Delay in Strategic Information Aggregation

Ettore Damiano
University of Toronto

Li, Hao
University of British Columbia

Wing Suen*
The University of Hong Kong

October 7, 2010

Abstract. We construct a model in which conflict between two agents prevents them from aggregating any of their private information if they must make a collective decision without delay. If the two agents vote repeatedly until they agree, in equilibrium they are increasingly more willing to vote their private information after each disagreement, and information is perfectly aggregated within a finite number of rounds. As delay becomes less costly, the two agents are less willing to vote their private information, and information aggregation takes longer. Even when the cost of delay becomes arbitrarily small, the ex ante welfare of the two agents is higher than when the decision is made without delay for moderate degrees of conflict.

Keywords: repeated voting, conflict resolution, endogenous information.

JEL codes: C72, D7, D82

* Li and Suen thank the Guanghua School of Management of Peking University, and especially Hongbin Cai, for the research support during their visit. Li thanks the Marshall School of Business of the University of Southern California for hosting his sabbatical when part of this paper was written. We have benefited from comments by Jacques Cremer, Jeff Ely, Roger Gordon, Jim Peck, Joel Sobel, Asher Wolinsky, and by other seminar participants at Georgetown University, Northwestern University, Ohio State University, Singapore Management University, University of California at Los Angeles and at San Diego, and University of Florida. Chris Chan and Eric Mak provided excellent research assistance. The research described in this paper is partially funded by the Research Grants Council of Hong Kong (Project No. HKU751508H), and by the Social Sciences and Humanities Research Council of Canada (Grant No. 410-2008-1032).

1. Introduction

Individuals may disagree with one another on a joint decision because they have conflicting preferences or because they have different private information. Often it is difficult to distinguish between these two types of disagreement because divergent preferences provide incentives for individuals to distort their information. Even though they may share a common interest in some states had the individuals known each other's private information, the strategic distortion of information can still cause disagreement in these states. When disagreements lead to delay in making decisions, it may seem that any decision is better than no decision and costly delay. We argue in this paper, however, that institutionalized delay in the decision making process can serve a useful purpose. In the context of a stylized model of repeated voting, the prospect of costly delay induces the parties to be more forthcoming with their private information. This enhances information aggregation and potentially improves the welfare of the agents relative to the case when the decision has to be made immediately. Even when the delay cost is arbitrarily small, there can be a significant welfare gain.

The constructive role of delay in strategic information aggregation is illustrated in the simplest environment that captures the distinction between preference-driven and information-driven disagreements. Although highly stylized, its only essential feature is that there exist states of nature in which agents would disagree when deciding on the basis on their own private information, but would agree if all information were public. Indeed, absent such states, either the agents continue to disagree after sharing their information, in which case there is no room for mutually beneficial information aggregation and delay can only be harmful, or the agents already agree based on their private information, in which case there is no reason to expect delay to occur at all. The particular configuration of preferences and information structures that captures the essential feature can be illustrated with the following example. Imagine that two managers of a corporation, of marketing and R&D divisions, must jointly decide how to enter an emerging market. The marketing division's strategy focuses on pushing existing products through an advertising campaign, and the R&D division's strategy mainly involves developing a new product that targets the emerging market. For some types of the market, one strategy is definitely more effective than the other, in which case the two managers both prefer the more effective strategy, but there are also other types of the market for which the two strategies are equally effective, in which case each manager prefers the strategy of his own division. Suppose that the marketing manager can distinguish the states for which the advertising strategy is more effective from the other states, and similarly the R&D manager can tell the states for which the product development strategy is more effective from the other states. In this environment, a marketing manager who knows that advertising is the more effective strategy has the incentive to convey this information to the R&D manager, who would benefit from such information. The problem is that a marketing manager with the opposite information also has the incentive to mislead the R&D manager into believing otherwise, provided his information indicates that it is sufficiently likely that two strategies are equally effective. When the product development strategy is in fact more effective, we have a situation in which the marketing manager disagrees with the R&D manager based on his own information but would agree with the latter if perfectly informed. This is the kind of environment we study, in which delay can potentially enhance information aggregation and improve welfare. In the above story, if instead the marketing manager can distinguish the states in which the rival strategy is more effective from the other states, and vice versa for the R&D manager, then either the managers immediately agree or they would never do. In either case, there is no information aggregation role for delay.

A similar configuration of preferences and private information arises naturally in other situations. In standard adoption games where competing firms are biased toward their own proprietary standard but would prefer a technologically superior competing standard, there is the same information aggregation difficulty if each firm knows the quality of its own standard but not that of the competitor's. In a recruiting committee, members are often better informed about the quality of the candidates in their own field of expertise and biased toward the same candidates, but they may still want to hire the the most promising one across the field.

In Section 2 we introduce a highly stylized framework of collective decision-making to analyze delay in information aggregation. There is a single conflict state in which the two agents prefer different alternatives, and two equally likely common interest states, one for each alternative, when their preferences coincide. Ex ante, each agent favors a different alternative, and the degree of conflict between the two agents is captured by the prior probability of the conflict state. In each common interest state, the agent who ex ante favors the mutually preferred alternative is perfectly informed, while the other agent is uninformed and knows only that the state is not the other common interest state. In the conflict state, both agents are uninformed and each knows only that his ex ante favorite alternative is not mutually preferred. The information structure and preferences are such that there is no incentive compatible outcome that Pareto dominates a coin flip between the two alternatives when the degree of conflict is high. Intuitively, to make the efficient decision, the informed agent must be given the incentive to persist with his favorite alternative while the uninformed must be discouraged from doing the same. This would be straightforward to accomplish if the two agents could commit to making side transfers between themselves; under the standard quasi-linear assumption on the total payoffs, it would suffice to choose an appropriate transfer from the agent that persists with his favorite alternative to the other agent. In many situations, however, transfer schemes are unrealistic, and it is in these situations that institutionalized delay in making the decision is a natural mechanism to mitigate the incentive problem in strategic information aggregation.

In Section 3 we introduce a game of repeated voting until the two sides agree, with each round of disagreement imposing an additive delay cost on both players. In the unique symmetric equilibrium of the repeated voting game, the informed type votes his ex ante favorite alternative in every round, while the uninformed type may randomize between the two alternatives. Even though the two players can in principle disagree indefinitely, we find that the information of the two sides is perfectly aggregated within a finite number of rounds. Further, uninformed types make increasingly large "concessions" by voting their ex ante favorite with a smaller probability after each round of disagreement, until either an agreement on the mutually preferred alternative is reached, or the negotiation breaks down because the state is revealed to be the conflict state. A decrease in the per-round delay cost causes the uninformed types to be less willing to make concessions, but increases their equilibrium payoff. By taking the limit as the delay cost goes to zero, we show that the ex ante equilibrium payoff of each agent in the repeated voting game is greater than what they would expect from an immediate coin flip when the degree of conflict is moderate.

The repeated voting game assumes that the agents cannot commit to a deadline for making the decision at the ex ante stage before they learn their types. If they can, an interesting "deadline play" by the uninformed emerges: when there are sufficiently few rounds remaining, the uninformed will stop concessions altogether, and persist with probability one until the last round when they compromise by switching to their opponent's favorite alternative. This deadline play has interesting implications. We show that welfare gains for moderate degrees of conflict relative to an immediate coin flip are robust with respect to the presence of a deadline. However, for a fixed deadline, this welfare gain disappears as the per-round delay cost vanishes, because the total possible payoff loss from delay goes to zero under a fixed deadline. If we instead interpret a finite deadline as the two agents committing to a fixed total of payoff loss from delay, so that the number of rounds becomes arbitrary large as the per-round delay cost decreases, then for moderate degrees of conflict an appropriate deadline not only leads to a strict improvement of the ex ante payoff over an immediate coin flip, but also does better than the repeated voting game without a deadline, even when the per-round delay cost goes to zero. The constructive role played by delay in improving the quality of information aggregation and welfare is discussed further in Section 4, where we also comment on how to extend the current framework to analyze the design of negotiation deadlines, and to study environments with more than two agents such as voting in elections.

The repeated voting game in this paper introduces the novel feature of gradual information aggregation to models of repeated bargaining and negotiation. In a pure bargaining model (Stahl 1972; Rubinstein 1982), the trade off between getting a bigger share of the pie but at a later date helps pin down a unique solution to the bargaining problem which is plagued by multiple equilibria in a one-shot model, even though delay does not occur in equilibrium. There are numerous extensions to the Stahl-Rubinstein model that can generate delay as part of the equilibrium outcome.[1] One strand of this literature relies

[1] See, for example, Admati and Perry (1987), Chatterjee and Samuelson (1987), Cho (1990), Cramton (1992), and Kennan and Wilson (1993). There are also bargaining models that generate equilibrium delay

on asymmetric information about the size of the pie that is being divided. In a model of strikes, for example, a firm knows its own profitability but the firm's unionized workforce does not. Strike or delay is a signaling device in the sense that the willingness to endure a longer work stoppage can credibly signal the firm's low profitability and help it to arrive at a more favorable wage bargain. These bargaining models are private-value problems, as each agent's gains from trade at a given price depend only on his own private information. We instead have a common-value problem: disagreement over the alternatives is not a pure bargaining issue, because agents in our model would sometimes agree on which is the best alternative had they known the true state. Put differently, voting outcomes in our setup determine the size as well as the division of the pie. We show that delay can play a constructive role in overcoming disagreements that arise from strategic considerations and improving the ex ante welfare of all agents. Avery and Zemsky (1994) argue that if players are allowed to wait for new information before accepting or rejecting offers, then there is an option value to delay. In our model, no new exogenous information arrives during the voting process. However, the way agents vote provides endogenous information that allows them to update their beliefs and reach better decisions.

Our paper is also related to the literature on debates (Austen-Smith 1990; Austen-Smith and Feddersen 2006; Ottaviani and Sorensen 2001) and voting (Li, Rosen and Suen 2001) in committees. Models of debate typically analyze repeated information transmission as cheap talk, while we emphasize the role of delay cost in multiple rounds of voting.[2] Our setup is the closest to Li, Rosen and Suen (2001). The focus there is on the impossibility of efficient information aggregation. Here, we skirt issues such as quality of private signals

---

through commitment to not accepting offers poorer than past rejected ones (Freshtman and Seidmann 1993; Li 2007), simultaneous offers (Sakovics 1993), multi-lateral negotiations (Cai 2000), and excessive optimism (Yildiz 2004). More closely related to the present paper are recent models of bargaining with interdependent values. See Deneckere and Liang (2006), and Fuchs and Skrzypacz (2010).

[2] Coughlan (2000) investigates conditions under which jurors vote their signals and aggregate their information efficiently in a model where a mistrial leads to a retrial by a new independent jury. He does not consider the issues of delay that are the focus of the present paper. Farrell (1987) introduces a model in which repeated cheap talk helps players coordinate to arrive at a correlated equilibrium of a battle-of-the-sexes game. There is no issue of efficient information aggregation in that model. More recently, in a dynamic cheap talk model with multiple senders and a receiver who may choose to wait, Eso and Fong (2007) show that when the senders are perfectly informed there is an equilibrium with full revelation with no delay. When the senders are imperfectly informed, Eso and Fong establish conditions under which there exist equilibria converging to full revelation with no delay as the noises in the senders' signals disappear.

and the trade-off between making the two different types of errors, and focus instead on how costly delay can help improve the quality of decisions and increase welfare.

2. The Model

Two players, called LEFT and RIGHT, have to make a joint choice between two alternatives, l and r. There are three possible states of the world: L, M, and R. We assume that the prior probability of state L and state R is the same, given by $\pi < 1/2$. The relevant payoffs for the two players are summarized in the following table:

	L	M	R
l	$(1, 1)$	$(1, 1-2\lambda)$	$(1-2\lambda, 1-2\lambda)$
r	$(1-2\lambda, 1-2\lambda)$	$(1-2\lambda, 1)$	$(1, 1)$

In each cell of this table, the first entry is the payoff to LEFT and the second is the payoff to RIGHT. We normalize the payoff from making the preferred decision to 1 and let the payoff from making the less preferred decision be $1 - 2\lambda$. The parameter $\lambda > 0$ is the loss from making the wrong decision relative to a coin toss. In state L both players prefer l to r, and in state R both prefer r to l. The two players' preferences are different when the state is M: LEFT prefers l while RIGHT prefers r. In this model there are elements of both common interest and conflict between these two players. Note that LEFT ex ante favors l, while RIGHT's ex ante favorite alternative is r.

The information structure is such that LEFT is able to distinguish whether the state is L or not, while RIGHT is able to distinguish whether the state is R or not. Such information is private and unverifiable. When LEFT knows that the state is L, or when RIGHT knows that the state is R, we say they are "informed;" otherwise, we say they are "uninformed."[3]

[3] It is not essential for our paper that the informed types are perfectly sure that the state is a common-interest state. The logic of our model remains the same as long as an informed and an uninformed type favor different alternatives on the basis on their private information only, but would recognize a mutually preferred alternative when information is shared. For example, suppose that each player observes a binary signal for or against his ex ante favorite alternative, and prefers his ex ante favorite if and only if there

Without information aggregation, the preference between l and r of an uninformed Left depends on the relative likelihood of state M versus state R. Let γ denote his belief that the state is M, given by

$$\gamma = \frac{1-2\pi}{1-\pi}.$$

We note that γ can be interpreted as the ex ante degree of conflict. When γ is high, an uninformed player perceives that his opponent is likely to have different preferences regarding the correct decision to be chosen.

2.1. An impossibility result

As a useful welfare benchmark, let us consider a single round of simultaneous voting game. Imagine that each player votes l or r, with the agreed alternative implemented immediately and any disagreement leading to an immediate fair coin toss between l and r and a payoff of $1-\lambda$ to each player. It is a dominant strategy for an informed player to vote for his ex ante favorite alternative. For uninformed Left or Right, the optimal strategy depends on the degree of conflict. If $\gamma < 1/2$, then the dominant strategy for the uninformed players is to vote against their favorite decisions. In states L and R, such equilibrium voting strategies lead to the mutually preferred alternative being chosen, while in state M, the decision is determined by flipping a coin, which is again Pareto efficient. In contrast, if $\gamma > 1/2$, then it is a dominant strategy for each uninformed player to vote for his ex ante favorite. The equilibrium outcome is that the two players disagree in every state, and the decision is always determined by flipping a coin, with a payoff of $1-\lambda$.

For future references, denote the uninformed player's probability of voting for his ex ante favorite alternative in the symmetric equilibrium of the benchmark game by $x^0(\gamma)$, which is given by

$$x^0(\gamma) \begin{cases} = 0 & \text{if } \gamma \in [0, 1/2), \\ \in [0,1] & \text{if } \gamma = 1/2, \\ = 1 & \text{if } \gamma \in (1/2, 1] \end{cases} \tag{1}$$

is at least one signal for it. Then, a player who receives a private signal for his ex ante favorite would be similar to an informed type in our setup, while a player who receives a signal against his ex ante favorite would be an uninformed type.

where γ is the belief of the uninformed. Note that when $\gamma = 1/2$, there is a continuum of equilibria, a feature that highlights the discontinuity at $\gamma = 1/2$ in information aggregation in one-round voting.[4] Let $U^0(\gamma)$ be the equilibrium payoff to the uninformed types, given by

$$U^0(\gamma) = \begin{cases} 1 - \lambda\gamma & \text{if } \gamma \in [0, 1/2), \\ \in [1-\lambda, 1-\lambda/2] & \text{if } \gamma = 1/2, \\ 1-\lambda & \text{if } \gamma \in (1/2, 1]. \end{cases} \tag{2}$$

Let $V^0(\gamma)$ be the equilibrium payoff to the informed types, which depends on the belief γ of the uninformed and is given by

$$V^0(\gamma) = \begin{cases} 1 & \text{if } \gamma \in [0, 1/2), \\ \in [1-\lambda, 1] & \text{if } \gamma = 1/2, \\ 1-\lambda & \text{if } \gamma \in (1/2, 1]. \end{cases}$$

Note that $U^0(1) = V^0(1)$ and $U^0(\gamma) \leq V^0(\gamma)$ for all γ.

The result that information aggregation is impossible for $\gamma > 1/2$ is a robust feature of our configuration of information structure and preferences. Indeed, this configuration is intentionally chosen to yield a stronger result that there is no incentive compatible outcome that Pareto dominates a coin toss when $\gamma > 1/2$. To see this, we apply the revelation principle and consider any direct mechanism that satisfies the incentive compatibility constraints for truthful reporting of private information. Since in a truth-telling equilibrium the true state can be recovered from the reports submitted by the two players, we can write q_R, q_M, and q_L as the probabilities of implementing alternative r when the true states are R, M, and L, respectively. Finally, let $\tilde{q}$ be the probability of implementing r when the reports are inconsistent, that is, when both report that they are informed. The incentive constraints for, respectively, the informed Right and the informed Left can be written as:

$$q_R + (1-q_R)(1-2\lambda) \geq q_M + (1-q_M)(1-2\lambda),$$
$$q_L(1-2\lambda) + (1-q_L) \geq q_M(1-2\lambda) + (1-q_M).$$

[4] If each player pays an additional penalty when the game ends in a disagreement, the continuum of equilibria at 1/2 is replaced by an interval of beliefs of the uninformed with each belief corresponding to a unique equilibrium probability of persisting by the uninformed. Our results in this paper can be extended to this case without any qualitative change.

Those for the uninformed RIGHT and the uninformed LEFT are similarly given by:

$$\gamma(q_M + (1 - q_M)(1 - 2\lambda)) + (1 - \gamma)(q_L(1 - 2\lambda) + (1 - q_L))$$
$$\geq \gamma(q_R + (1 - q_R)(1 - 2\lambda)) + (1 - \gamma)(\tilde{q}(1 - 2\lambda) + (1 - \tilde{q})),$$
$$\gamma(q_M(1 - 2\lambda) + (1 - q_M)) + (1 - \gamma)(q_R + (1 - q_R)(1 - 2\lambda))$$
$$\geq \gamma(q_L(1 - 2\lambda) + (1 - q_L)) + (1 - \gamma)(\tilde{q} + (1 - \tilde{q})(1 - 2\lambda)).$$

The first two incentive constraints imply that $q_R \geq q_M$ and $q_M \geq q_L$; the last two imply that $(1 - \gamma)(\tilde{q} - q_L) \geq \gamma(q_R - q_M)$ and $(1 - \gamma)(q_R - \tilde{q}) \geq \gamma(q_M - q_L)$, and thus

$$(1 - \gamma)(q_R - q_L) \geq \gamma(q_R - q_L).$$

The above is inconsistent with $\gamma > 1/2$ unless $q_R - q_L = 0$. It follows that $q_R = q_M = q_L$ when $\gamma > 1/2$ in any incentive compatible outcome.[5] Since the two players are ex ante symmetric, it is natural to focus on the outcome of $q_R = q_M = q_L = 1/2$, which is equivalent to a fair coin toss. The equilibrium payoff of $1 - \lambda$ in the one-round voting game is a natural welfare benchmark for comparison with the games that allow delay in Section 3 when $\gamma > 1/2$.

2.2. Delay mechanisms

The impossibility of information aggregation when $\gamma > 1/2$ assumes that there are no side transfers between the two players. It is straightforward to show that this incentive problem in strategic information aggregation can be overcome if the two players can commit to making side transfers between themselves. Suppose that each player's final payoff is simply the sum of the payoff from the decision and the transfer he receives. Consider a symmetric mechanism where each player chooses between his favorite alternative and the opponent's favorite, and if he chooses the former, he makes a fixed payment of τ to his opponent. If the two players agree then the agreed decision is implemented; otherwise the

[5] This result does not depend on the symmetry assumption that the probabilities of the two agreement states are the same. No information aggregation is possible if both probabilities are less than the prior probability of state M. In the present model of strategic information aggregation, the signal structure of each player is partitional and binary. This feature is responsible for the result that information aggregation is either ex post efficient, or impossible. In a more general model, ex post inefficiency does not necessarily take the form of impossibility of information aggregation. See Li, Rosen and Suen (2001).

decision is made by a coin flip. For this mechanism to implement the efficient decision, we need τ to be large enough such that the uninformed Left weakly prefers to concede to his opponent's favorite alternative:

$$\gamma(1-\lambda)+(1-\gamma)(1+\tau)\geq\gamma(1-\tau)+(1-\gamma)(1-\lambda).$$

At the same time, we need τ to be small enough such that the informed Left weakly prefers to persist with his favorite alternative:

$$1-\tau\geq 1-\lambda.$$

It is easy to see that any τ between $(2\gamma-1)\lambda$ and λ will do.

In situations where side transfers are unrealistic, costly delay is a natural mechanism that plays a similar role of providing the incentive for the informed players to persist with their favorite alternative and at the same time discouraging the uninformed from doing the same. This is one of the main points of the present paper, made through analyzing the specific game of repeated voting until an agreement. To motivate this game, we note that another implicit assumption behind the impossibility of information aggregation is that the choice between the two alternative must be made without delay. It is not difficult to see that efficient information aggregation can be achieved regardless of γ if a sufficiently large payoff loss can be imposed on both players when their reports are inconsistent. Consider a symmetric mechanism where each player chooses between his favorite alternative and the opponent's favorite. If the two players agree then the agreed decision is implemented; if both players concede to their opponent's favorite alternative then the decision is made by a coin flip; and finally, if both players persist with their own favorite then again the decision is made by a coin flip but only after each makes a payment of Δ to a third party. One can alternatively interpret the payment Δ to the third party as representing the payoff loss due to a delay in flipping the coin. This payment should be sufficiently large to make the uninformed Left weakly prefer to concede than to persist:

$$\gamma(1-\lambda)+(1-\gamma)\geq\gamma+(1-\gamma)(1-\lambda-\Delta).$$

Since the informed Left has no incentive to concede regardless of Δ, any value of Δ satisfying the above inequality ensures that the efficient decision is made. Furthermore, in equilibrium the payoff loss Δ is never incurred by the players, so the first best outcome is attained.

One unrealistic feature of the above delay mechanism is that it treats two kinds of disagreement differently: the delay penalty is imposed only after a "regular" disagreement when both players persist with their favorite alternative, but not after a "reverse" disagreement when both players concede to their opponent's favorite. In the situations we are interested in, it is more natural that any disagreement between the two players, regardless of whether it is regular or reverse, leads to delay and payoff loss. We are then led to consider an anonymous, symmetric delay mechanism, which differs from the one just analyzed in that a reverse disagreement also leads to a penalty of Δ on both players and then a coin flip. We claim that Δ can be chosen appropriately so that there is an equilibrium under this anonymous mechanism in which the uninformed players randomize between persisting with their own favorite alternative and conceding to their opponent's alternative, and further, the first best outcome can be approximated when γ is just above 1/2. Let $x \in (0,1)$ be the probability that uninformed RIGHT persists with r. For uninformed LEFT to be indifferent between persisting with l and conceding to r, we need

$$\gamma(x(1-\lambda-\Delta)+(1-x))+(1-\gamma)(1-\lambda-\Delta) = \gamma(x(1-2\lambda)+(1-x)(1-\lambda-\Delta))+(1-\gamma).$$

It is easy to see that for any $\gamma > 1/2$, if $\Delta > (2\gamma-1)\lambda$, the indifference condition implies a unique value for $x \in (0,1)$, given by

$$x = \frac{(2\gamma-1)(\lambda+\Delta)}{2\gamma\Delta}.$$

Note this indifference condition implies that informed LEFT strictly prefers persisting to conceding:

$$x(1-\lambda-\Delta)+(1-x) > x(1-2\lambda)+(1-x)(1-\lambda-\Delta).$$

This verifies that for any $\Delta > (2\gamma-1)\lambda$, it is an equilibrium for the informed players to persist with favorite alternative and for the uninformed players to persist with probability x determined by the indifference condition and concede with probability $1-x$. Further, since the lower bound on Δ and the value of x both go to 0 as γ goes to 1/2, the equilibrium outcome approaches the first best.[6]

Although the above delay mechanisms perform quite well in overcoming or mitigating the incentive problem in information aggregation, they rely on the critical assumption that the players can commit to paying the delay penalty Δ in whole. In the applications we are interested in, the cost of delay reflects the time and expenses of setting up another round of meeting and negotiations, and as such, is at least to some extent divisible. Then, the assumption that in the event of a disagreement the players can commit to paying the delay penalty in whole amounts to the commitment to not renegotiating with each other before the coin flip. In our view, it is unrealistic to assume that players can have this much commitment power. More importantly, by moving away from such assumption, we are able to study the role of delay in strategic information aggregation in a natural dynamic environment. In the next section, we first consider the game of repeated voting until agreement, with each player incurring an additive cost of delay $\delta > 0$ between two adjacent rounds of voting.[7] In the equilibrium analysis we hold δ as fixed; thus, even though the two players cannot commit to not voting again in the next round, they are prevented from starting the renegotiation immediately after casting each round of votes. In the comparative statics analysis, we consider the effects of δ becoming arbitrarily small. This has the interpretation of assuming that the two players cannot commit to not renegotiating very frequently. Finally, we introduce a game of repeated voting with a deadline. The interpretation is that the players can commit to ending the process with a coin flip within a maximum number of rounds, even though they cannot commit to not renegotiating in the next round after a disagreement before the deadline.

---

[6] For any $\gamma > 1/2$, we can solve for the value of Δ that maximizes the ex ante payoff of each player (before he knows his type). This corresponds to the "optimal" anonymous mechanism in our model. The solution is $\Delta_* = \lambda\sqrt{(2\gamma - 1)/(1 - 2(1 - \gamma)^2)}$ for $\gamma \in (1/2, (9 - \sqrt{17})/8)$, and $\Delta_* = 0$ otherwise. Note that the latter case corresponds to making the decision with a coin flip without delay.

[7] An alternative way to model delay cost is to apply a multiplicative discount factor to the payoffs if the decision is implemented in the next round. In this case, delaying a preferred decision is more costly than delaying an inferior decision. Consequently the analysis of the discounting case is slightly more cumbersome than the fixed cost case. However, the basic insights of this paper do not depend on which of these two assumptions is used.

3. Repeated Voting

We refer to LEFT voting his ex ante favorite alternative l, and RIGHT voting r, as "persisting," and refer to the opposite as "conceding." In each voting round, disagreement can be either regular or reverse. When delay cost is large, the inferior alternative can be better than the preferred alternative with delay. In that case, even an informed RIGHT would prefer to vote l if he knows that LEFT will vote l. The strategic situation is analogous to a "battle-of-the-sexes" game and the main economic issue is that of coordination to one of the two asymmetric outcomes (all voting for l or all voting for r) to avoid the large delay cost. Clearly these two outcomes cannot be Pareto ranked. Since our main concern in this paper is to study whether and how delay can improve the payoff of both players by making information aggregation more efficient, we focus on equilibria in which the strategies of RIGHT and LEFT, for both informed and uninformed types, are symmetric to each other, and the informed types always persist in each round. The notion of equilibrium we use in the ensuing analysis is perfect Bayesian equilibrium.

3.1. Equilibrium construction and characterization

Since the game is symmetric and since the uninformed types vote for their ex ante favorites with the same probability on the equilibrium path, they have the same belief about the state being the conflict state after any observed sequence of disagreeing votes. For the equilibrium constructed below, it is sufficient to consider equilibrium play when the uninformed types hold the same beliefs. For each such common belief $\gamma \in [0, 1]$ that the uninformed types hold regarding the conflict state M, we denote by $x(\gamma) \in [0, 1]$ the equilibrium probability that the uninformed types persist. Let $U(\gamma)$ and $V(\gamma)$ be the equilibrium expected payoffs of the uninformed and informed types respectively.[8]

To construct an equilibrium, first we identify an equilibrium play when the uninformed players believe that the state is M with probability 1, in which they persist with probability

[8] We have implicitly restricted to stationary strategies that depend only on the belief. There are no non-stationary symmetric equilibria in which the informed types always persist. In particular, it cannot be an equilibrium in which the uninformed types coordinate to persist in some given time period, followed by randomizing between persisting and conceding. This follows because each uninformed type would have unilateral incentive to concede in the time period when both uninformed types are supposed to persist.

$x(1)$. It follows from the indifference condition for the uninformed types that

$$U(1) = x(1)(-\delta + U(1)) + (1 - x(1)) = x(1)(1 - 2\lambda) + (1 - x(1))(-\delta + U(1)).$$

Solving these two equations gives a unique pair of equilibrium values

$$U(1) = 1 - \lambda - \sqrt{\delta^2 + \lambda^2}; \quad x(1) = \frac{-\delta + \lambda + \sqrt{\delta^2 + \lambda^2}}{2\lambda}. \tag{3}$$

We note that $x(1) \in (1/2, 1)$ and $U(1) < 1 - 2\lambda$.

Next, we identify an equilibrium play when $\gamma = 0$. Since the uninformed RIGHT believes that the state is L and his opponent (who is informed) votes l, voting l to obtain the preferred decision is strictly better than voting r. Thus, we have $x(0) = 0$ and $U(0) = 1$. Given this, we claim that it is an equilibrium when γ is positive but sufficiently small for the uninformed types to concede with probability 1. To see this, note that $x(\gamma) = 0$ implies that the updated belief upon a regular disagreement is $\gamma' = 0$. Therefore, the payoff to the uninformed RIGHT from voting r is $\gamma + (1-\gamma)(-\delta + U(0))$, and his payoff from voting l is $\gamma(-\delta + U(1)) + (1 - \gamma)$. Conceding is strictly preferred to persisting if and only if

$$\gamma < \frac{\delta}{(1 + \delta - U(1)) + \delta} \equiv G_1. \tag{4}$$

The corresponding equilibrium payoff of the uninformed types takes the linear form of

$$U(\gamma) = 1 - (1 + \delta - U(1))\gamma. \tag{5}$$

We refer to the interval $[0, G_1]$ as the "conceding region."

For γ just above G_1, we construct an equilibrium in which $x(\gamma)$ is such that the one-step updated belief γ' falls into the conceding region. We will then try to identify a one-step interval $[G_1, G_2]$, and so on. That is, there exists an infinite sequence, $G_0 < G_1 < G_2 < \ldots$, with $G_0 = 0$ and $\lim_{k\to\infty} G_k = 1$, such that if $\gamma \in (G_k, G_{k+1}]$ for $k = 1, 2, \ldots$, then $x(\gamma) \in (0, 1)$ is such that the updated belief after a regular disagreement satisfies

$$\gamma' = \frac{\gamma x(\gamma)}{\gamma x(\gamma) + 1 - \gamma} \in (G_{k-1}, G_k].$$

Furthermore, the payoff function for the uninformed is piecewise linear of the form

$$U(\gamma) = a_k - b_k\gamma \tag{6}$$

for $\gamma \in (G_k, G_{k+1}]$, with $a_0 = 1$ and $b_0 = 1 + \delta - U(1)$ from equation (5). We construct the sequences of $\{G_k\}$ and $\{(a_k, b_k)\}$ recursively, starting from G_1 and (a_0, b_0).

Fix any $\gamma \in (G_k, G_{k+1}]$ for $k \geq 1$. Assuming that the continuation payoff is given by equation (6), the expected payoff to the uninformed from persisting is

$$(\gamma x + 1 - \gamma)(-\delta + a_{k-1} - b_{k-1}\gamma') + \gamma(1-x) = (\gamma x + 1 - \gamma)(-\delta + a_{k-1}) - \gamma x b_{k-1} + \gamma(1-x).$$

The payoff from conceding is

$$\gamma(x(1-2\lambda) + (1-x)(-\delta + U(1))) + (1-\gamma).$$

The uninformed is indifferent between persisting and conceding when x is given by

$$x(\gamma) = \frac{\gamma b_0 - (1-\gamma)(1+\delta - a_{k-1})}{\gamma(b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda)}. \tag{7}$$

Using Bayes' rule

$$\frac{G_{k+1}x(G_{k+1})}{G_{k+1}x(G_{k+1}) + 1 - G_{k+1}} = G_k,$$

with $x(G_{k+1})$ given in equation (7), we can define G_{k+1} as follows:

$$G_{k+1} = \frac{1 + \delta - a_{k-1} + G_k(b_0 + b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)}. \tag{8}$$

Note that $x(\gamma)$ is increasing in γ from equation (7), implying that the updated belief γ' after a regular disagreement falls in the interval $(G_{k-1}, G_k]$. Finally, substituting equation (7) into the expression for the payoff from voting r, we can verify that $U(\gamma)$ is indeed piece-wise linear of the form given in equation (6), where

$$a_k = 1 - \frac{(1+\delta - a_{k-1})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda}; \quad b_k = 2\lambda + \frac{(b_0 - 2\lambda)(b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda}. \tag{9}$$

The above is a pair of difference equations for the sequence $\{(a_k, b_k)\}$. We have the following preliminary results regarding the sequences $\{G_k\}$ and $\{(a_k, b_k)\}$. The proof is in the appendix.

LEMMA 1. *(i) $a_k \leq 1$ and $b_k > 2\lambda$ for all k; (ii) both a_k and b_k are decreasing in k; (iii) $\lim_{k\to\infty} a_k$ exists and is given by $a_\infty = 1 + \lambda - \sqrt{\delta^2 + \lambda^2}$, and $\lim_{k\to\infty} b_k$ exists and is $b_\infty = 2\lambda$; (iv) $0 < G_k < G_{k+1} < 1$ for all $k \geq 1$; and (v) $\lim_{k\to\infty} G_k = 1$.*

The above piece-wise construction of $x(\gamma)$ and $U(\gamma)$ ensures that the strategy of the uninformed types is consistent with equilibrium. It remains to verify that the informed types have no incentive to deviate by voting against their ex ante favorite alternatives. This is established below by showing that given the equilibrium strategy of the uninformed types, the informed types have stronger incentives than the uninformed types to vote for their ex ante favorite alternative.

PROPOSITION 1. *There exists an equilibrium of the repeated voting game in which the strategy of the uninformed types is given by $x(\gamma)$ and their payoff is given by $U(\gamma)$.*

PROOF. First, for $\gamma = 1$, since his opponent is persisting with probability $x(1)$, the informed type is indifferent between persisting and conceding, and his equilibrium payoff is $V(1) = U(1)$.

Next, for $\gamma \in [G_0, G_1]$, since his opponent is conceding with probability 1, the payoff for the informed type from persisting is 1, while his payoff from conceding is $-\delta + V(1) < 1$, implying $V(\gamma) = 1 \geq U(\gamma)$, with equality only if $\gamma = 0$.

Finally, for $\gamma \in (G_1, 1)$, we first establish by induction that $V(\gamma) > U(\gamma)$ for all $\gamma < 1$, as follows. Consider any $\gamma \in [G_k, G_{k+1}]$ and $k \geq 1$, with $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma) \in (G_{k-1}, G_k]$. We obtain

$$\begin{aligned} V(\gamma) &> (\gamma x(\gamma) + 1 - \gamma)(-\delta + V(\gamma')) + \gamma(1 - x(\gamma)) \\ &> (\gamma x(\gamma) + 1 - \gamma)(-\delta + U(\gamma')) + \gamma(1 - x(\gamma)) = U(\gamma), \end{aligned}$$

where the first inequality follows from the fact that $x(\gamma) < \gamma x(\gamma) + 1 - \gamma$, the second inequality follows from the induction hypothesis, and the last equality follows because the uninformed types are indifferent between persisting and conceding for $\gamma \in [G_k, G_{k+1}]$ for $k \geq 1$. Moreover, from the indifferent condition of the uninformed types we obtain

$$\gamma \left[x(\gamma)(-\delta + U(\gamma') - 1 + 2\lambda) + (1 - x(\gamma))(1 + \delta - U(1)) \right] + (1 - \gamma)(-\delta + U(\gamma') - 1) = 0.$$

Note that the last term is strictly negative, and so the expression in the square bracket is strictly positive. Since $V(1) = U(1)$, and $V(\gamma') > U(\gamma')$, this implies that

$$x(\gamma)(-\delta + V(\gamma')) + 1 - x(\gamma) > x(\gamma)(1 - 2\lambda) + (1 - x(\gamma))(-\delta + V(1)).$$

The left-hand-side of the above inequality is the equilibrium payoff for the informed type from persisting. The right-hand-side is the deviation payoff from conceding, because after a reverse disagreement the uninformed types are convinced that the state is M. Thus, the informed type strictly prefers persisting to conceding. *Q.E.D.*

The equilibrium represented by equations (7) and (6) is continuous and monotone with respect to the degree of conflict γ. Note that the continuity of $x(\gamma)$ in γ is not required for the construction to be an equilibrium. Nor it is automatic from the construction, because the equilibrium strategy to the left and inclusive of $\gamma = G_k$ is constructed in the interval $(G_{k-1}, G_k]$ while $x(\gamma)$ just to the right of G_k is separately constructed in the next step of $[G_k, G_{k+1})$. The continuity and monotonicity of $x(\gamma)$ is indirectly established below by showing that the payoff function U is continuous. See the appendix.

PROPOSITION 2. *The equilibrium strategy $x(\gamma)$ in the repeated voting game is continuous and increasing for all $\gamma \in [0, 1]$.*

The monotonicity result of Proposition 2 provides an intuitive description of the equilibrium behavior. In each round of voting, there are four possible outcomes: an immediate agreement on r, an immediate agreement on l, a regular disagreement, or a reverse disagreement. We interpret a reverse disagreement as a breakdown of the negotiation process. Once a reverse disagreement occurs, it is revealed that what is a good decision for one player is necessarily an inferior decision for the other player. The continuation game is a version of a war of attrition game, where each uninformed player chooses the stationary strategy represented by $x(1)$ until they reach a decision.[9] Upon a regular disagreement, on the other hand, the uninformed player becomes more convinced that he is playing against an informed type. The informed type continues to vote for his favorite alternative, but the uninformed player will "soften" his position as $x(\gamma') < x(\gamma)$. In a sense, the negotiation between the two players is making progress, because the probability of choosing the mutually preferred alternative rises if the state is L or R. Moreover, for any γ not arbitrarily

[9] In our version of the war of attrition game, "stopping" corresponds to voting against one's ex ante favorite alternative. Unlike the standard war of attrition game, when both players vote against their favorite, we have a reverse disagreement and the game continues.

close to 1, it only takes a finite number of rounds of regular disagreement before the uninformed player yields to his opponent completely by switching to voting against his ex ante favorite (i.e., $x(\gamma) = 0$), provided there is no breakdown of negotiation before that. Once the game reaches this conceding region, there is either an agreement on the mutually preferred alternative, or the negotiation breaks down and the two uninformed players engage in a war of attrition by adopting the strategy of voting for his ex ante favorite alternative with probability $x(1)$.

The equilibrium in Proposition 1 is unique. That is, if there is a function $y(\gamma)$ defined on $\gamma \in [0, 1]$ such that it is an equilibrium for the uninformed types with belief γ to persist with probability $y(\gamma)$, then $y(\gamma) = x(\gamma)$ for all $\gamma \in [0, 1]$. To see this, first note that the argument leading to (3) establishes that $y(1) = x(1)$ in any equilibrium. Second, we argue that in any equilibrium $y(\gamma) = 0$ for any $\gamma \in [0, G_1]$. This follows because regardless of the continuation plays, when γ is sufficiently small, the payoff to the uninformed from voting to persist is strictly lower than the payoff from voting to concede regardless of the strategy of the opposing uninformed type. Third, for any $\gamma > G_1$, if the equilibrium $y(\gamma)$ is such that there is a unique continuation value $U(\gamma')$ as given by Proposition 1, then $y(\gamma)$ equals $x(\gamma)$ because the latter is the only value that simultaneously satisfies the equilibrium indifference condition of the uninformed types and Bayes' rule. Finally, because $y(\gamma) = 1$ is never part of equilibrium strategy, in any equilibrium the uninformed types are indifferent between conceding and persisting whenever their belief is strictly higher than G_1. It follows that in any equilibrium $y(\gamma)$ is bounded away from 1. Then the second claim and third claim above imply that $y(\gamma) = x(\gamma)$ for all γ.

3.2. Equilibrium welfare and comparative statics

To analyze the welfare properties of the equilibrium constructed in the repeated voting game, we first derive the payoff function of the informed types. Recall that for any belief γ of the uninformed, $V(\gamma)$ is the equilibrium expected payoff of the informed types. Given the equilibrium strategy $x(\gamma)$ of the uninformed, $V(\gamma)$ satisfies the following recursive formula:

$$V(\gamma) = x(\gamma)(V(\gamma') - \delta) + 1 - x(\gamma), \tag{10}$$

where $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma)$ is the updated belief of the uninformed after a regular disagreement. Using the characterization of $x(\gamma)$ in Proposition 1, we have the following result about V.

LEMMA 2. *There exists a sequence* $\{(c_k, d_k)\}$, *with* $c_k \leq 1$ *decreasing in* k *and* $\lim_{k\to\infty} c_k = U(1)$, *and* $d_k \geq 0$ *increasing in* k *and* $\lim_{k\to\infty} d_k(1 - G_k)/G_k = 0$, *such that*

$$V(\gamma) = c_k + d_k \frac{1-\gamma}{\gamma} \tag{11}$$

for any $\gamma \in (G_k, G_{k+1}]$, $k \geq 1$.

The proof of the lemma is in the appendix, where we establish the following system of difference equations for $\{(c_k, d_k)\}$:

$$c_k = 1 - \frac{b_0(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}; \; d_k = d_{k-1} + \frac{(1+\delta-a_{k-1})(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}. \tag{12}$$

By the proof of Proposition 2, the payoff function $V(\gamma)$ is continuous for all $\gamma \in [0,1]$, as is $U(\gamma)$. However, while $U(\gamma)$ is decreasing and piece-wise linear in γ, and is convex because b_k decreases with k, the payoff function $V(\gamma)$ is piece-wise convex but since d_k is increasing in k, at each kink G_k, the left derivative is smaller than the right derivative. Further, from the proof of Proposition 1 we know that the two payoff functions satisfy $V(\gamma) \geq U(\gamma)$ for all $\gamma \in [0,1]$, with equality only at $\gamma = 0$ and $\gamma = 1$.[10]

The equilibrium welfare of the informed and uninformed types depend on the delay cost δ. In the following proposition, we establish that as δ decreases, the conceding region becomes smaller; further, the equilibrium voting strategy by the uninformed types becomes tougher for any degree of conflict. Correspondingly, for any initial degree of conflict, as δ decreases, it takes a greater number of regular disagreements to reach the conceding region. However, in spite of the tougher positions taken by the uninformed types, their equilibrium expected payoffs increase unambiguously because the direct impact of a lower cost of delay per-round dominates. The proof is in the appendix.

[10] While the limit of d_k as k goes to infinity does not exist, the product $d_k(1-\gamma)/\gamma$ converges to 0 because γ goes to 1 as k grows arbitrarily large, which is why $V(1) = U(1)$.

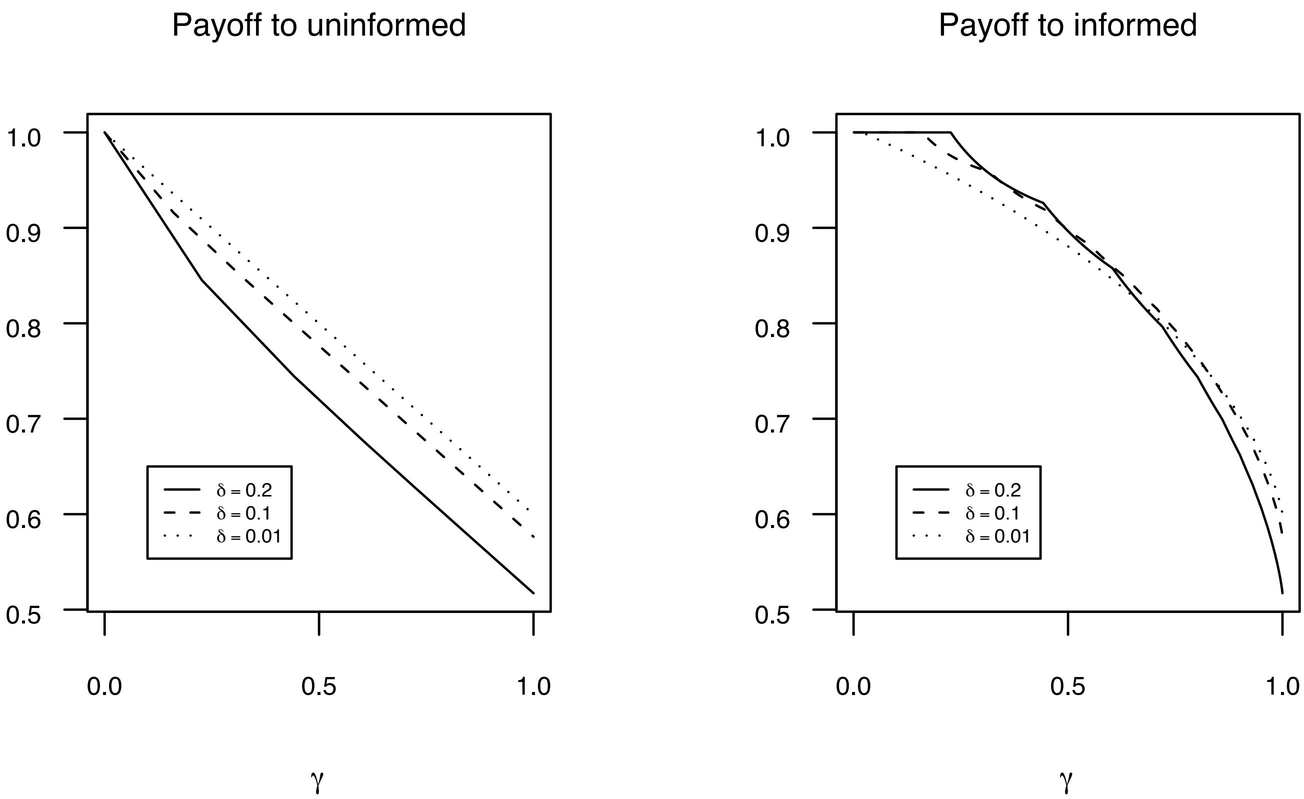

Figure 1

PROPOSITION 3. *In the symmetric equilibrium of the repeated voting game, as δ decreases, G_k strictly decreases for each $k \geq 1$, $x(\gamma)$ strictly increases for all $\gamma \in (G_1, 1]$, and $U(\gamma)$ strictly increases for all $\gamma \in (0, 1]$.*

For the informed types, the effect of a decrease in the delay cost δ turns out to be generally ambiguous. The uninformed types toughen their positions, which means longer delays before the mutually preferred alternative is chosen, but each round of disagreement is less costly. The difference equations (9) and (12) allow an explicit calculation of the payoff functions of the informed and the uninformed. In Figure 1, we plot these functions for $\delta = 0.2$, $\delta = 0.1$, and $\delta = 0.01$ (holding λ fixed at 0.2). Consistent with Proposition 3, we see that $U(\gamma)$ is decreasing in δ for any γ. However, $V(\gamma)$ is not monotone in δ.

We now compare the equilibrium payoffs of the uninformed and informed types with their corresponding benchmarks when there is no possibility of delay. For the uninformed, it is immediate that there is no possibility of welfare gain relative to the benchmark of one-round voting regardless of the degree of conflict γ or the delay cost δ. This follows from (2), and because $U(\gamma)$ is a decreasing function, with a slope b_k that is strictly larger than 2λ in absolute value by Lemma 1, implying that

$$U(\gamma) \leq 1 - 2\lambda\gamma \leq U^0(\gamma).$$

On the other hand, Figure 1 shows that welfare gains are possible for the informed types. In this figure, $V(1/2)$ is greater than $1 - \lambda = 0.8$ for the various values of δ indicated.

The ex ante payoff function before the realization of informed or uninformed types is given by

$$W(\gamma) = \frac{1}{2-\gamma} U(\gamma) + \frac{1-\gamma}{2-\gamma} V(\gamma).$$

A simple characterization of the condition for $W(\gamma)$ to exceed the benchmark welfare $W^0(\gamma)$ in one-round voting is not easy because the payoff functions $U(\gamma)$ and $V(\gamma)$ have infinitely many kink points at $G_1, G_2, \ldots$. However, Figure 1 suggests that U and V approach smoothly differentiable functions as δ becomes small. We therefore undertake to study the welfare comparison through the limiting case of arbitrarily small delay cost. The limit functions $\lim_{\delta\to 0} U(\gamma)$ and $\lim_{\delta\to 0} V(\gamma)$ are well-defined because the equilibrium given in Proposition 1 is continuous in δ.

Proposition 4. *There exists a threshold value* $\overline{\gamma} \in (1/2, 1)$ *such that for any initial belief* $\gamma \in (1/2, \overline{\gamma})$, $W(\gamma) > W^0(\gamma)$ *when the delay cost* δ *is sufficiently small.*

Proof. From equation (3), we obtain $\lim_{\delta\to 0} U(1) = 1 - 2\lambda$, implying that $\lim_{\delta\to 0} b_0 = 2\lambda$. It follows from the difference equations (9) that $\lim_{\delta\to 0} a_k = 1$ and $\lim_{\delta\to 0} b_k = 2\lambda$ for any k. It is then straightforward to show from (4) that $\lim_{\delta\to 0} G_1 = 0$, and from (8) by induction that, for any $k \geq 1$,

$$\lim_{\delta\to 0}(G_{k+1} - G_k) = 0.$$

For any $\delta > 0$ and $\gamma \in (0, 1)$, let $\kappa(\delta; \gamma)$ represent the smallest integer k such that $G_k \geq \gamma$. Since the U function is decreasing, the value of $U(\gamma)$ is bounded above by $a_{\kappa(\delta;\gamma)} - b_{\kappa(\delta;\gamma)} G_{\kappa(\delta;\gamma)-1}$, and is bounded below by $a_{\kappa(\delta;\gamma)} - b_{\kappa(\delta;\gamma)} G_{\kappa(\delta;\gamma)}$. Since $\lim_{\delta\to 0} a_{\kappa(\delta;\gamma)} = 1$ and $\lim_{\delta\to 0} b_{\kappa(\delta;\gamma)} = 2\lambda$, and since $\lim_{\delta\to 0} G_{\kappa(\delta;\gamma)} = \lim_{\delta\to 0} G_{\kappa(\delta;\gamma)-1} = \gamma$, the upper bound and the lower bound converge to the same limiting value of

$$\lim_{\delta\to 0} U(\gamma) = 1 - 2\lambda\gamma.$$

For the informed types, we note from the difference equation (12) for $\{c_k\}$ and the limit values of a_k and b_k that $\lim_{\delta\to 0} c_k = 1-2\lambda$ for any k. To calculate the limit value of $d_{\kappa(\delta;\gamma)}$, we write

$$d_{\kappa(\delta;\gamma)} = d_1 + \sum_{k=2}^{\kappa(\delta;\gamma)} \frac{d_k - k_{k-1}}{G_k - G_{k-1}}(G_k - G_{k-1}).$$

Letting $v_k = 1+\delta-a_k$ and $s_k = b_k - 2\lambda$, and using the difference equations (12) and (8), this can be expresses as:

$$d_{\kappa(\delta;\gamma)} = d_1 + \sum_{k=2}^{\kappa(\delta;\gamma)} \frac{1+\delta-c_{k-1}}{1-G_{k-1}} \frac{b_0+v_{k-2}+G_{k-1}s_{k-2}}{b_0+v_{k-1}+s_{k-1}} \frac{v_{k-1}}{v_{k-2}+G_{k-1}s_{k-2}}(G_k - G_{k-1}).$$

As δ goes to 0, the first term in the summand goes to $2\lambda/(1-G_{k-1})$, the second term goes to 1, and the third term goes to 1 (for any $k \geq 2$, the limit of v_{k-2}/v_{k-1} is 1 and the limit of s_{k-2}/v_{k-1} is 0 as δ goes to 0). Moreover, d_1 goes to 0 as δ goes to 0. Therefore

$$\lim_{\delta\to 0} d_{\kappa(\delta;\gamma)} = \lim_{\delta\to 0} \sum_{k=2}^{\kappa(\delta;\gamma)} \frac{2\lambda}{1-G_{k-1}}(G_k - G_{k-1}) = \int_0^\gamma \frac{2\lambda}{1-G}\mathrm{d}G = -2\lambda\ln(1-\gamma),$$

where the second equality uses the definition of the Riemann integral. Since $V(\gamma)$ is bounded above by $V(G_{\kappa(\delta;\gamma)-1})$ and is bounded below by $V(G_{\kappa(\delta;\gamma)})$, and these two bounds converge to the same limiting value as δ goes to 0, we have

$$\lim_{\delta\to 0} V(\gamma) = 1-2\lambda-2\lambda\frac{1-\gamma}{\gamma}\ln(1-\gamma).$$

It follows immediately from the limiting payoff functions that

$$\lim_{\delta\to 0} W(1/2) = \frac{2}{3}(1-\lambda)+\frac{1}{3}\left(1-2\lambda(1-\ln 2)\right) > 1-\lambda.$$

Moreover, the limit of $\lim_{\delta\to 0} W(\gamma)$ as γ goes to 1 is $1-2\lambda$, which is strictly less than $1-\lambda$. Since the limit function $\lim_{\delta\to 0} W(\gamma)$ is decreasing in γ, there exists a $\overline{\gamma} \in (1/2,1)$ such that $\lim_{\delta\to 0} W(\gamma) > W^0(\gamma)$ if and only if $\gamma \in (1/2,\overline{\gamma})$. The proposition follows by continuity of the ex ante welfare function in the delay cost δ. *Q.E.D.*

Proposition 3 establishes that $U(\gamma)$ increases as δ decreases. When δ goes to 0, $U(1/2)$ approaches a limiting value of $1-\lambda$, implying that the uninformed types get the same payoff in equilibrium as the benchmark payoff when $\gamma = 1/2$. We have shown in the proof of Proposition 1 that the payoff to the informed is strictly higher than the payoff to the uninformed for any $\gamma \in (0, 1)$. This already suggests that the informed types will be better off in the repeated voting game than in the benchmark one-round voting game when $\gamma = 1/2$ and δ is small. Proposition 4 makes this precise by deriving an explicit characterization of the limit functions $\lim_{\delta \to 0} U(\gamma)$ and $\lim_{\delta \to 0} V(\gamma)$, ensuring that $V(\gamma) - U(\gamma)$ does not go to 0 as δ goes to 0.

In this repeated voting game, as in the case of the two-round voting game, ex ante welfare gains relative to the benchmark one-round voting game exist only for values of γ close enough to and greater than 1/2. If the degree of conflict γ is too large, since $V(1) = U(1) < 1 - \lambda$, the continuity of V implies that $V(\gamma)$ is smaller than the benchmark expected payoff of the informed types for γ close to 1. Also, as in the case of the two-round voting game, the delay cost δ cannot be too great for ex ante welfare gains to exist. From equations (4) and (8) we can verify that for δ sufficiently great, $G_1 < 1/2 < G_2$, and then from (7) we can verify that $x(1/2)$ is bounded away from 0 for sufficiently great δ, implying that $V(1/2)$ falls below the benchmark payoff of $1 - \lambda$ if δ is sufficiently great.

3.3. Finite deadlines

We have motivated the game of repeated voting until agreement by assuming that the two players cannot commit to not renegotiating after any disagreement. There are in fact two aspects of this assumption: first, after each disagreement and after the two players each incur the payoff loss δ, the two players cannot commit to not voting again in the next round; and second, at the ex ante stage before the two players learn their types and start playing the game, they cannot commit to ending the negotiation within a fixed number of rounds. We now discuss the implications of relaxing just the second aspect of the no-commitment assumption, by studying a repeated voting game with a finite deadline. We model a deadline as a maximum number, T, of voting rounds and assume that after T rounds of disagreement the game ends with the decision determined by a coin toss. Formally, any history of T rounds of disagreement is now a terminal history with the payoff to the two players being $1 - \lambda - \delta T$, and all other terminal histories are unchanged.[11]

[11] The benchmark single-round voting game corresponds to $T = 1$.

A full characterization of the equilibrium for any deadline T, any delay cost δ and any initial belief γ of the uninformed is beyond the scope of the present paper. The main difficulty arises from the non-stationary nature of the equilibrium play of the uninformed types. Unlike the game of repeated voting until agreement, the equilibrium play of the uninformed depends both on his belief that the state is M and on the number of rounds remaining until the deadline.[12] Our modest goal in this subsection is to provide a partial equilibrium characterization and use it to further illustrate the constructive role of costly delay in strategic information aggregation.

We construct an equilibrium of the finite deadline game when the initial belief of the uninformed is just above 1/2. As in the game without deadline, the informed types persist in each round of voting. The uninformed types randomize between persisting and conceding in the first and the last round of voting and, unlike the case with no deadline, persist with probability 1 for each round in between. The main departure from the game without deadline is that, when there are sufficiently few rounds remaining, it is optimal for the uninformed to stop concessions altogether until the last round. This equilibrium construction requires that the deadline line is not too long and the belief of the uninformed types is sufficiently close to 1/2. Before presenting this equilibrium characterization result, we establish in the following lemma that when the initial belief of the uninformed is exactly 1/2, and the deadline is sufficiently short, there is an equilibrium in which the uninformed persist with probability 1 until the last round. Let T_* be the largest integer smaller than or equal to $\lambda/(2\delta)$.

LEMMA 3. *Suppose that $\delta \in (0, \lambda/2)$. In the repeated voting game with any deadline T satisfying $2 \leq T \leq T_*$ and initial belief $\gamma = 1/2$ of the uninformed types, there is an equilibrium in which the uninformed types persist with probability 1 for $T-1$ rounds and persist with probability smaller than or equal to $1 - 2(T-1)\delta/\lambda$ in the last round.*

PROOF. First recall from equation (1) that with the belief γ equal to 1/2 the uninformed types are indifferent between persisting and conceding in the final round. Next, when

[12] This is because the continuation equilibrium after a reverse disagreement depends on the number of the rounds remaining. Such issue does not arise in a continuous-time model because reverse disagreements are irrelevant. See Damiano, Li and Suen (2009) for further detail.

there are $T' \leq T$ rounds remaining, the expected payoff of the uninformed types under the proposed equilibrium is

$$-(T'-1)\delta + \gamma(x^0(1-\lambda) + (1-x^0)) + (1-\gamma)(1-\lambda),$$

where x^0 is the probability of the uninformed persisting in the last round, satisfying $x^0 \leq 1 - 2(T-1)\delta/\lambda$. By construction, with $\gamma = 1/2$, the above reaches the minimum value of $1-\lambda$ when $T' = T$ and $x^0 = 1-2(T-1)\delta/\lambda$. Since $\gamma = 1/2$ and the opposing uninformed type persists with probability 1, the deviation payoff from conceding when there are T' rounds remaining is $1-\lambda$. This verifies the equilibrium condition for the uninformed types. For the informed, the equilibrium payoff when there are T' rounds left is

$$-(T'-1)\delta + x^0(1-\lambda) + (1-x^0),$$

which is always strictly greater than the equilibrium payoff of the uninformed types. We have just shown that the latter is at least $1-\lambda$, which is greater than the deviation payoff of $1-2\lambda$ from conceding for the informed. *Q.E.D.*

The intuition behind the deadline play by the uninformed types is that when there are few rounds left, the deadline payoff from playing an equilibrium of the benchmark one-round voting game becomes too attractive for the unformed to contemplate conceding.[13] The deadline play has the same role as the conceding region of $[0, G_1]$ in the repeated voting game without deadline, and can be now used to construct an equilibrium for initial belief γ just above 1/2. In this equilibrium, the uniformed types concede with a positive probability in the first around so that the updated belief is 1/2. The equilibrium play then continues as described in Lemma 3, with a selection of the uninformed conceding probability x^0 in the final round to make them indifferent in the first round. Define

$$\gamma_* = \frac{(T_*+1)\delta + \lambda}{2(T_*\delta + \lambda)},$$

[13] Although the equilibrium constructed in Lemma 3 is not unique, it can be shown that in any equilibrium the uninformed types must persist for the initial $T-1$ rounds. Furthermore, the equilibrium constructed in Proposition 5 below is unique subject to the informed types always persisting. See Damiano, Li and Suen (2009) for details.

which is strictly between 1/2 and 1. We establish the following result in the Appendix.

PROPOSITION 5. *Suppose that $\delta \in (0, \lambda/2)$. In the repeated voting game with any deadline T satisfying $2 \leq T \leq T_*$ and any initial belief $\gamma \in [1/2, \gamma_*]$, there is an equilibrium in which the uninformed types persist with probability $x = (1-\gamma)/\gamma$ in the first round.*

The first observation we can make from Proposition 5 is that welfare gains for moderate degrees of conflict are possible also in the presence of a deadline. To see this, fix some $\delta < \lambda/2$, a deadline $T \leq T_*$ and an initial belief of the uninformed arbitrarily close to 1/2. The equilibrium payoff of the uninformed types, given by the expected payoff from conceding[14]

$$2(1-\gamma)(1-\lambda) + (2\gamma - 1)(-(T-1)\delta + (1-\lambda)), \tag{13}$$

is arbitrarily close to $1-\lambda$, which is the equilibrium payoff $U^0(\gamma)$ in the benchmark one-round voting game for any $\gamma > 1/2$. As γ becomes close to 1/2, the probability x of the uninformed persisting in the first round approaches 1, while the probability x^0 of the uninformed persisting in the last round approaches $1-2(T-1)\delta/\lambda$. Thus, the equilibrium payoff of the informed types converges to

$$-(T-1)\delta + (1-x^0) + x^0(1-\lambda) = 1-\lambda+(T-1)\delta, \tag{14}$$

which is strictly greater than the informed equilibrium payoff $V^0(\gamma) = 1-\lambda$ in the benchmark one-round voting game for any $\gamma > 1/2$.

A second observation is that the welfare gain illustrated above disappears as the per-round delay cost δ vanishes. More precisely, as δ goes to 0, the upper-bound γ_* on the initial belief of the uninformed in our equilibrium construction converges to 1/2. For any fixed deadline $T \leq T_*$, by (13), the equilibrium payoff of the uninformed types converges to $1-\lambda$ as δ goes to 0. As γ approaches 1/2, the equilibrium payoff of the informed converges to the expression in (14), which also goes to $1-\lambda$ as δ goes to 0. Thus, the equilibrium payoffs of both the uninformed and informed types converge to their equilibrium payoff in the benchmark one-round game. The disappearance of the welfare gain when δ goes to 0

[14] In the proof of Proposition 5 we establish that after a reverse disagreement the uninformed types update their belief to 1 and persist with probability 1 in all rounds including the last.

for a fixed deadline confirms our impossibility result in Section 2. One interpretation is that costless straw polls or other forms of cheap talk cannot bring about any improvement in information aggregation or welfare, which is simply another illustration that in the environment of the present model information aggregation is impossible in any incentive compatible outcome without costly delay.

The welfare gain under a finite deadline reappears if we allow the number of voting rounds T to go to infinity as δ goes to 0. We already know, from Proposition 4 that without deadline the welfare gain persists as the delay cost vanishes. The additional observation is that keeping constant the maximum total loss from delay — given by $(T-1)\delta$ for a fixed deadline T — suffices to yield welfare gain when the cost of delay vanishes. One can think of this as a commitment to ending the game by a coin flip if the players fail to reach an agreement after a maximum number of rounds T not exceeding $\Delta/\delta + 1$, where Δ is the cap on the total loss from delay. As above, the upper-bound γ_* on the initial belief of the uninformed in our equilibrium construction converges to $1/2$ as δ goes to 0, and since γ is arbitrarily close to $1/2$, the equilibrium payoff of the informed again approaches the expression in (14). In this case, however, since $T\delta$ converge to a positive limit Δ the informed types can be made strictly better off than in the benchmark one-round game. The logic behind the reappearance of the welfare gain under a finite deadline is in fact the same as why there are welfare gains in our repeated voting game without deadline. Even though in equilibrium the expected duration of disagreement is finite, the assumption of voting until agreeing creates a strictly positive payoff loss from delay as δ goes to 0. This payoff loss reflects the constructive role played by costly delay, even as the per-round delay cost goes to 0.

As a final observation, considering the limit case when the per-round delay cost δ goes to 0 but the maximum total loss from delay is fixed, we can show that a deadline can strictly improve the ex ante payoff of each player relative to no deadline. To make this point it suffices to set the deadline equal to T_*, so that the total loss from delay converges to $\lambda/2$ when δ goes to 0. In our equilibrium construction, as γ goes to zero, the payoff of the uninformed types goes to $1-\lambda$ from (13) while the equilibrium payoff of the informed converges to $1-3\lambda/2$ from (14). In the proof of Proposition 4 we have established that in the game of repeated voting without deadline, as δ goes to 0, the limit value of $U(1/2)$ is $1-\lambda$, and the limit value of $V(1/2)$ is $1-2\lambda(1-\ln 2)$. The uninformed types are unaffected with or without a deadline, but the informed types are strictly better off with the deadline. Thus, for delay cost δ sufficiently small and initial belief γ of uninformed types sufficiently close to 1/2, imposing the deadline T_* strictly improves the ex ante welfare of the players. The logic behind the improvement in the ex ante welfare through a finite deadline lies in the deadline play mentioned earlier. The relatively attractive payoff from the single round game in the last round motivates the uninformed types to stop conceding and wait for the deadline to arrive when there are few rounds remaining. Under an appropriately chosen deadline, the uninformed types are willing to compromise in the last round. Even though the uninformed types do not benefit from such deadline play, the informed types do because the latter care more about making the correct decision.

4. Concluding Remarks

The constructive role of costly delay is robust to the game form in the repeated voting game. Imagine a repeated voting game with costly delay which differs from our game only in that after a reverse disagreement the game ends with an immediate coin toss. Analysis for this game follows in a parallel fashion as what we have done in our paper. It turns out that in the limit of the delay cost converging to zero, this new game has the same equilibrium outcome as our game. The same is true for any game defined by replacing the equilibrium continuation payoff after a reverse disagreement with any feasible continuation payoff. In a sense, the critical part of our construction has to do with the costly delay that arises after a regular disagreement in which the uninformed types vote their ex ante favorite alternative in hope of persuading each other to switch, rather than the costly delay that happens after a reverse disagreement resulting from each tentatively agreeing with the other side. The fact the continuation equilibrium after a reverse disagreement becomes irrelevant when the delay cost goes to zero is the major analytical advantage of using a continuous-time framework. In a follow-up paper (Damiano, Li and Suen 2009), we use such framework to study the issue of optimal deadline. We are able to provide a complete characterization of the equilibrium for any deadline, which allows us to establish that the deadline that maximizes the ex ante payoff of each player, when positive, is the shortest time ensuring efficient information aggregation.

The constructive role of costly delay is illustrated in the present paper with a model most suitable for small committees and bilateral negotiations. In an on-going project (Damiano, Li and Suen 2010), we extend the idea of using costly delay to aggregate information to large elections. We consider two-candidate elections, allowing any number of privately informed voters, and any election rule that requiring a super majority to elect a candidate in the first round and a simple majority in the second and final round after a costly delay. In this extension, there are two states of the world, corresponding to which of the two candidates is the "right" one for all voters, and voters receive binary signals that are independent conditional on the state. We specify preferences and information structures of the voters in such a way that a voter that receives a signal supporting his preference bias votes according to his bias in each round, while a voter that receives the opposite signal chooses his vote by considering pivotal events. In addition to the standard kind of pivotal events in which the vote changes the outcome in the first round of voting (see, e.g., Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996), there is another kind of pivotal events: the vote causes delay and re-voting without affecting the eventual outcome, or avoids delay. As in the present paper, costly delay and re-voting can have a positive impact on the voters' ex ante welfare by improving information aggregation. This is accomplished through careful choices of the super majority rule in the first round and the delay cost, so that all informed voters cast their votes sincerely according to their signals. We can thus arbitrarily closely approximate efficient information aggregation in an environment where aggregation uncertainty about the size of uninformed partisan voters makes information aggregation impossible under the standard plurality voting.

Appendix

PROOF OF LEMMA 1

(i) For $k = 0$, we have $a_0 = 1$ and $b_0 = \delta + \lambda + \sqrt{\delta^2 + \lambda^2} > 2\lambda$. Next, if $a_{k-1} \leq 1$ and $b_{k-1} > 2\lambda$, the two fractions that appear in the difference equation (9) are both positive. Hence $a_k \leq 1$ and $b_k > 2\lambda$ by induction

(ii) For the monotonicity of b_k, we can subtract b_{k-1} from both sides of the second equation in (9) to get:

$$b_k - b_{k-1} = -\frac{1 + \delta - a_{k-1} + b_{k-1}}{b_0 + 1 + \delta - a_k + b_k - 2\lambda}(b_{k-1} - 2\lambda) < 0.$$

To establish the monotonicity of a_k, we use induction. First, it is easy to see that $a_1 < a_0 = 1$. Next, assume that $a_{k-1} < a_{k-2}$. We can write:

$$\begin{aligned} a_k - a_{k-1} &= \frac{(1 + \delta - a_{k-2})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-2} + b_{k-2} - 2\lambda} - \frac{(1 + \delta - a_{k-1})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda} \\ &< \frac{(1 + \delta - a_{k-2})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-2} + b_{k-2} - 2\lambda} - \frac{(1 + \delta - a_{k-1})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + b_{k-2} - 2\lambda} \\ &< \frac{(1 + \delta - a_{k-2})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-2} + b_{k-1} - 2\lambda} - \frac{(1 + \delta - a_{k-2})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-2} + b_{k-2} - 2\lambda} = 0, \end{aligned}$$

where the first inequality follows from $b_{k-1} < b_{k-2}$, and the second inequality follows from the induction hypothesis and the fact that the second term is decreasing in a_{k-1}.

(iii) Solving for the steady state version of the difference equation (9), we obtain the steady state values $a_\infty = 1 + \lambda - \sqrt{\delta^2 + \lambda^2}$ and $b_\infty = 2\lambda$. By the monotonicity of a_k and b_k, these steady state values are also the limit values of the sequence $\{(a_k, b_k)\}$.

(iv) By definition, we have $G_1 \in (0, 1)$. Since $a_{k-1} \leq 1$ and $b_{k-1} > 2\lambda$, an induction argument establishes that $G_k \in (0, 1)$ for all $k \geq 1$. Next, subtracting G_k from both sides of (8), we obtain

$$G_{k+1} - G_k = \frac{1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)}(1 - G_k) > 0.$$

(v) Since G_k is an increasing and bounded sequence, it has a limit value. By part (iii) established above, the limit is 1. *Q.E.D.*

PROOF OF PROPOSITION 2

We first establish the continuity of $U(\gamma)$ for all $\gamma < 1$. For each $k \geq 0$, the function $U(\gamma)$ is trivially continuous at any $\gamma \in (G_k, G_{k+1})$. We show by induction that $U(\gamma)$ is continuous at each G_{k+1}, that is,

$$a_{k+1} - b_{k+1}G_{k+1} = a_k - b_k G_{k+1}.$$

For $k = 0$, we have

$$a_1 - a_0 = -\frac{\delta(b_0 - 2\lambda)}{b_0 + \delta + b_0 - 2\lambda}; \quad b_1 - b_0 = -\frac{(b_0 + \delta)(b_0 - 2\lambda)}{b_0 + \delta + b_0 - 2\lambda}.$$

Therefore,

$$\frac{a_1 - a_0}{b_1 - b_0} = \frac{\delta}{b_0 + \delta} = G_1.$$

Next, denote $w_k = 1 + \delta - a_k + b_k - 2\lambda$. We have

$$a_{k+1} - a_k = \frac{b_0 - 2\lambda}{w_k w_{k-1}}((a_k - a_{k-1})(b_0 + b_{k-1} - 2\lambda) + (1 + \delta - a_{k-1})(b_k - b_{k-1}));$$
$$b_{k+1} - b_k = \frac{b_0 - 2\lambda}{w_k w_{k-1}}((a_k - a_{k-1})(b_{k-1} - 2\lambda) + (b_0 + 1 + \delta - a_{k-1})(b_k - b_{k-1}).$$

Therefore,

$$\frac{a_{k+1} - a_k}{b_{k+1} - b_k} = \frac{1 + \delta - a_{k-1} + G_k(b_0 + b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)} = G_{k+1},$$

where the first equality follows from the induction hypothesis and the second equality follows from the law of motion of the sequence $\{G_k\}$ (equation 8). To show that $U(\gamma)$ is continuous at $\gamma = 1$, we note that $\lim_{k\to\infty} G_k = 1$ and $a_\infty - b_\infty = U(1)$. The continuity and monotonicity of $x(\gamma)$ follows immediately. *Q.E.D.*

PROOF OF LEMMA 2

¿From the proof of Proposition 1, $V(\gamma) = 1$ for $\gamma \in [0, G_1]$. Let $c_0 = 1$ and $d_0 = 0$. We derive difference equations for c_k and d_k by induction. For any $\gamma \in (G_k, G_{k+1}]$, $k \geq 1$, we can write

$$V(\gamma) = x(\gamma)\left(-\delta + c_{k-1} + d_{k-1}\frac{1-\gamma}{\gamma}\frac{1}{x(\gamma)}\right) + 1 - x(\gamma).$$

Using the formula (7) for $x(\gamma)$, we can verify the functional form of V and obtain a pair of difference equations in (c_k, d_k):

$$c_k = 1 - \frac{b_0(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}; \quad d_k = d_{k-1} + \frac{(1+\delta-a_{k-1})(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}.$$

It is straightforward to show by induction that $c_k \leq 1$ and $d_k \geq 0$ for all k. This implies that $\{d_k\}$ is an increasing sequence. Let $w_k = 1+\delta-a_k+b_k-2\lambda$; that $\{c_k\}$ is a decreasing sequence follows immediately by induction if we establish that w_k is decreasing in k. To prove the latter claim, combine equations (9) to obtain

$$w_k = \delta + \frac{(b_0-2\lambda)w_{k-1}}{b_0+w_{k-1}}. \tag{A.1}$$

The derivative of the right-hand-side with respect to w_{k-1} is positive. So $w_{k-1} < w_{k-2}$ implies $w_k < w_{k-1}$. Now,

$$w_1 - w_0 = \delta - \frac{(1+\delta-a_0+b_0)w_0}{b_0+w_0} = -\frac{b_0(b_0-2\lambda)}{2b_0+\delta-2\lambda} < 0.$$

An induction argument then establishes the claim.

The limit value of c_k as k goes to infinity can be verified by using the difference equation for c_k and the limit values of a_k and b_k given in Lemma 1. To verify that the limit value of $d_k(1-G_k)/G_k$, we multiply both sides of the difference equation for d_k by $(1-G_k)/G_k$, and then use Bayes' rule for G_k and $\lim_{k\to\infty} G_k = 1$. *Q.E.D.*

Proof of Proposition 3

We first establish a lemma.

Lemma A. *As δ decreases, for any k: (i) $(1+\delta-a_k+b_k-2\lambda)/b_0$ decreases; (ii) a_k increases; (iii) b_k decreases; (iv) $(1+\delta-a_k)/b_0$ decreases; and (v) $(1+\delta-a_k)/(b_0+1+\delta-a_k+b_k-2\lambda)$ decreases.*

Proof. (i) Let $v_k = 1+\delta-a_k$, $w_k = v_k+b_k-2\lambda$, and $u_k = b_0+w_k$. Recall that $b_0 = 1+\delta-U(1)$, and therefore $\mathrm{d}b_0/\mathrm{d}\delta = 1+\delta/\sqrt{\delta^2+\lambda^2}$. Also, from the proof of Lemma 2 we know that w_k is decreasing in k.

First, we show that w_k is increasing in δ for each k. Take derivative of equation (A.1) to get

$$\frac{\partial w_k}{\partial \delta} = 1 + \frac{w_{k-1}(w_{k-1}+2\lambda)}{(b_0+w_{k-1})^2}\frac{\mathrm{d}b_0}{\mathrm{d}\delta} > 0; \qquad \frac{\partial w_k}{\partial w_{k-1}} = \frac{b_0(b_0-2\lambda)}{(b_0+w_{k-1})^2} > 0.$$

Now, we have

$$\frac{\mathrm{d}w_k}{\mathrm{d}\delta} = \frac{\partial w_k}{\partial \delta} + \frac{\partial w_k}{\partial w_{k-1}}\frac{\mathrm{d}w_{k-1}}{\mathrm{d}\delta}.$$

An induction argument establishes that $\mathrm{d}w_k/\mathrm{d}\delta > 0$ if we can show that $\mathrm{d}w_0/\mathrm{d}\delta > 0$, which is true because $w_0 = \delta + b_0 - 2\lambda$ is increasing in δ.

To establish part (i) of the lemma, we write $f_k = w_k/(b_0+w_k)$. Use equation (A.1) for w_k to write:

$$f_k = \frac{\delta + f_{k-1}(b_0-2\lambda)}{b_0+\delta+f_{k-1}(b_0-2\lambda)}.$$

The partial derivative $\partial f_k/\partial\delta$ has the same sign as

$$b_0 + (2f_{k-1}\lambda - \delta)\frac{\mathrm{d}b_0}{\mathrm{d}\delta}.$$

Since w_k is decreasing in k, we have that f_k is decreasing in k. Therefore, this expression is greater than

$$b_0 + (2f_\infty\lambda - \delta)\frac{\mathrm{d}b_0}{\mathrm{d}\delta},$$

which is positive, where $f_\infty = 1/2(1-\lambda/(1-\lambda+\delta-U(1)))$ is the limit value of f_k as k goes to infinity. It is also easy to see that f_k is increasing in f_{k-1}. The claim then follows if we show $\mathrm{d}f_0/\mathrm{d}\delta > 0$, which we can verify by using the definition of f_0 and taking derivatives with respect to δ.

(ii) We claim that $(b_0-2\lambda)/u_k$ is increasing in δ for each k. To prove it, let $t_k = w_k + 2\lambda$. Write the difference equation for w_k in the form:

$$\frac{t_k}{b_0-2\lambda+t_k} = \frac{(\delta+2\lambda)u_{k-1} + (b_0-2\lambda)(t_{k-1}-2\lambda)}{(b_0+\delta)u_{k-1} + (b_0-2\lambda)(t_{k-1}-2\lambda)}.$$

Let $g_k = (b_0-2\lambda)/u_k = 1 - t_k/u_k$. Then the above equation can be transformed into:

$$g_k = \frac{b_0-2\lambda}{\delta+2b_0-2\lambda-b_0g_{k-1}}.$$

It is clear that $\partial g_k/\partial g_{k-1} > 0$. Moreover, $\partial g_k/\partial \delta$ has the same sign as:

$$-(b_0 - 2\lambda) + (\delta + 2\lambda - 2\lambda g_{k-1})\frac{\mathrm{d}b_0}{\mathrm{d}\delta}.$$

Since g_k is increasing in k, the above expression is greater than

$$-(b_0 - 2\lambda) + (\delta + 2\lambda - 2\lambda g_\infty)\frac{\mathrm{d}b_0}{\mathrm{d}\delta} > 0,$$

where $g_\infty = 1/2(1-\lambda/(1-\lambda+\delta-U(1)))$ is the limit value of g_k as k goes to infinity. So an induction argument will establish the monotonicity of g_k with respect to δ if we establish that $\mathrm{d}g_0/\mathrm{d}\delta > 0$, which we can verify by using the definition of g_0 and taking derivatives with respect to δ.

To establish part (ii) of the lemma, we write the difference equation for a_k as:

$$a_k = 1 - g_{k-1}(1 + \delta - a_{k-1}).$$

Thus,

$$\frac{\mathrm{d}a_k}{\mathrm{d}\delta} = -g_{k-1} - (1 + \delta - a_{k-1})\frac{\mathrm{d}g_{k-1}}{\mathrm{d}\delta} + g_{k-1}\frac{\mathrm{d}a_{k-1}}{\mathrm{d}\delta}.$$

Since $\mathrm{d}a_0/\mathrm{d}\delta = 0$, an induction argument establishes that $\mathrm{d}a_k/\mathrm{d}\delta \leq 0$ for each k.

(iii) We write the difference equation for b_k as:

$$b_k = 2\lambda + g_{k-1}(b_{k-1} - 2\lambda),$$

implying that

$$\frac{\mathrm{d}b_k}{\mathrm{d}\delta} = (b_{k-1} - 2\lambda)\frac{\mathrm{d}g_{k-1}}{\mathrm{d}\delta} + g_{k-1}\frac{\mathrm{d}b_{k-1}}{\mathrm{d}\delta}.$$

We have already shown that $\mathrm{d}g_{k-1}/\mathrm{d}\delta > 0$. Moreover, $\mathrm{d}b_0/\mathrm{d}\delta > 0$. So an induction argument shows that $\mathrm{d}b_k/\mathrm{d}\delta > 0$ for each k.

(iv) From part (ii) we have v_k is increasing in δ for each k. Write the difference equation for a_k as:

$$\frac{v_k}{b_0} = \frac{\delta}{b_0} + g_{k-1}\frac{v_{k-1}}{b_0}.$$

Note that $v_0/b_0 = \delta/b_0$ is increasing in δ. Also, g_{k-1} is increasing in δ. So an induction argument establishes the claim.

(v) First, we claim that $v_k/(b_0 - \lambda)$ is increasing in δ for each k. To prove it, write the difference equation for a_k as:

$$\frac{v_k}{b_0 - \lambda} = \frac{\delta}{b_0 - \lambda} + g_{k-1}\frac{v_{k-1}}{b_0 - \lambda}.$$

Note that $v_0/(b_0 - \lambda) = \delta/(b_0 - \lambda)$ is increasing in δ. So an induction argument establishes the claim.

Next, we show that $(b_k - \lambda)/(b_0 - \lambda)$ is decreasing in δ for each k. We can write the difference equation for b_k as:

$$\frac{b_k - \lambda}{b_0 - \lambda} = \frac{\lambda}{b_0 - \lambda} + \frac{(b_0 - 2\lambda)((b_{k-1} - \lambda) - \lambda)}{(b_0 - \lambda) + (1 + \delta - a_{k-1}) + (b_{k-1} - \lambda)}.$$

Define $p_k = (b_k - \lambda)/(b_0 - \lambda)$ and $q_k = (1 + \delta - a_k)/(b_0 - \lambda)$. Then we can write

$$p_k = (1 - g_{k-1}) + \frac{b_0 - 2\lambda}{b_0 - \lambda}\frac{p_{k-1}}{1 + q_{k-1} + p_{k-1}}.$$

Note that

$$\begin{aligned}
\frac{\partial p_k}{\partial p_{k-1}} &= \frac{b_0 - 2\lambda}{b_0 - \lambda}\frac{1 + q_{k-1}}{(1 + q_{k-1} + p_{k-1})^2} > 0,\\
\frac{\partial p_k}{\partial q_{k-1}} &= -\frac{b_0 - 2\lambda}{b_0 - \lambda}\frac{p_{k-1}}{(1 + q_{k-1} + p_{k-1})^2} < 0,\\
\frac{\partial p_k}{\partial g_{k-1}} &= -\frac{\lambda}{b_0 - \lambda} < 0,\\
\frac{\partial p_k}{\partial b_0} &= -\frac{\lambda}{(b_0 - \lambda)^2}\frac{1 + \delta - a_{k-1} + \lambda}{u_{k-1}} < 0.
\end{aligned}$$

Now,

$$\frac{\mathrm{d}p_k}{\mathrm{d}\delta} = \frac{\partial p_k}{\partial b_0}\frac{\mathrm{d}b_0}{\mathrm{d}\delta} + \frac{\partial p_k}{\partial g_{k-1}}\frac{\mathrm{d}g_{k-1}}{\mathrm{d}\delta} + \frac{\partial p_k}{\partial q_{k-1}}\frac{\mathrm{d}q_{k-1}}{\mathrm{d}\delta} + \frac{\partial p_k}{\partial p_{k-1}}\frac{\mathrm{d}p_{k-1}}{\mathrm{d}\delta}.$$

Since $\mathrm{d}b_0/\mathrm{d}\delta > 0$, $\mathrm{d}g_{k-1}/\mathrm{d}\delta > 0$, $\mathrm{d}q_{k-1}/\mathrm{d}\delta > 0$ and $\mathrm{d}p_0/\mathrm{d}\delta = 0$, an induction argument establishes that $\mathrm{d}p_k/\mathrm{d}\delta < 0$ for each k.

To establish the last part of the lemma, we divide both the denominator and numerator of v_k/u_k by $b_0 - \lambda$ to get:

$$\frac{v_k}{u_k} = \frac{q_k}{1 + q_k + p_k}.$$

Since q_k is increasing in δ and p_k is decreasing in δ, the result follows. *Q.E.D.*

We are now ready to prove the proposition. Fix any $k \geq 1$. Let $w_k = 1+\delta-a_k+b_k-2\lambda$. For the effects on G_k, rewrite the difference equation for G_k as:

$$\frac{G_{k+1}}{1-G_{k+1}} = \frac{1+\delta-a_{k-1}}{b_0} + \frac{b_0+w_{k-1}}{b_0}\frac{G_k}{1-G_k}.$$

From part (i) and part (iv) of Lemma A, both w_k/b_0 and $(1+\delta-a_k)/b_0$ are increasing in δ. It is also clear that G_{k+1} is increasing in G_k. Finally, note that $G_1 = \delta/(b_0+\delta)$ is increasing in δ. An induction argument then establishes that G_k is strictly increasing in δ for each $k \geq 1$.

Next, for the effects on $x(\gamma)$, fix any γ and let

$$x_k(\gamma) = \frac{b_0}{b_0+w_{k-1}} - \frac{1-\gamma}{\gamma}\frac{1+\delta-a_{k-1}}{b_0+w_{k-1}}.$$

Since $x_k(G_{k+1}) = x_{k+1}(G_{k+1})$, and since

$$\frac{\partial x_k(\gamma)}{\partial\gamma} = \frac{1}{\gamma^2}\frac{1+\delta-a_{k-1}}{b_0+w_{k-1}} < \frac{1}{\gamma^2}\frac{1+\delta-a_k}{b_0+w_k} = \frac{\partial x_{k+1}(\gamma)}{\partial\gamma}$$

by part (v) of Lemma A, we obtain $x_k(\gamma) \geq x_{k+1}(\gamma)$ for all $\gamma \leq G_{k+1}$. Iterating the argument establishes that $x_k(\gamma) \geq x_{\tilde{k}}(\gamma)$ for all $\gamma \leq G_{k+1}$ and all $\tilde{k} \geq k$. The same argument also proves that $x_k(\gamma) \geq x_{\tilde{k}}(\gamma)$ for all $\gamma \geq G_k$ and all $\tilde{k} \leq k$. Combining these two results, we have $x_k(\gamma) \geq x_{\tilde{k}}(\gamma)$ for all $\tilde{k}$ if $\gamma \in (G_k, G_{k+1}]$. Now, for any $\tilde{\delta} > \delta$, denote the corresponding equilibrium strategy as $\tilde{x}(\gamma)$, and define $\tilde{x}_k(\gamma)$ analogously. Then, for any $\gamma \in (G_k, G_{k+1}]$,

$$x(\gamma) = x_k(\gamma) \geq x_{\tilde{k}}(\gamma) > \tilde{x}_{\tilde{k}}(\gamma) = \tilde{x}(\gamma),$$

where the first inequality follows because $\gamma \in (G_k, G_{k+1}]$, and the second inequality comes from part (i) and part (v) of Lemma A. Thus, $x(\gamma)$ is decreasing in δ for all γ.

Finally, for the effects on $U(\gamma)$, let $\tilde{d} > d$. Denote the sequence of threshold values of γ corresponding to $\tilde{d}$ as $\{\tilde{G}_k\}$, and denote the corresponding sequence of coefficients of the payoff function U as $\{(\tilde{a}_k, \tilde{b}_k)\}$. Suppose that $\gamma \in (G_k, G_{k+1}]$ while $\gamma \in (\tilde{G}_{\tilde{k}}, \tilde{G}_{\tilde{k}+1}]$. Then

$$\tilde{a}_{\tilde{k}} - \tilde{b}_{\tilde{k}}\gamma < a_{\tilde{k}} - b_{\tilde{k}}\gamma \leq a_k - b_k\gamma,$$

where the first inequality follows from part (ii) and part (iii) of Lemma A, and the second inequality follows from the convexity of $U(\gamma)$. Thus, $U(\gamma)$ is decreasing in δ. *Q.E.D.*

Proof of Proposition 5

We first verify that by using the deadline play constructed in Lemma 3 as the continuation equilibrium, we can make the uninformed indifferent between persisting and conceding in the first round. Under the proposed equilibrium, in the first round the expected payoff to the uninformed from persisting is

$$\gamma(x(-\delta + U^{T-1}(1/2)) + (1-x)) + (1-\gamma)(-\delta + U^{T-1}(1/2)),$$

where $x = (1-\gamma)/\gamma$ is the equilibrium probability of persisting by the opposing uninformed type, and

$$U^{T-1}(1/2) = -(T-2)\delta + 1 - (1+x^0)\lambda/2$$

with $x^0 \leq 1 - 2(T-2)\delta/\lambda$, is the continuation equilibrium payoff after a regular disagreement due to Lemma 3. The expected payoff from conceding in the initial round is

$$\gamma(x(1-2\lambda) + (1-x)(-\delta + U^{T-1}(1))) + (1-\gamma),$$

where $U^{T-1}(1)$ is the continuation payoff to the uninformed after a reverse disagreement, given by

$$U^{T-1}(1) = -(T-2)\delta + (1-\lambda).$$

The above follows because $T \leq T_*$ implies that after the uninformed type updates the belief to 1 upon a reverse disagreement, it is an equilibrium to persist for the rest of the game. Using $x = (1-\gamma)/\gamma$ we can write the indifference condition as

$$2(1-\gamma)(1-\lambda+\delta-U^{T-1}(1/2)) = (2\gamma-1)(1+\delta-U^{T-1}(1)).$$

By Lemma 3, the minimum value of $U^{T-1}(1/2)$ is $1-\lambda$, achieved when $x^0 = 1-2(T-2)\delta/\lambda$. If $x^0 = 0$, then

$$U^{T-1}(1/2) = -(T-2)\delta + 1 - \lambda/2 \geq -(T_*-2)\delta + 1 - \lambda/2 \geq 1 - \lambda + 2\delta.$$

It follows that for any initial belief $\gamma \in [1/2, \gamma^T]$, with γ^T given by

$$\gamma^T = \frac{1}{2} + \frac{\delta}{2(\delta + (1+\delta-U^{T-1}(1)))},$$

there is a unique value of $x^0 \leq 1 - 2(T-2)\delta/\lambda$ such that the above indifference condition is satisfied. Note that γ^T is decreasing in T, and is equal to γ_* when $T = T_*$. Thus, for any deadline T satisfying $2 \leq T \leq T_*$ and any initial belief $\gamma \in [1/2, \gamma_*]$, the equilibrium condition of the uninformed types is satisfied if they persist with probability $x = (1-\gamma)/\gamma$ in the first round.

It remains to argue that given the prescribed strategy of the uninformed, the informed types never want to concede. From Lemma 3 we already know that the informed types have no incentive to concede after the first round. In the first round, the expected payoff in the proposed equilibrium is

$$(1-x) + x(-\delta - (T-2)\delta + (1-x^0) + x^0(1-\lambda)),$$

where $x = (1-\gamma)/\gamma$. The deviation payoff is

$$(1-x)(-\delta - (T-2)\delta + (1-\lambda)) + x(1-2\lambda),$$

because $T \leq T_*$ implies that after a reverse disagreement it is optimal for the informed types to persist for the remainder of the game. It is straightforward to verify that persisting is optimal for the informed types in the first round. *Q.E.D.*

References

Austen-Smith, David and Jeffrey Banks, "Information Aggregation, Rationality and the Condorcet Jury Theorem," *American Political Science Review* 90, 1996, pp. 34–45.

Admati, Anat and Motty Perry, "Strategic Delay in Bargaining," *Review of Economic Studies* 54, 1987, pp. 342–364.

Austen-Smith, David, "Information Transmission in Debate," *American Journal of Political Science* 34(1), 1990, pp. 124–152.

Austen-Smith, David and Timothy J. Feddersen, "Deliberation, Preference Uncertainty, and Voting Rules," *American Political Science Review* 100(2), May 2006, pp. 209–217.

Avery, Christopher and Peter Zemsky, "Option Values and Bargaining Delays," *Games and Economic Behavior* 7, 1994, pp. 139–153.

Cai, Hong-bin, "Delay in Multilateral Bargaining under Complete Information," *Journal of Economic Theory* 93, pp. 260-276.

Chatterjee, Kalyan and Larry Samuelson, "Bargaining with Two-sided Incomplete Information: An Infinite Horizon Model with Alternating Offers," *Review of Economic Studies* 54(2), April 1987, pp. 175–192.

Cho, In-Koo, "Uncertainty and Delay in Bargaining," *Review of Economic Studies* 57(4), October 1990, pp. 575–595.

Cho, In-Koo and David Kreps, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics* 102, pp. 179–221.

Coughlan, Peter J., "In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting," *American Political Science Review* 94(2), June 2000, pp. 375–393.

Cramton, Peter C., "Strategic Delay in Bargaining with Two-Sided Uncertainty," *Review of Economic Studies* 59(1), January 1992, pp. 205–225.

Deneckere, Raymond and Meng-Yu Liang, "Bargaining with Interdependent Values," *Econometrica* 74(5), 2006, pp. 1309-1364.

Damiano, Ettore, Hao Li and Wing Suen, "Optimal Deadlines for Agreements," University of Toronto working paper, 2009.

Damiano, Ettore, Hao Li and Wing Suen, "Sincere Voting in Large Elections," University of Toronto working paper, 2010.

Eso, Peter and Yuk-fai Fong, "Wait and See," Northwestern University working paper, 2007.

Farrell, Joseph, “Cheap Talk, Coordination, and Entry,” *RAND Journal of Economics* 18(1), 1987, pp. 34–39.

Feddersen, Timothy and Wolfgang Pesendorfer, “The Swing Voter’s Curse,” *American Economic Review* 86, 1996, pp. 408–424.

Fershtman, Chaim and Daniel Seidmann, “Deadline Effects and Inefficient Delay in Bargaining with Endogenous Commitment,” *Journal of Economic Theory* 60, 1993, pp. 306–321.

Fuchs, William and Andrzej Skrzypacz, “Bargaining with Arrival of New Traders,” *American Economic Review* 100, 2010, 802–836.

Kennan, John and Robert Wilson, “Bargaining with Private Information,” *Journal of Economic Literature* 31(1), March 1993, pp. 45–104.

Li, Duozhe, “Bargaining with History Dependent Preferences,” *Journal of Economic Theory* 136(1), 2007, pp. 695–708.

Li, Hao, Sherwin Rosen and Wing Suen, “Conflicts and Common Interests in Committees,” *American Economic Review* 91(5), December 2001, pp. 1478–1497.

Ottaviani, Marco and Peter Sorensen, “Information Aggregation in Debate: Who Should Speak First?” *Journal of Public Economics* 81, 2001, pp. 393–421.

Rubinstein, Ariel, “Perfect Equilibrium in a Bargaining Model,” *Econometrica* 50, 1982, pp. 97–109.

Sakovics, Jozsef, ”Delay in Bargaining Games with Complete Information,” *Journal of Economic Theory* 59, 1993, pp. 78–95.

Stahl, Ingolf, *Bargaining Theory,* Stockholm: Economics Research Institute, Stockholm School of Economics, 1972.

Yildiz, Muhamet, “Waiting to Persuade,” *Quarterly Journal of Economics* 119, 2004, pp. 223–248.