Eco220 Exercise Set 9/10.

In the first five questions let $y_i = \gamma x_i + e_i$, $i = 1,..,n$ where y_i and x_i are observed variables, γ is an unobserved parameter and the e_i $i = 1,..,n$ are unobserved random variables with a zero mean and variance $\sigma^2 > 0$ for all i.

1) Derive the ordinary least squares estimator of γ.

2) Stating any necessary additional assumptions prove that the estimator in 1) is unbiased and derive its variance.

3) Stating any further additional assumptions derive a test statistic and its critical value for H_0 $\gamma \geq 1$ against H_1 $\gamma < 1$.

4)The following set of data (y_i, x_i) corresponds to logarithms of expenditures on a commodity and income of individual i respectively so that in the above model γ corresponds to the income elasticity of demand for that comodity. Test the hypothesis that income elasticity of demand is elastic (>1) against the alternative that it is inelastic. Set the size of the test at .05.

i	1	2	3	4	5	6	7	8	9	10
y_i	2	3	2	4	1	3	3	2	1	4
x_i	1.1	1.4	1	2.2	.9	1.5	1.6	1.2	.8	2.1

5). When $\Sigma x_i \neq 0$ an alternative to the OLS estimator you derived in question 1 is given by $\Sigma y_i/\Sigma x_i$, show that this is unbiased and derive its variance. Comment on its efficiency relative to your OLS estimator (hint $\Sigma x_i^2 > n^{-1}(\Sigma x_i)^2$).

In the next set of questions let $y_i = \zeta + \gamma x_i + e_i$, $i = 1,..,n$ where y_i and x_i are observed variables, ζ and γ are unobserved parameters and the e_i $i = 1,..,n$ are unobserved random variables with a zero mean and variance $\sigma^2 > 0$ for all i.

6) Derive the ordinary least squares estimators of ζ and γ.

7) Stating any necessary additional assumptions prove that the estimators in 6) are unbiased and derive their variances.

8) Stating any further additional assumptions derive a test statistic and its critical value for H_0 $\gamma \geq 1$ against H_1 $\gamma < 1$.

9) Stating any further additional assumptions derive a test statistic and its critical value for H_0 $\zeta \geq 0$ against H_1 $\zeta < 0$.

10) Using the data in question 4) test the same hypothesis for this model.

11). Given n observations on the pairs (y_i,x_i) $i = 1,..,n$ with means y and x, three alternative representations of the OLS estimator of the slope of a regression function follow, show that they are all the same estimator.

$$\frac{\sum_{i=1}^{n}(x_i-x)y_i}{\sum_{i=1}^{n}(x_i-x)x_i} \quad ; \quad \frac{\sum_{i=1}^{n}(y_i-y)x_i}{\sum_{i=1}^{n}(x_i-x)^2} \quad ; \quad \frac{\sum_{i=1}^{n}x_iy_i-nxy}{\sum_{i=1}^{n}x_i^2-nx^2}$$

12. 20 identical wheat fields were randomly allocated to one of three fertilizer treatments, the yields (in bushels) in the fields were as follows:
Fields under treatment 1: 40, 42, 39, 39, 37, 43
Fields under treatment 2: 43, 44, 44, 45, 44, 42, 46.
Fields under treatment 3: 35, 37, 35, 38, 38, 32, 36.
Test the hypothesis that the different treatments had no distinguishable effects at the 5% level.

13 For the following data test H_0: $\beta \geq 0$ against H_1: $\beta < 0$ in the regression $Y_i = \alpha + \beta X_i + e_i$, state clearly what assumptions underlay your testing procedure.

Y_i	22	30	25	26	21	26
X_i	10	16	13	14	10	12

14. A regression of the log of earnings (Y) ON AGE (X_1), AGE^2 (X_2) and the number of children (X_3) yielded the following results (standard errors of the parameter estimates in brackets):
For women with no schooling

$$\underset{(0.6006)}{Y = 1.8294} + \underset{(0.3389)}{0.1063\ X_1}, - \underset{(0.0003)}{0.0010\ X_2} - \underset{(0.0494)}{0.0562\ X_3}$$

n=93, $R^2 = .2123$

For women with post graduate experience

$$\underset{(1.6348)}{Y = 3.4360} + \underset{(0.0485)}{0.2226\ X_1}, - \underset{(0.0006)}{0.0023\ X_2} - \underset{(0.0668)}{0.2267\ X_3}$$

n=228, $R^2 = .1632$

Comment on the significance of the impact of children on the respective earnings profiles and the joint significance of the explanatory variables X_1, X_2 and X_3. What do the slopes of the earnings profiles look like at age 25 for the two groups of women? When would you expect their earnings capacity to peak?