ECO220 Exercise Set 7/8.

1. Let x and y be 2 discrete random variables. Using the definitions of E (x) and V (x) prove that:

$$E(a + bx) = a + b\ E(x)$$
$$V(a + bx) = b^2\ V(x)$$
$$V(x + y) = V(x) + V(y) + 2\ COV(xy)$$

2. A bicycle salesman thought that there was some relationship between the colour of bicycle purchased and the gender of the purchaser. He kept data on his sales with the following results. Test the hypothesis that choice of colour is independent of gender. Let the size of the test be .05.

	Red	Green	Blue
Male	48	52	50
Female	42	20	38

3) Letting E(X) be the expected value of X and V(X) be the variance of X for 2 random variables X and Y prove that:

$$V(X-Y) = V(X) + V(Y) - 2COV(X,Y)$$

What does this imply when X and Y are independent.

4. A group of 15 business students were randomly assigned to one of three types of course instruction and their final marks on a common exam were recorded. Test the hypothesis that the instruction method had an influence on their final mark (set the size of the test at .05).

Method	Marks				
Computer Module Based	78	65	44	72	80
Classroom Based	82	78	76	81	76
Experiential Based	65	67	66	64	61

5. Prove that the Total Sum of Squares less the Error Sum of Squares is equal to the Treatment Sum of Squares.

6. By relating the test to Pearson's Goodness of Fit Test show that the degrees of freedom for the χ^2 independence test is (r-1)(c-1) where r and c are the respective number of categories of the two characteristics in question.

7.The chance of getting a driving license is supposed to be the same across all regions. Results from four regions follow, test the hypothesis that this is so letting the size of the test be .05.

	Region 1	Region 2	Region 3	Region 4
Number of Applications	450	150	230	170
Number of Approvals	165	45	65	55

(Hint, set the test up as a test of the independence between licence approval and region.)

8. A farmer employed three different types of fertilizer to various fields on his farm, the yields per acre of the fields under each type of fertilizer were as follows:
Fertilizer 1: 24, 28, 23, 27, 27, 27.
Fertilizer 2: 25, 22, 22, 23.
Fertilizer 3: 29, 27, 28.
Test the hypothesis that the fertilizers had different effects. Set the size of the test at .05.

9. A survey of 200 movie goers discriminated by age and asked whether the soundtrack was too loud with the following results:

	< 24 years	24 to 44 years	> 44 years
Not loud enough	30	29	12
O.K.	20	28	16
Too Loud	14	27	24

Are attitudes towards loudness of the sound track independent of age (let $\alpha = .05$).

11). In an analysis of variance problem there are T treatments indexed t = 1,..,T with n_t observations indexed i = 1,..,n_t on treatment t so that Y_{it} is the i'th observation on the t'th treatment so that the total sum of squares (TOSS) and error sum of squares (ERSS) are given by:

$$TOSS=\sum_{t=1}^{T}\sum_{i=1}^{n_t}(Y_{it}-\overline{Y})^2;\quad ERSS=\sum_{t=1}^{T}\sum_{i=1}^{n_t}(Y_{it}-\overline{Y}_t)^2$$

where:

$$\overline{Y}=\frac{\sum_{t=1}^{T}\sum_{i=1}^{n_t}Y_{it}}{\sum_{t=1}^{T}n_t};\quad \overline{Y}_t=\frac{\sum_{i=1}^{n_t}Y_{it}}{n_t}$$

derive the Treatment Sum of Squares and its degrees of freedom.