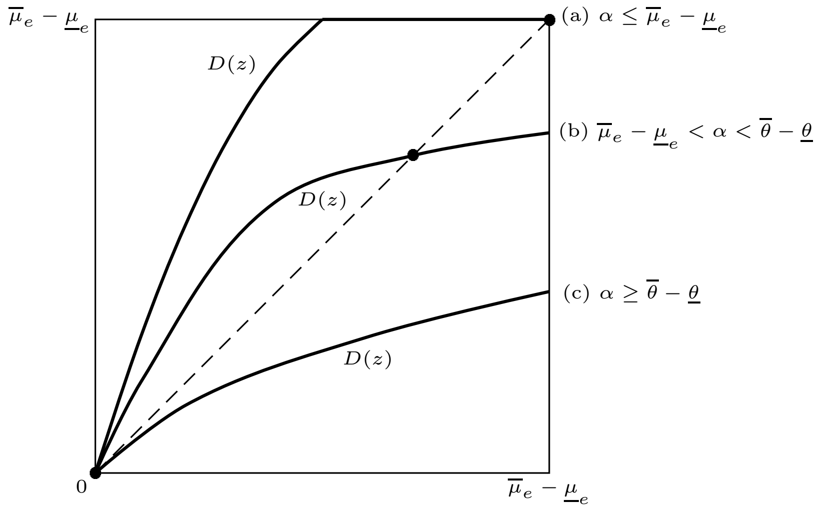

Figure 2

existence part can be easily extended to the case when relative ranking and average ability are not perfect substitutes. As we have remarked about Lemma 1, when the individual utility function is instead given by equation (3), the support sets remain overlapping intervals. In general there will not be an even split of types that are allocated to both organizations, but we can follow the same procedure as above to construct a stable sorting equilibrium. For any z in the permissible range of $[0, \overline{\mu}_e - \underline{\mu}_e]$, we can determine the threshold types x and y by $v(F(x) - 1, m_A) = v(1, m_B)$ and $v(0, m_A) = v(F(y), m_B)$ respectively, while indifference conditions $v(H_A(\theta), m_A) = v(H_B(\theta), m_B)$ define the allocation (H_A, H_B) between y and x. A sorting equilibrium is then defined as before, as a fixed-point of equation (8). A stable equilibrium always exists: either $D'(0) \leq 1$, in which case perfect mixing ($z = 0$) is a stable sorting equilibrium (because $D(0) = 0$); or $D'(0) > 1$, in which case an interior stable equilibrium exists if $D(\overline{\mu}_e - \underline{\mu}_e) < \overline{\mu}_e - \underline{\mu}_e$; or otherwise perfect segregation ($z = \overline{\mu}_e - \underline{\mu}_e$) is a stable equilibrium.

Since an agent of a higher type has more choices of organization than an agent of a lower type, equilibrium utility increases with type. In a perfectly mixed equilibrium allocation, the two organizations are identical and equilibrium utility increases at the rate of $\alpha f(\theta)$. In a perfectly segregated equilibrium, equilibrium utility increases twice as fast, at the rate of $2\alpha f(\theta)$ in both organizations, with a discontinuity at $\theta = \theta_e$ since the median type strictly prefers A to B. In a partial-overlapping allocation, equilibrium utility is continuous at the cutoffs y and x, and increases at the rate of $2\alpha f(\theta)$ for the segregating types in $[\underline{\theta}, y)$ and $(x, \overline{\theta}]$, and at the rate of $\alpha f(\theta)$ for the mixing types in $[y, x]$. Therefore, for any fixed type distribution, equilibrium utility is more sensitive to agent type for segregated types. Inequality among agent types is more pronounced in a more segregated outcome.

4. Sorting with Transfers

The utility function $V_i(\theta) = \alpha r_i + m_i$ is clearly inadequate for welfare analysis, because it would imply that aggregate utility is the same for any allocation. We therefore resort to the more general formulation of equation (2).

Equation (2) may be justified by a model where in each firm workers need to be matched with tasks, and complementarity exists between the skill of a worker and the productivity of a task. Imagine that two firms, A and B, need to hire a unit mass of workers each. In each firm, there is a continuum of tasks and each employed worker must be matched with a task. Tasks differ by the level of capital investment k_i in each firm i, $i = A, B$. The productivity of task k_i is $\alpha k_i + m_i$, where m_i is the average skill level of workers in i. The output of a worker with skill level θ matched with task k_i is given by

$$l(\theta)(\alpha k_i + m_i). \tag{9}$$

When the level of capital investment k_i is uniformly distributed, we can assume without loss of generality that $k_i \in [0, 1]$, and identify a task with capital investment k_i as a position of rank k_i. Then this production function (9) provides a rationale for the individual utility function in equation (2). We assume that the preference function of each agent is quasi-linear, given by the difference between the agent's utility (equation 2) and the agent's payment. As remarked earlier, unlike in previous analysis, r_i in equation (2) is now a choice variable that ranges from 0 to 1. To avoid confusion, we refer to r_i as the level of a position, and use $H_i(\theta)$ as the quantile ranking of type θ in organization i. We call a feasible assignment of agents between the two organizations and a pairwise matching of agents with positions in each organization simply as an allocation.

4.1. Competitive sorting

We define a competitive equilibrium as an allocation together with two price schedules for positions, such that each agent chooses an organization and a position in the organization to maximize his utility, and markets clear with each agent getting exactly one position. Formally, let $p_i(r_i)$ be the price of position level r_i in organization i, $i = A, B$. We have the following definition.

Definition 3. *A competitive equilibrium is a feasible allocation* (H_A, H_B) *and price schedules* p_A *and* p_B*, such that (a) if type* θ *is in the support set of* H_i*,* $i = A, B$*, then* i *and* $H_i(\theta)$ *solves* $\max_{j=A,B}\left\{\max_{r_j} l(\theta)(\alpha r_j + m_j) - p_j(r_j))\right\}$*; and (b)* $m_i = \int_{\underline{\theta}}^{\overline{\theta}} t \; dH_i(t)$*.*

The above definition implicitly assumes that in any competitive equilibrium higher types are matched to higher positions in each organization. As a result, the market clearing condition is automatically satisfied. Note that since $l(\theta)$ is strictly increasing, the positive assortative matching of types to positions is implied by individual optimization and market clearing, because for any two positions $r_i > \tilde{r}_i$ in organization i, if a type weakly prefers r_i to $\tilde{r}_i$ then any higher type strictly prefers r_i to $\tilde{r}_i$.

We now argue that any sorting equilibrium allocation can be supported as a competitive equilibrium. Suppose that (H_A, H_B) is a sorting equilibrium, with threshold types x and y. Without loss of generality, we assume that $m_A \geq m_B$. Then, the equilibrium ranking $H_A(\theta)$ of any type θ in A satisfies equation (7), with the equilibrium ranking of θ in B given by $H_B(\theta) = F(\theta) - H_A(\theta)$. The following proposition constructs price schedules and verifies that Definition 3 is satisfied. The reverse is also true: only sorting equilibrium allocations can be supported as competitive equilibrium. This is established by first showing that in any competitive equilibrium the two support sets T_A and T_B are intervals. Then, using the condition that types allocated to both organizations are indifferent, we show that these types must be evenly split between A and B. Since any competitive equilibrium must satisfy the condition that the average type in each organization taken as given by each agent is precisely the average type that results from the organization choices of all agents (condition (b) in Definition 3), the support sets in the competitive equilibrium must be overlapping intervals and the corresponding allocation is a sorting equilibrium allocation. The proof of Proposition 2 is in the appendix.[10]

PROPOSITION 2. *A feasible allocation can be supported as a competitive equilibrium if and only if it is a sorting equilibrium.*

The correspondence between sorting equilibrium without transfers and competitive equilibrium with transfers can be better understood if we think of the relative positions in each organization as different match types. Without transfers, positive assortative matching emerges in a sorting equilibrium because higher type agents have the priority for higher relative positions, or higher ranks as referred to in our analysis of sorting without transfers. With transfers, the complementarity between the agent type and the relative position as captured in equation (9) allows higher type agents to outbid lower types for high relative positions, resulting in the same positive assortative matching.[11] Competitive equilibrium thus provides a foundation for our intuitive concept of sorting equilibrium in the absence of transfers.

4.2. Efficient sorting

To understand the welfare properties of the equilibrium, let us consider a planner's problem of choosing an assignment of types to organizations and positions to maximize aggregate utility. We refer to the solution to the planner's problem as the *efficient allocation*. Since l is an increasing function, given any allocation, the total utility of each organization is maximized by matching positions and agent types positive assortatively, as the average type m_i is fixed, and complementarity exists between the type of agent and the level of position. Thus, for any feasible allocation (H_A, H_B), type θ will be matched to position

[10] Proposition 2 holds so long as the marginal rate of substitution is type-independent, as in the case where the individual production function is given by $l(\theta)v(r_i, m_i)$.

[11] In our model there is also the issue of sorting agents into two organizations, besides the problem of allocating agents to different relative positions within an organization. Proposition 2 establishes the equivalence between sorting equilibrium without transfers and competitive equilibrium with transfers in this two-stage matching problem. As we will show in Proposition 3 below, equilibrium is inefficient due to the two-stage nature of our model.

$r_A = H_A(\theta)$ in organization A and the maximal total utility in A is

$$Q(H_A) = \alpha \int_{\underline{\theta}}^{\overline{\theta}} l(\theta) H_A(\theta) \, \mathrm{d}H_A(\theta) + \left(\int_{\underline{\theta}}^{\overline{\theta}} l(\theta) \, \mathrm{d}H_A(\theta) \right) \left(\int_{\underline{\theta}}^{\overline{\theta}} \theta \, \mathrm{d}H_A(\theta) \right). \qquad (10)$$

Aggregate utility in the two organizations is $Q(H_A) + Q(H_B)$, where $H_B = F - H_A$.

The efficient allocation can be easily characterized for extreme values of α. In particular, perfect segregation is efficient when α is sufficiently small, while perfect mixing is efficient for sufficiently large α.[12] For intermediate values of α a characterization of the efficient allocation is difficult without any restriction on l. In the remainder of this subsection we assume that l is linear, and equal to θ without loss of generality. In the appendix we prove the following necessary condition for efficient allocations.[13]

Lemma 2. *Assume that l is linear. Any efficient allocation takes the overlapping interval form with an even split of types allocated to both organizations.*

In the proof of Lemma 2, we use a local variation argument to show that mixing of types in the neighborhood of type θ implies that[14]

$$\alpha(H_A(\theta) - H_B(\theta)) = 2(m_B - m_A). \qquad (11)$$

Compare the above to equation (6). The next proposition follows immediately.

Proposition 3. *Assume that l is linear. There exists a unique efficient allocation for each α, and it is identical to the sorting equilibrium corresponding to $\alpha/2$.*

---

[12] To see this, when α is sufficiently small, the efficient allocation maximizes the second term in equation (10). The maximum is achieved when m_A and $\int l(\theta)\mathrm{d}H_A(\theta)$ are made both as large as possible (or as small as possible), implying perfect segregation. When α is sufficiently large, the efficient allocation maximizes the first term in equation (10). Using integration by parts, we can show that the solution is $H_A(\theta) = H_B(\theta)$ for all θ, implying perfect mixing.

[13] If relative ranking and average ability are imperfect substitutes in individual utility function, types that are allocated to both organizations need not be evenly split in an efficient allocation. However, as long as the marginal rate of substitution between relative ranking and average ability is type-independent, as when the individual utility function is given by $\theta v(r_i, m_i)$, the support sets in any efficient allocation remain intervals. Thus, the no-gap result is a robust feature of efficiency, as is true in both sorting equilibrium and competitive equilibrium allocation.

[14] This result relies on the linearity assumption on l. The general condition for any θ in a mixing interval is given by $\alpha(H_B(\theta) - H_A(\theta)) = (m_A - m_B) + (l_A - l_B)/l'(\theta)$, where $l_i = \int l(\theta)\mathrm{d}H_i(\theta)$, $i = A, B$. The splitting rule for the efficient allocation is generally not even.

From Proposition 2, an immediate implication of Proposition 3 is that there is too little segregation in the sorting equilibrium. The inefficiency of the sorting equilibrium can be understood as follows.[15] In a sorting equilibrium, agents choose organizations based on their individual utility, without any regard for the external effects of their decisions on the utility of other agents. Therefore, types that are allocated to both organizations in equilibrium must be indifferent (see equations (4) and (6)). In particular, the equilibrium threshold x is the highest type that weakly prefers the lower average type organization. In contrast, since there is more segregation at the efficient allocation than in the corresponding sorting equilibrium, we have

$$\alpha(2 - F(x)) > m_A - m_B,$$

so that the threshold type x strictly prefers B, the lower average type organization. The efficient allocation requires a lower threshold type x, and hence less mixing, because of the need to internalize the externalities. When the threshold x is increased and some agents of types around x are reallocated from the higher average type organization A to B, it may appear that two kinds of externalities on other agents need to be internalized to maximize aggregate utility. One is that the agents with types between y and x now take on higher level positions in organization A and lower level positions in organization B (while those with types above x and below y keep their positions in the two organizations). This externality is not responsible for the discrepancy between the competitive allocation and the efficient allocation, because the utility gain of each type in A exactly cancels the utility loss of the same type in B. The other externality is that the average type in organization A is decreased while the average type in B is increased, making all agents in A worse

[15] The inefficiency of competitive equilibrium does not arise from the assumption of continuum of agents. The proof of Proposition 2 shows that the indirect utilities of the types that are mixing between two organizations must have the same slope at each type, which yields the same indifference condition of equation (4) as in sorting equilibrium. This differs from the mixing condition of equation (11) in the efficient sorting derived from a local variation argument in Lemma 2. In a discrete model, a counterpart of equation (4) obtains because transfers have to be such that positive assortative matching between type and rank within each organization is an equilibrium, with some modifications because when an agent moves from one organization to the other he will change the transfer schedule and the mean in the target organization. However, unlike the local variation argument, the modified equation (4) cannot capture all the externalities generated by an agent moving from one organization to the other. In particular, an agent moving from A to B does not internalize the decrease in the average type in A.

off and all agents in B better off. This is the externality that fails to be internalized in a sorting equilibrium when agents with types around x move from A to B. Due to the complementarity between agent type and the average ability in an organization, agents of the threshold type x must be discouraged from such a move to maximize aggregate welfare.

5. Comparative Statics and Welfare

The analysis leading to Proposition 1 shows that the equilibrium allocation can be reduced to a single variable z, which in turn depends on the parameter α and the distribution function F. As the equilibrium value of z increases from 0 to $\overline{\mu}_e - \underline{\mu}_e$, the threshold type x decreases from $\overline{\theta}$ to θ_e and y increases from $\underline{\theta}$ to θ_e. Thus, the equilibrium allocation becomes more segregated both in terms of a greater quality difference between the two organizations, and in terms of a smaller range of mixing.[16]

In this section, we consider comparative statics regarding the degree of segregation. Part of this comparative statics analysis follows directly from the characterization of the unique stable sorting equilibrium in Section 3. In this case there is no ambiguity regarding how to measure the degree of segregation. In other cases we need to suitably extend the main model, by generalizing the overlapping interval form of allocation.[17] Throughout this section we will identify the degree of segregation with the equilibrium quality difference between the two organizations.

Due to the equivalence between the sorting equilibrium without transfers and the competitive equilibrium with transfers established in Proposition 2, we will use the aggregate utility given in equation (10) as the welfare measure. For quantitative statements about welfare effects of parameter changes, we adopt the assumption that $l(\theta)$ is linear, which

[16] Kremer and Maskin (1996) define a segregation index as the ratio of between-organization variance to the sum of between-organization variance and within-organization variance. In our model, it can be shown the between-organization variance decreases with x, and the within-organization variance increases in x. Thus, the value of the Kremer-Maskin segregation index falls with x.

[17] In an earlier version of the present paper, we also consider how the degree of segregation is affected when the relative size of the two organizations changes, when higher types care less about the pecking order effect, and when there are more than two organizations.

further restricts (10). Finally, if a parameter change affects the aggregate utility in an efficient allocation as characterized by Proposition 3, we will use the ratio of the equilibrium aggregate utility to the efficient aggregate utility as the relative welfare measure.

5.1. Egalitarianism and segregation

Suppose the pecking order effect becomes more important relative to the peer effect. This change is represented by a rise in α. Then, according to Proposition 1, perfect segregation becomes less likely to be the stable equilibrium while perfect mixing becomes more likely to be the stable equilibrium. Furthermore, in an equilibrium with a partial overlapping of support sets, equations (4) imply that the threshold type x increases and y decreases for any fixed z, with more types evenly split between y and x. As a result, the whole function $D(z)$ shifts down as α increases. More precisely, we have

$$\frac{\partial D(z)}{\partial \alpha} = -\frac{1}{\alpha}(x-y)(2-F(x)) < 0.$$

Thus the equilibrium z decreases. We state this result as a proposition.

PROPOSITION 4. *An increase in the weight of the pecking order effect relative to the peer effect reduces the extent of segregation between two organizations.*

The above result can be generalized to the case where relative ranking and average ability are not perfect substitutes in individual utility functions. As we have remarked on Proposition 1, stable sorting equilibria can be shown to exist when the individual utility function is given by equation (3). At any such equilibrium with allocation (H_A, H_B) and quality difference z, when $\partial v(r_i, m_i)/\partial r$ increases at every r_i and m_i, the equilibrium quality difference z decreases. To see this, note that the indifference conditions $v(H_A(\theta), m_A) = v(H_B(\theta), m_B)$ in the overlapping interval can be rewritten as

$$\int_{m_B}^{m_A} \frac{\partial v(H_A(\theta), m)}{\partial m}\, \mathrm{d}m = \int_{H_A(\theta)}^{H_B(\theta)} \frac{\partial v(r, m_B)}{\partial r}\, \mathrm{d}r.$$

When $\partial v(r,m)/\partial r$ increases at every r and m (while $\partial v(r,m)/\partial m$ remains unchanged), organization B becomes more attractive for each type θ. Since v is an increasing function of r, to restore the indifference condition $v(H_A(\theta), m_A) = v(H_B(\theta), m_B)$, we need to increase

$H_A(\theta)$ and decrease $H_B(\theta)$. Similarly, x increases and y decreases. As a result, $m_A(z)$ (and hence $D(z)$) decreases at each z, and the equilibrium value of z decreases.

Proposition 4 has important implications regarding the effects of egalitarian policies in organizations.[18] Suppose, for example, that a school board decides that all schools within its district adopt a more egalitarian approach to education. This means that top students within a school receive less attention and fewer scholarships or prizes, while students lower down the ladder of academic ability have greater access to the limited opportunities that help make a valuable educational experience (e.g., representing the school in external competitions). A more egalitarian policy can also be achieved by a more compressed distribution of grades, so that outsiders cannot so easily distinguish the top students from the bottom ones. Such a change in policy would lead to a fall in α. As the advantages from being at the top of a school diminishes, students will compete harder to enroll in the school with higher average student ability. Paradoxically, our analysis suggests that a policy toward greater egalitarianism within a school may lead to an outcome with greater segregation across schools.

The Texas top-ten-percent law mentioned in the introduction takes education policies in the opposite direction of egalitarianism. Cullen, Long and Reback (2005) study the residential-school choices of families with eighth-graders that for some exogenous reasons have moved to a Texas school district affected by the law. They find that any student who chooses a high school other than the one that would have been chosen before the law is likely to choose one in which the student expects to be in the top ten percent. As a result of these strategic school choices, the Texas law was not only a success in terms of achieving the policy goal of restoring the racial mix in the Texas public university system, but it also led to increased mixing of high school graduates by test scores, consistent with our comparative statics analysis above.

[18] In some professions, the peer effect comes from the fact that being associated with high ability colleagues tends to enhance one's human capital formation. To the extent that the pecking order effect arises from the consumption motive (e.g., self-esteem) while the peer effect arises from the investment motive, younger workers are expected to have a smaller relative weight α than do older workers. Proposition 4 then suggests that the distribution of talent across firms is more concentrated for younger cohorts of workers than it is for older cohorts.

To study the welfare effects of a policy change such as the Texas law, we need to distinguish between the direct effect of an increase in α on the aggregate utility as given in (10), with $l(\theta) = \theta$, and the indirect effect through increased segregation in equilibrium. Further, an increase in α also changes the aggregate utility at the efficient allocation through a direct effect (the indirect effect is zero by the definition of efficiency), so we need to measure the welfare effects by the ratio of the equilibrium aggregate utility to the efficient aggregate utility. It immediately follows from Proposition 1 and Proposition 3 that the welfare ratio is equal to 1, both when $\alpha \leq \overline{\mu}_e - \underline{\mu}_e$, in which case both the equilibrium and the efficient allocations involve perfect segregation, and when $\alpha \geq 2(\overline{\theta} - \underline{\theta})$, in which case both allocations involve perfect mixing. For $\alpha \in (\overline{\mu}_e - \underline{\mu}_e, \overline{\theta} - \underline{\theta})$, the equilibrium allocation has a partial overlapping interval form, and the indirect effect of an increase in α on the welfare ratio is negative, because there is more segregation by Proposition 4, which exacerbates the inefficiency of the equilibrium by Proposition 3. For $\alpha \in (\overline{\theta} - \underline{\theta}, 2(\overline{\theta} - \underline{\theta}))$, there is perfect mixing in equilibrium, and the indirect effect is nil. To study the direct effects of an increase in α on the welfare ratio, for any given overlapping interval allocation represented by (7), we rewrite the aggregate utility $Q(H_A) + Q(H_B)$ as

$$\alpha \left(\int_{\underline{\theta}}^{y} \theta F(\theta) f(\theta) \, \mathrm{d}\theta + \int_{y}^{x} \frac{1}{2} \theta F(\theta) f(\theta) \, \mathrm{d}\theta + \int_{x}^{\overline{\theta}} \theta (F(\theta) - 1) f(\theta) \, \mathrm{d}\theta \right)$$
$$+ \left(\int_{y}^{x} \frac{1}{2} \theta f(\theta) \, \mathrm{d}\theta + \int_{x}^{\overline{\theta}} \theta f(\theta) \, \mathrm{d}\theta \right)^2 + \left(\int_{\underline{\theta}}^{y} \theta f(\theta) \, \mathrm{d}\theta + \int_{y}^{x} \frac{1}{2} \theta f(\theta) \, \mathrm{d}\theta \right)^2,$$

where the first term (α times the sum in the bracket) is the contribution to the aggregate utility from the concerns for the pecking order effect, and the second term (including both squared terms) is the contribution from the peer effect. Using equation (5), we can take $Q(H_A) + Q(H_B)$ as a function of x only. By taking derivatives, we can easily show that the first term is increasing in x while the second term is decreasing in x. Since in equilibrium the highest type in the lower organization B is greater than the corresponding type in an efficient allocation by Proposition 3, an increase in α puts a greater weight on the first term, which is greater in the equilibrium allocation than in the efficient allocation, and simultaneously a smaller weight on the second term, which is smaller in the equilibrium allocation. Thus, the direct effect of an increase in α on the welfare ratio is positive.

Whether the positive direct effect or the negative indirect effect dominates depends on α: for α close to $\overline{\mu}_e - \underline{\mu}_e$ the indirect effect of greater and more inefficient segregation dominates, so that a policy such as the Texas law that reduces α makes the welfare relatively higher, while for α close to $\overline{\theta} - \underline{\theta}$ the direct effect dominates, as the contribution to the aggregate utility from the pecking order effect gets a greater weight.

5.2. Meritocracy

In the last subsection we have considered the effect of across-the-board changes in α, but in reality organization policies are often not coordinated. Now we examine the effects of relative changes in α. Consider an increase in α_B, the weight on relative ranking in the lower quality organization, while α_A remains unchanged. This makes organization B more attractive for all agents. In order to isolate the effect on equilibrium sorting arising from changes in B that alter the trade-off between the pecking order effect and the peer effect in B, from the effect arising from changes that make B universally more attractive, we look at a "compensated" change in α_B. To do so, we assume that the utility for type θ from each organization $i = A, B$ is

$$V_i(\theta) = \alpha_i \left(r_i(\theta) - \frac{1}{2} \right) + m_i. \tag{12}$$

Under this formulation, an increase in α_B means that all agents in organization B ranked above the median become better off while those ranked below are made worse off.[19] For example, a compensated increase in α_B occurs if organization B adopts a more meritocratic personnel policy without increasing the overall level of compensation to its employees.

When $\alpha_A \neq \alpha_B$, we can still apply a similar argument as in Lemma 1 to show that the support sets of the two organizations are intervals. However, in equilibrium one interval may strictly contain the other one. To see this, assume $\alpha_A > \alpha_B$ without loss of generality. The highest type, $\overline{\theta}$, prefers A to B if

$$\frac{1}{2}(\alpha_A - \alpha_B) + z > 0,$$

[19] For simplicity we choose the median as the reference rank. From the following analysis we can see that the choice of the reference does not affect the mixing of types. Further, when $\alpha_A = \alpha_B$, the reference rank does not affect the range of mixing either. Thus the equilibrium under equation (12) is the same as the equilibrium under equation (1) when $\alpha_A = \alpha_B$.

while the lowest type, $\underline{\theta}$, prefers B to A if

$$-\frac{1}{2}(\alpha_A - \alpha_B) + z < 0.$$

Thus, if in equilibrium the quality difference z is between $-\frac{1}{2}(\alpha_A - \alpha_B)$ and $\frac{1}{2}(\alpha_A - \alpha_B)$, the very high types will all choose A, while the very low types are forced to stay with A,[20] giving rise to a generalized form of overlapping intervals.

We restrict our attention to comparative static analysis of the partial overlapping interval equilibria. Starting from one such equilibrium with $z > 0$, when α_B increases, there is a stronger incentive for the threshold type x to switch from A to B. This tends to increase x, resulting in a decrease in z. We refer to this negative effect on the quality difference as the "threshold effect." When α_B increases, there is also an "allocation effect" on the quality difference, which is positive. To see this, note that for types $\theta \in [y, x]$ to be indifferent between the two organizations, we must have a constant difference $\alpha_A H_A(\theta) - \alpha_B H_B(\theta)$. This implies a mixing rule of a fraction $\alpha_B/(\alpha_A + \alpha_B)$ of each type θ going to A and the rest going to B. Thus, an increase in α_B reduces the fraction of types in $[y, x]$ allocated to organization B. This tends to reduce B's average ability (for fixed thresholds x and y). The following proposition provides a sufficient condition for the threshold effect to dominate the allocation effect, and therefore for an increase in α_B to narrow the degree of segregation between organizations A and B.

PROPOSITION 5. *Suppose that the density function f is symmetric about the median. At a partial-overlapping equilibrium with $z > 0$ and $\alpha_A \geq \alpha_B$, a compensated increase in α_A lowers the equilibrium average ability m_A of organization A, while a compensated increase in α_B raises the equilibrium average ability m_B of organization B.*

The proof of this proposition is in the appendix, where we also establish a sufficient condition on α_A and α_B for the existence of a partial-overlapping interval equilibrium. The intuition behind this result follows from the partial-overlapping structure. Recall that

[20] In any sorting equilibrium agents of type $\underline{\theta}$ will never end up in B, which they strictly prefer. Otherwise, the lowest type in organization A would strictly prefer to switch to B. See the proof of Proposition 5 in the appendix for a characterization of all sorting equilibria.

types below y do not have the option of switching from organization B to A. Reallocating resources away from these low types toward agents higher up in the hierarchy therefore makes organization B more attractive to agents with relatively higher ability. Similarly, types above x strictly prefer to stay in organization A. Reallocating resources away from these individuals toward agents lower down in the pecking order helps organization A retain its talent. In a partial-overlapping allocation, since individuals with intermediate levels of ability are indifferent across organizations, competition for talent is most intense in this segment of the market. Thus, if the objective of organizations is to improve the overall ability of their incumbents, they should try to reallocate internal resources in order to attract this specific group of individuals. In a low quality organization, people with intermediate levels of ability are relatively high in the pecking order. So a more meritocratic approach to personnel policies that appeals to this group will enhance the quality of the organization. In a high-quality organization, however, meritocratic personnel policies are not appealing to people with intermediate levels of ability, since they are relatively low in the pecking order. Policies that are suitable for low quality organizations can be counterproductive for high quality organizations.

The attempt by Princeton University to combat grade inflation in its undergraduate programs, and the concerns and controversies it generated provide a cautionary tale of how to, and how not to, use a meritocratic policy to compete for talent. In April 2004, Princeton University became the first major university to ration A's in its undergraduate courses, from then 46% down to 35%. In discussions about the wisdom of this move, Princeton students and officials have raised the concern that Princeton graduates risk losing out to graduates from other elite universities in applying for jobs and to graduate schools (*USA Today*, March 27, 2007). If anticipated, this concern may have a lasting effect on Princeton's admissions, as talented high school graduates who receive other offers may decide not to come to Princeton, which would lower the average student ability at Princeton and justify the negative bias against Princeton graduates on the job market and in graduate school applications. Although it seems early to assess the impact of Princeton's grade reform, the fact that no other elite college so far has followed Princeton's lead suggests that the potential negative effect on admissions is a real concern.

5.3. Organizational biases, snowballing and tipping

So far we have assumed that the quality of an organization depends solely on the average type of agents who choose it. But some organizations may be preferred for exogenous reasons, such as geographical location, physical endowments, and so on. Consider therefore a modification of the basic model in Section 2. Suppose that preferences for the high quality organization A are instead given by

$$V_A(\theta) = \alpha r_A(\theta) + m_A + \delta,$$

where δ is a parameter representing an attribute of organization A that makes it more appealing relative to organization B.[21] Modifying equations (4) and (6) with the addition of δ, we can easily see that in an equilibrium with partial overlapping intervals, an increase in δ reduces x and raises y, without affecting the result of an even split between y and x. More precisely, we have

$$\frac{\partial D(z)}{\partial \delta} = \frac{1}{\alpha}(x - y) > 0.$$

Thus, the whole function $D(z)$ shifts down, resulting in a greater value of equilibrium z. This result is stated as the following proposition.

Proposition 6. *An increase in the preference bias for the high quality organization increases the extent of segregation between two organizations.*

The comparative statics result remains valid when relative ranking and average ability are not perfect substitutes in individual utility functions, as in equation (3). With the preference bias δ for organization B, the equations that determine the threshold types become $v(F(x) - 1, m_A) = v(1, m_B + \delta)$ and $v(0, m_A) = v(F(y), m_B + \delta)$. Since v is an increasing function of m, an increase in δ raises x and reduces y. Moreover, the indifference condition for a type $\theta \in (y, x)$ at a stable equilibrium with allocation (H_A, H_B) becomes $v(H_A(\theta), m_A) = v(H_B(\theta), m_B + \delta)$. An increase in δ makes organization B more attractive

[21] We allow δ to be either positive or negative. A similar argument as below can be used to show that the sorting equilibrium exists as long as the bias parameter for the (equilibrium) high organization δ is not too negative.

for θ. To restore the indifference, we need to increase $H_A(\theta)$ and decrease $H_B(\theta)$. As a result, $D(z)$ decreases pointwise and the equilibrium value of z decreases.

Proposition 6 can be strengthened to yield a "multiplier effect" in our sorting model. From the equilibrium condition (8) we have

$$\frac{\mathrm{d}z}{\mathrm{d}\delta} = \frac{\partial D(z)/\partial \delta}{1 - D'(z)} > \frac{\partial D(z)}{\partial \delta},$$

where the inequality follows because $D'(z) \in (0,1)$ by the local stability of the equilibrium with partial overlapping. Thus, an increase in the preference bias for A not only increases the equilibrium quality difference z between A and B, but also increases z by a greater amount that what is directly implied by the increase in the bias. A heuristic pseudo-dynamic story is that, at the stable sorting equilibrium with partial overlapping, a small increase in δ causes the top ranked agents in B to defect to A, which further increases the quality difference between the two organizations and yet more defections from B to A. Thus, there is a multiplier effect, or "snowballing," in our model. However, our sorting equilibrium is also locally stable, so that the snowballing caused by an increase in δ eventually stops as fewer and fewer top ranked agents from B defect to A as a result of previous defections. In other words, there is no "tipping" in the sense that a small increase in δ cannot lead to complete segregation with all types above the median going to A.

That the local stability of our sorting equilibrium is simultaneously responsible for both the presence of snowballing and the absence of tipping makes the present model a parsimonious framework to understand the phenomenal transformation of Silicon Valley from a decidedly low-tech "Valley of the Heart's Delight," and the more recent slowdown in its growth. Although Silicon Valley had a long tradition of research and innovations associated with the U.S. Navy and Stanford University, this initial "hardware" advantage alone cannot explain the explosive growth in the concentration of the high-tech industry in the area since the 1970's. Instead, it was the decisions of high-tech talents to come to Silicon Valley that created a lasting advantage over other potential high-tech centers. Further, even though the hardware advantage appeals to all high-tech talents equally, only the top talents have the priority in taking advantage of it, as in our sorting equilibrium in the non-transferable model. Equivalently, as in a competitive equilibrium with transfers, the top talents outbid lower talents for scarce high relative positions due to the complementarity between talent and relative position, and they are the ones that fueled the initial growth of Silicon Valley. As the growth continued, the newly arrived had less impact, and eventually the growth would halt in the absence of a fresh impetus. Conversely, as the housing market price skyrocketed and the traffic congestion went from bad to worse, or when the dot com bubble burst in the early 2000's, it was the less talented in Silicon Valley who were first squeezed out. From 2002 to 2006, middle-wage jobs—paying up to \$80,000 a year—fell from 52% to 46% of the work force (*The New York Times*, February 19, 2008). The same force that eventually put a break on the explosive growth of Silicon Valley is also at the heart of its strength and resilience to keep on reinventing itself.

5.4. Heterogeneous biases

Different agents may not value the other organizational attributes in the same way. For example, agents typically have different locational preferences. Let us consider a model where the utility of a type θ agent from choosing organization A is given by

$$V_A(\theta) = \alpha r_A(\theta) + m_A + \sigma\delta, \tag{13}$$

where σ is a positive parameter representing the degree of heterogeneity in tastes, and δ is a random variable distributed according to G, with support $[\underline{\delta}, \overline{\delta}]$ and a continuous density function. For simplicity, we consider the case where the median of the distribution G is zero, so that there is no aggregate bias for either organization.

Under equation (13), the equilibrium allocation takes the form of a generalized version of overlapping interval structures. When σ is relatively large, then the highest type agents will be present in both organizations. In particular, if $\sigma\underline{\delta} < \underline{\mu}_e - \overline{\mu}_e$ and $\sigma\overline{\delta} > \overline{\mu}_e - \underline{\mu}_e$, the equilibrium allocation will not involve high type agents choosing exclusively the high quality organization. More generally, for each type θ, the fraction of agents choosing the high quality organization A increases in the quality difference z and decreases in the ranking difference $r_B(\theta) - r_A(\theta)$. In the appendix, we show how to characterize the sorting equilibrium in this model, and establish the following result regarding the degree of heterogeneity in the idiosyncratic preferences for other organizational attributes.

Proposition 7. *At a stable equilibrium, the degree of segregation decreases (z decreases) as the degree of heterogeneity in the idiosyncratic preferences increases (σ increases).*

The logic of the above result is that a greater degree of heterogeneity in the idiosyncratic preferences reduces the importance of the trade-off between average quality and relative ranking to agents' organizational choices. For any quality difference z there are more agents who prefer the low quality organization B for idiosyncratic reasons as σ increases. The equilibrium quality difference shrinks as a result.

The above result has an interesting implication for the design of benefit packages for organizations. Suppose an organization can choose between raising salaries by some total amount or providing in-kind benefits that cost the same amount. Suppose that different employees value the in-kind benefits differently. Then, our analysis suggests that the in-kind benefits are more effective for the lower quality organization to attract talent, whereas cash wages are more effective for the higher quality organization to retain talent. Again, the intuition for this result follows from the overlapping interval allocation. Top talents in organization A strictly prefer staying in A to moving to B under given cash wages. When the form of compensation switches to in-kind benefits, some of these top talents become better off because the benefits are worth more than the cash wages, but these individuals will stay in A regardless. For those who value the in-kind benefits less than the cash wages, however, they will consider moving to organization B. On balance, therefore, increasing the variance of the value of benefits hurts the high-quality organization.

The above comparative statics analysis may be part of the logic behind Harvard University's recent decision to increase tuition aid to students of middle class and upper-middle class families. Starting from the academic year of 2008, Harvard will offer generous tuition cuts to students with household incomes from \$120,000 to \$180,000, increasing the university's spending on financial aid from \$98 million to \$120 million (*The New York Times*, December 10, 2007). Given that the stated goal of this financial aid initiative is to make Harvard more affordable and more attractive to high-achieving high school graduates, an outside observer may question whether at least part of the increase in the spending could have been used to subsidize specific academic and extracurricular activities that these students care about. Examples of these activities include highly valuable but unpaid research opportunities with professors, unpaid summer internships and overseas studies, which Harvard officials are concerned that often only the wealthy students can afford. In reality, as in our model, it is prohibitively costly to fine tune the financial aid program to match specific subsidies to talented high school graduates Harvard wants to attract, so that the real choice is between a blanket program of the kind that Harvard just adopted and a broad collection of individual subsidy programs that may suit the needs of some students but not those of others. The comparative statics analysis following Proposition 7 suggests that a blanket program is more suitable for a university, like Harvard, that is in the enviable position of having to keep its talented freshman class from being poached by competing universities. Our analysis also suggests that the more effective response to the Harvard initiative is not to imitate it, but to take the opposite approach of using targeted individual subsidy programs to appeal to those students who would enjoy Harvard less due to the pecking order effect.

6. Conclusion

We have presented a sorting model with heterogeneous types where the equilibrium allocation of agents to organizations is determined by agents' concerns for average ability comparisons across organizations and for their relative ranking within the organizations. Sorting of agents is shown to result in an overlapping interval structure in the type space that allows coexistence of segregation and mixing. This result enables us to easily characterize the degree of segregation (measured by mean ability difference) across organizations. When transfers are possible, we construct a model of endogenous task allocation with complementarity between agents and task as well as complementarity among agents in the same organization. In such an environment, the competitive equilibrium in which people bid for tasks in organizations corresponds precisely to our stable sorting equilibrium. The efficient sorting of agents to organizations in this environment also takes the form of an overlapping interval allocation, but efficiency requires a greater degree of segregation than in the sorting (or competitive) equilibrium. In comparative statics analysis, we show that a greater emphasis on relative ranking within organizations leads to less segregation across organizations. Since the equilibrium allocation involves too little segregation, a greater weight on the pecking order effect may lead to a further welfare loss compared to the efficient allocation. Finally, because agents with intermediate ability are the most mobile across organizations, personnel policies that cater to this group (i.e., the low status members in high-quality organizations, and the high-status members in low-quality organizations) are particularly effective for raising the average ability of the organization.

The main limitations of the present paper are the assumptions of capacity constraints and one-dimensional characteristics, and the absence of objective functions for organizations. Each of the three is worth further investigation. The assumption of capacity constraints may be profitably replaced by participation constraints on the side of agents or the side of organizations. The assumption of one-dimensional characteristics is standard in the sorting literature, but needs to be relaxed to enhance the applicability of our model. Finally, modeling the objective functions and strategies of organizations can further our understanding of equilibrium distribution of talents among organizations. Extension of the present model in these directions is beyond the scope of this paper, but the model presented here can prove fruitful in investigating these issues in sorting.

Appendix

Proof of Proposition 2. (i) Suppose that (H_A, H_B) is a sorting equilibrium. For each organization $i = A, B$ and each position $r_i \in [0, 1]$, let the price schedule p_i be defined by

$$p_i'(r_i) = \alpha l\left(H_i^{-1}(r_i)\right),$$

up to an integration constant to be determined below. Then, for any type $\theta \in T_i$, the first order condition of the optimization problem $\max_{r_i} l(\theta)(\alpha r_i + m_i) - p_i(r_i)$ is satisfied at $r_i = H_i(\theta)$. Since p_i is convex by construction, the second order condition of the above optimization problem is satisfied. Thus, it is optimal for type $\theta \in T_i$ to choose the position of rank $H_i(\theta)$ among all positions in i. The implied indirect utility of θ is $U_i(\theta) = l(\theta)(\alpha H_i(\theta) + m_i) - p_i(H_i(\theta))$, implying that

$$U_i'(\theta) = l'(\theta)(\alpha H_i(\theta) + m_i). \tag{A.1}$$

Given any $p_A(0)$ and $p_B(0)$, the indirect utility functions $U_A(\theta)$ (for $\theta \in [y, \overline{\theta}]$) and $U_B(\theta)$ (for $\theta \in [\underline{\theta}, x]$) are thus well-defined and continuously differentiable. Choose $p_A(0)$ and $p_B(0)$ such that $U_A(y) = U_B(y)$, and $U_B(\underline{\theta}) \geq 0$. Since (H_A, H_B) is a sorting equilibrium, equation (6) is satisfied, and so it follows from equation (A.1) that $U_A(\theta) = U_B(\theta)$ for all $\theta \in [y, x]$.

It remains to argue that no type θ in T_i but not T_j will deviate from i to j. Consider some type $\theta > x$ in A. The deviation utility $\tilde{U}_B(\theta)$ of this type in organization B is maximized at $r_B = 1$. This is because $\tilde{U}_B(\theta)$ is concave, and its derivative evaluated at $r_B = 1$ has the same sign as $\alpha(l(\theta) - l(x))$, which is positive. To show that type θ will not deviate to $r_B = 1$, we need $U_A(\theta) > l(\theta)(\alpha + m_B) - p_B(1)$. This holds because by construction $U_A(x) = l(x)(\alpha + m_B) - p_B(1)$, and $U'_A(\theta) = l'(\theta)(\alpha H_A(\theta) + m_A)$, which is strictly greater than $l'(\theta)(\alpha + m_B)$. The argument for why it is not optimal for type $\theta < y$ to deviate from B to A is similar.

(ii) Let (H_A, H_B) be a feasible allocation and suppose that (H_A, H_B) can be supported by a competitive equilibrium allocation, with price schedules p_A and p_B. Note that in any competitive equilibrium allocation each price schedule p_i, $i = A, B$, is continuous; otherwise, at a point of discontinuity, say r_i, there would be an excess supply for positions just above r_i. To show that the two support sets are both intervals, suppose without loss of generality that there is a gap in T_A, with types on the interval $[\theta, \tilde{\theta}]$ allocated to organization B and at least some types just below θ and just above $\tilde{\theta}$ are allocated to A. Since $U_B(\theta) = U_A(\theta)$ by continuity, a necessary condition for the gap is $U'_B(\theta) \geq \tilde{U}'_A(\theta)$, where U_B is the indirect utility for types between θ and $\tilde{\theta}$ and $\tilde{U}_A$ is the deviation utility for the same types.[22] Since each type t on the interval $(\theta, \tilde{\theta})$ must find it optimal to choose its equilibrium position $H_B(t)$ among all positions in B ranked between $H_B(\theta)$ and $H_B(\tilde{\theta})$, the envelope theorem implies that $U'_B(t) = l'(t)(\alpha H_B(t) + m_B)$. Since a deviating type $t \in (\theta, \tilde{\theta})$ can always choose the position in A corresponding to type θ, we have $\tilde{U}'_A(t) \geq l'(t)(\alpha H_A(\theta) + m_A)$. Thus,

$$l'(\theta)(\alpha H_B(\theta) + m_B) \geq l'(\theta)(\alpha H_A(\theta) + m_A).$$

[22] Following the standard mechanism design literature (e.g., Stole, 1996), we can show that the indirect utility functions are differentiable almost everywhere.

Similarly, using the deviation condition of type $\tilde{\theta}$, we have

$$l'(\tilde{\theta})(\alpha H_A(\tilde{\theta}) + m_A) \geq l'(\tilde{\theta})(\alpha H_B(\tilde{\theta}) + m_B).$$

However, since $H_A(\theta) = H_A(\tilde{\theta})$ and $H_B(\tilde{\theta}) > H_B(\theta)$, the above two inequalities contradict each other, implying that there cannot be a gap in T_A.

If the support sets of a competitive equilibrium allocation (H_A, H_B) overlap on some interval $[y, x]$, then we have equation (A.1) from individual optimization of types on the interval within each i among the positions ranked between $H_i(y)$ and $H_i(x)$. Since each such type θ is indifferent between A and B, we have the same indifference conditions (6) as in a sorting equilibrium. Thus, types on the interval $[y, x]$ are evenly split. It follows from condition (b) in Definition 3 that (H_A, H_B) is a sorting equilibrium. *Q.E.D.*

PROOF OF LEMMA 2. First, we show that if for some $\epsilon > 0$ and some type θ, H_B is strictly increasing on $(\theta, \theta + \epsilon)$ and H_A is strictly increasing on $(\theta - \epsilon, \theta)$, then

$$\alpha(H_A(\theta) - H_B(\theta)) \leq 2(m_B - m_A).$$

For any sufficiently small $\epsilon > 0$, let $\gamma(\epsilon)$ solve

$$H_A(\theta) - H_A(\theta - \gamma(\epsilon)) = H_B(\theta + \epsilon) - H_B(\theta).$$

Since H_B is strictly increasing just above θ and H_A is strictly increasing just below θ, we have that $\lim_{\epsilon \to 0} \gamma(\epsilon) = 0$. Define a new allocation $(H_A^\epsilon, H_B^\epsilon)$ such that

$$H_A^\epsilon(t) = \begin{cases} H_A(t), & \text{if } t \notin (\theta - \gamma(\epsilon), \theta + \epsilon) \\ H_A(\theta - \gamma(\epsilon)), & \text{if } t \in (\theta - \gamma(\epsilon), \theta) \\ H_A(\theta - \gamma(\epsilon)) + F(t) - F(\theta), & \text{if } t \in [\theta, \theta + \epsilon) \end{cases}$$

and $H_B^\epsilon = F - H_A^\epsilon$. The allocation $(H_A^\epsilon, H_B^\epsilon)$ modifies (H_A, H_B) by redistributing all agents of type on the interval $(\theta - \gamma(\epsilon), \theta)$ to organization B and all types on the interval $[\theta, \theta + \epsilon)$ to organization A. Let $\Delta(\epsilon)$ be the difference in aggregate utility between $(H_A^\epsilon, H_B^\epsilon)$ and

(H_A, H_B). We have

$$\begin{aligned}\Delta(\epsilon) =& \alpha\left(\int_{\theta}^{\theta+\epsilon} t\left(H_A(\theta-\gamma(\epsilon)) + F(t) - F(\theta)\right)\, \mathrm{d}F(t) - \int_{\theta-\gamma(\epsilon)}^{\theta} tH_A(t)\, \mathrm{d}H_A(t)\right)\\ &+ \alpha\left(\int_{\theta-\gamma(\epsilon)}^{\theta} t\left(F(t) - H_A(\theta-\gamma(\epsilon))\right)\, \mathrm{d}F(t) - \int_{\theta}^{\theta+\epsilon} tH_B(t)\, \mathrm{d}H_B(t)\right)\\ &+ \left(m_A + \int_{\theta}^{\theta+\epsilon} t\, \mathrm{d}H_B(t) - \int_{\theta-\gamma(\epsilon)}^{\theta} t\, \mathrm{d}H_A(t)\right)^2 - m_A^2\\ &+ \left(m_B + \int_{\theta-\gamma(\epsilon)}^{\theta} t\, \mathrm{d}H_A(t) - \int_{\theta}^{\theta+\epsilon} t\, \mathrm{d}H_B(t)\right)^2 - m_B^2.\end{aligned}$$

Evaluating $\Delta(\epsilon)$ and its first two derivative at $\epsilon = 0$ we have $\Delta(0) = \Delta'(0) = 0$ while $\Delta''(0)$ is proportional to $\alpha(H_A(\theta) - H_B(\theta)) - 2(m_B - m_A)$. Thus, if $\alpha(H_A(\theta) - H_B(\theta)) > 2(m_B - m_A)$, it is possible to increase aggregate utility by redistributing agents of type close to θ according to $(H_A^\epsilon, H_B^\epsilon)$.

Next, we claim that the support set of each organization is an interval in the efficient allocation. To see this, suppose there is a gap in T_A. Then, there exist two types θ and $\tilde{\theta}$, with $\theta < \tilde{\theta}$, such that all types between θ and $\tilde{\theta}$ are allocated to B, and at least some types just below θ and just above $\tilde{\theta}$ are allocated to A. Then, by the result we have just established, we have

$$\alpha(H_B(\theta) - H_A(\theta)) \geq 2(m_A - m_B);$$
$$\alpha(H_B(\tilde{\theta}) - H_A(\tilde{\theta})) \leq 2(m_A - m_B).$$

These two inequalities contradict each other, because $H_A(\theta) = H_A(\tilde{\theta})$ and $H_B(\theta) < H_B(\tilde{\theta})$. The lemma follows immediately, because types allocated to both A and B must be evenly split, and a unit mass must be allocated to each organization. *Q.E.D.*

Proof of Proposition 5. First, we provide a characterization of stable sorting equilibrium. Without loss of generality assume $\alpha_A \geq \alpha_B$. We allow z to range from $\underline{\mu}_e - \overline{\mu}_e$ to $\overline{\mu}_e - \underline{\mu}_e$. For any such z, we can determine the support sets T_A and T_B, as follows. Suppose for now $\frac{1}{2}(\alpha_A - \alpha_B) < \overline{\mu}_e - \underline{\mu}_e$. For any $z > \frac{1}{2}(\alpha_A - \alpha_B)$, we have $T_A = [y, \overline{\theta}]$ and

$T_B = [\underline{\theta}, x]$, where x and y solve

$$\begin{aligned} \alpha_A \left((F(x) - 1) - \frac{1}{2} \right) - \frac{1}{2}\alpha_B + z = 0; \\ -\frac{1}{2}\alpha_A - \alpha_B \left(F(y) - \frac{1}{2} \right) + z = 0. \end{aligned} \tag{A.2}$$

For z between $-\frac{1}{2}(\alpha_A - \alpha_B)$ and $\frac{1}{2}(\alpha_A - \alpha_B)$, we have $T_A = [\underline{\theta}, \overline{\theta}]$ and $T_B = [y, x]$, where

$$\begin{aligned} \alpha_A \left((F(x) - 1) - \frac{1}{2} \right) - \frac{1}{2}\alpha_B + z = 0; \\ \alpha_A \left(F(y) - \frac{1}{2} \right) + \frac{1}{2}\alpha_B + z = 0. \end{aligned}$$

Finally, for $z < -\frac{1}{2}(\alpha_A - \alpha_B)$, we have $T_A = [\underline{\theta}, x]$ and $T_B = [y, \overline{\theta}]$, where

$$\begin{aligned} \frac{1}{2}\alpha_A - \alpha_B \left((F(x) - 1) - \frac{1}{2} \right) + z = 0; \\ \alpha_A \left(F(y) - \frac{1}{2} \right) + \frac{1}{2}\alpha_B + z = 0. \end{aligned}$$

If $\frac{1}{2}(\alpha_A - \alpha_B) \geq \overline{\mu}_e - \underline{\mu}_e$, then only the second case arises. In each of the three cases the resulting quality difference $D(z)$ is given by $2(m_A(z) - \mu)$, where $m_A(z)$ is the average ability in organization A using the corresponding thresholds and the mixing rule of a fraction $\alpha_B/(\alpha_A + \alpha_B)$ of each type in $[y, x]$ going to A and the rest going to B. As before, a stable sorting equilibrium is then a z such that $D(z) = z$ and $D'(z) < 1$.

Next, we provide a sufficient condition for the existence of a partial-overlapping stable sorting equilibrium with $z > 0$ when $\alpha_A > \alpha_B$. Start with $\alpha_A = \alpha_B = \alpha$. We argue that if α is between $2(\overline{\mu}_e - \mu)$ and $2(\overline{\theta} - \mu)$, then an overlapping equilibrium with $z > 0$ exists when α_A marginally increases or α_B decreases marginally. To see this, note that $D(z) < z$ at $z = \overline{\mu}_e - \underline{\mu}_e$, because $\alpha_A, \alpha_B > 2(\overline{\mu}_e - \mu)$ implies $x > \theta_e$ by equation (A.2). At $z = \frac{1}{2}(\alpha_A - \alpha_B)$, from equation (A.2) we have $F(\hat{x}) = 1 + \alpha_B/\alpha_A$ and $F(\hat{y}) = 0$. Thus

$$D\left(\frac{1}{2}(\alpha_A - \alpha_B)\right) = 2\left(\mu - \int_{\underline{\theta}}^{\hat{x}} \frac{\theta f(\theta)}{F(x)}\, \mathrm{d}\theta\right).$$

By taking derivatives with respect to α_A or α_B at $\alpha_A = \alpha_B = \alpha$, we can show that $D\left(\frac{1}{2}(\alpha_A - \alpha_B)\right) > \frac{1}{2}(\alpha_A - \alpha_B)$ because $\alpha < 2(\overline{\theta} - \mu)$. Thus there exists at least one stable equilibrium with partial overlapping.

Finally, for comparative statics when $\alpha_A \geq \alpha_B$, note that at any stable sorting equilibrium, the sign of $\mathrm{d}z/\mathrm{d}\alpha_A$ is the same as $\partial m_A/\partial\alpha_A$. We have

$$m_A(z) = \int_y^x \frac{\alpha_B}{\alpha_A+\alpha_B} tf(t)\,\mathrm{d}t + \int_x^{\overline{\theta}} tf(t)\,\mathrm{d}t,$$

where y and x depend on z and α_A through equation (A.2). The sign of $\partial m_A/\partial\alpha_A$ is the same as

$$\left(F(x)-\frac{3}{2}\right)x + \frac{1}{2}y - \frac{\alpha_B}{\alpha_A+\alpha_B}\int_y^x tf(t)\,\mathrm{d}t.$$

Upon integration by parts, and using equation (A.2), the above can be rearranged to

$$\frac{x-y}{\alpha_A+\alpha_B}\left(F(x)-2+\frac{\alpha_A-\alpha_B}{2\alpha_A}\right) + \frac{\alpha_B}{\alpha_A+\alpha_B}\int_y^x (F(t)-F(\theta_e))\,\mathrm{d}t. \tag{A.3}$$

We know that at $z=\frac{1}{2}(\alpha_A-\alpha_B)$, the threshold types satisfy $F(\hat{x}) = 1+\alpha_B/\alpha_A$ and $F(\hat{y})=0$. At any partial-overlapping interval equilibrium with $z>0$, we have $x<\hat{x}$, and hence the bracketed term in (A.3) is less than $F(\hat{x})-2+\frac{1}{2}(\alpha_A-\alpha_B)/\alpha_A \leq 0$. Furthermore, (A.2) and the symmetry of f imply that $x-\theta_e \leq \theta_e - y$ when $\alpha_A \geq \alpha_B$. Therefore, the integral in (A.3) is also negative. We thus have $\mathrm{d}z/\mathrm{d}\alpha_A < 0$ when $\alpha_A > \alpha_B$. Similar calculations show that $\mathrm{d}z/\mathrm{d}\alpha_B > 0$ under the same conditions. *Q.E.D.*

Proof of Proposition 7. Without loss of generality, assume $m_A \geq m_B$. The permissible range of z is $[0, \overline{\mu}_e - \underline{\mu}_e]$. If $\sigma\underline{\delta} < \underline{\mu}_e - \overline{\mu}_e$ and $\sigma\overline{\delta} > \overline{\mu}_e - \underline{\mu}_e$, a positive fraction, $1-G(-z/\sigma)$, of the highest type agents will choose A and the rest will choose B. When $z > \sigma\overline{\delta}$ all agents of the highest type $\overline{\theta}$ will choose A.

Let $H_A(\theta)$ be the type distribution function in organization A. Then for any $\theta < \overline{\theta}$, a fraction $1-G(-(z+\alpha(2H_A(\theta)-F(\theta)))/\sigma)$ of type θ agents prefer A to B. Thus,

$$H'_A(\theta) = \left(1-G\left(-z/\sigma - \alpha(2H_A(\theta)-F(\theta))/\sigma\right)\right)f(\theta). \tag{A.4}$$

The above is a differential equation in H_A, with the boundary condition $H_A(\overline{\theta})=1$. A unique solution to the differential equation (A.4) exists; see, for example, Verhulst (1996). Let $H_A(\theta;z)$ be the solution to the differential equation. We assume that the solution satisfies the capacity constraint of A; otherwise if the solution is negative for some θ, redefine $H_A(\theta; z)$ as 0. Note that the solution to the differential equation cannot exceed $F(\theta)$ at any θ, hence the capacity constraint of B is never violated.

We have thus established that for any given z, there is a unique allocation of types between the two organizations consistent with the quality gap z. As before, an equilibrium with z is defined by $D(z) = z$, and the equilibrium is stable if $D'(z) < 1$. Note that $D(0) = 0$ because $H'_A(\overline{\theta}) = \frac{1}{2} f(\overline{\theta})$ and therefore the solution to (A.4) is given by $H_A(\theta; 0) = \frac{1}{2} F(\theta)$ for all θ. At the other end, we have $D(\overline{\mu}_e - \underline{\mu}_e) \leq \overline{\mu}_e - \underline{\mu}_e$, with strict inequality if and only if $\sigma\overline{\delta} > \overline{\mu}_e - \underline{\mu}_e$. Thus, a stable sorting equilibrium exists.

Finally, we argue that an increase in σ shifts $H_A(\theta; z)$ downward in that $H_A(\theta; z)$ is ordered by first order stochastic dominance for any $z \in (0, \overline{\mu}_e - \underline{\mu}_e)$. To establish this claim, first note that $H_A(\overline{\theta}; z) = 1$ for all σ, and the slope is smaller for the solution corresponding to a greater σ. Next note that no two solutions corresponding to different σ's can cross at any other point, since at any crossing the solution corresponding to a greater σ has a smaller slope by equation (A.4). An increase in σ then shifts down the function D. Since $D'(z) < 1$ at a stable equilibrium, the quality difference z decreases. *Q.E.D.*

References

Aigner, Dennis J. and Glen G. Cain (1977), "Statistical Theories of Discrimination in the Labor Market," *Industrial and Labor Relations Review* 30, 175–187.

Arnott, Richard and John Rowse (1987), "Peer Group Effects and Educational Attainment," *Journal of Public Economics* 32, 287–305.

Athey, Susan, Christopher Avery and Peter Zemsky (2000), "Mentoring and Diversity," *American Economic Review* 90, 765–786.

Benabou, Roland (1993), "Workings of a City: Location, Education, and Production," *Quarterly Journal of Economics*,108, 619–653.

De Bartolome, Charles A.M. (1990), "Equilibrium and Inefficiency in a Community Model with Peer Group Effect," *The Journal of Political Economy* 98, 110–133.

Becker, Gary S. (1973), "A Theory of Marriage: Part I," *Journal of Political Economy* 81, 813–846.

Becker, Gary S. and Kevin M. Murphy (2000), *Social Economics,* Cambridge, Harvard University Press.

Cole, Harold L., George J. Mailath and Andrew Postlewaite (1992), "Social Norms, Savings Behavior, and Growth," *Journal of Political Economy* 100, 1092–1125.

Coleman, J. S., E.Q. Campbell, C.J. Hobson, J. McPartland, A.M. Mood, F.D. Weinfeld and R.L. York (1966), *Equality of Educational Opportunity,* Washington, Government Printing Office.

Cullen, J. B., M.C. Long, and R. Reback (2005), "Jockeying for Position: High School Student Mobility and Texas' Top-Ten Percent Rule," Manuscript, UC San Diego.

Damiano, Ettore and Hao Li, "Competing Matchmaking," forthcoming *Journal of the European Economic Association.*

Duesenberry, James (1949), *Income, Saving, and the Theory of Consumer Behavior,* Cambridge, Harvard University Press.

Epple, Dennis and Richard E. Romano (1998), "Competition between Private and Public schools, Vouchers, and Peer-group Effects," *American Economic Review* 88, 33–62.

Fernandez, Raquel and Richard Rogerson (1996), "Income Distribution, Communities, and the Quality of Public Education," *Quarterly Journal of Economics* 111, 135–164.

Frank, Robert H. (1985), *Choosing the Right Pond,* Oxford, Oxford University Press.

Gamoran, A., M. Nystrand, M. Berends and P.C. LePore (1995), "A Organizational Analysis of the Effects of Ability Grouping," *American Educational Research Journal* 32, 687–715.

Garicano, Luis, and Rossi-Hansberg, Esteban (2004), "Inequality and the Organization of Knowledge," *American Economic Review* 94, 197–202.

Hartwick, John M. and Yoshitsugu Kanemoto (1984), "Formation of Convoys, Tennis Ladders, Colleges and Related Groups," Discussion Paper 589, Economics Department, Queen's University.

Kremer, Michael (1993), "The O-Ring Theory of Economic Development," *Quarterly Journal of Economics* 108, 551–576.

Kremer, Michael and Eric Maskin (1996), "Wage Inequality and Segregation by Skill," NBER working paper 5718.

Lazear, Edward P. (2001), "Educational Production," *Quarterly Journal of Economics* 116, 777–803.

Legros, Patrick and Andrew F. Newman (2002), "Monotone Matching in Perfect and Imperfect Worlds," *Review of Economic Studies* 69, 925–942.

Legros, Patrick and Andrew F. Newman (2007), "Beauty is a Beast, Frog is a Prince: Assortative Matching in a Non-transferable World," *Econometrica* 75, 1073-1102.

Lou, Y., P.C. Abrami and J.C. Spence (2000), "Effects of Within-Class Grouping on Student Achievement: An Exploratory Model," *Journal of Educational Research* 94, 101–112.

Marsh, Herbert W. and John W. Parker (1984), "Determinants of Student Self-Concept: Is It Better to Be a Relatively Large Fish in a Small Pond if You Don't Swim as Well?" *Journal of Personality and Social Psychology* 47, 213–231.

Marsh, Herbert W., Chit-Kwong Kong and Kit-Tai Hau (2000), "Longitudinal Multilevel Models of the Big-Fish–Little-Pond Effect on Academic Self-Concept: Counterbalancing Contrast and Reflected-Glory Effects in Hong Kong Schools," *Journal of Personality and Social Psychology* 78, 337–349.

The New York Times, "Silicon Valley Losing Middle-wage Jobs," February 19, 2008.

The New York Times, "Harvard to Aid Students High in Middle Class," December 10, 2007.

Postlewaite, Andrew (1998), "The Social Basis of Interdependent Preferences," *European Economic Review* 42, 779–800.

Roth, Alvin and Marilda Sotomayor (1990), *Two-sided Matching: A Study in Game-theoretic Modeling and Analysis*, Cambridge University Press: Cambridge.

Sacerdote, Bruce (2001), "Peer Effects with Random Assignment: Results for Dartmouth Roommates," *Quarterly Journal of Economics* 116, 681–704.

Slavin, R.E. (1987), "Ability Grouping and Student Achievement in Elementary Schools: A Best-Evidence Synthesis," *Review of Educational Research* 57, 293–336.

Solnick, Sara and David Hemenway (1998), "Is More Always Better: A Survey on Positional Goods," *Journal of Economic Behavior and Organization* 37, 373–383.

Stole, L. (1996), "Lectures on the Theory of Contracts and Organizations," mimeo, University of Chicago.

Summers, A. and B. Wolfe (1977), "Do Schools Make a Difference?" *American Economic Review* 67, 639–652.

USA Today, "Princeton Leads in Grade Deflation," March 27, 2007.

Verhulst, Ferdinand (1996), *Nonlinear Differential Equations and Dynamic Systems*, Berlin, Springer-Verlag.

Zeidner, Moshe and Esther Jane Schleyer (1998), "The Big-Fish–Little-Pond Effect for Academic Self-Concept, Test Anxiety, and School Grades in Gifted Children," *Contemporary Educational Psychology* 24, 305–329.