ECO220 Exercise Set 6.

1. Derive the formula for determining the sample size n when one wishes to test $H_0 : \mu \geq \mu_0$ against H_1 $\mu < \mu_0$ with size α and type 2 error probability β where μ_1 is the value of interest in the alternative and σ is known. Suppose $\sigma^2 = 100$, $\mu_0 = 0$, $\mu_1 = -1$, and $\alpha = \beta = .05$ what would the sample size be?.

2.a) In harness racing horses are drawn randomly for starting positions, position 1 is on the inside track, position 6 is on the outside track. In 180 six horse races, position 1 won 24 times, position 2 won 36 times, position 3 won 39 times and positions 4, 5 and 6 each won 27 times. Test the hypothesis that there is no advantage to starting position (i.e. that each starting position has the same chance of winning) setting the size of the test at 0.05.
2b) Suppose that the following season exactly the same pattern of winning positions occurred, perform the same test at the same size based upon the 360 races. What is the conclusion now? Explain the difference between this and your answer to part a).

3. In a two sectioned economics course one section with 19 students obtained an average mark of 74 with a sample variance of 81, the other section with 14 students obtained an average mark of 70 with a sample variance of 100.
a) Test the hypothesis that the marks variance in the first class is the same as the marks variance in the second class.
b) Test the hypothesis that the true abilities of the two classes were the same, setting the size of the test at .05.

4. A random sample of size 21 yields a sample mean of 10 and a sample variance of 9, test the hypothesis that the true mean is equal to 12.5 set the size of the test at .05.

5. A six sided die is rolled 180 times, 1 occurred 24 times, 2 occurred 36 times, 3 occurred 39 times and 4, 5 and 6 each occurred 27 times. Test the hypothesis that the die is fair (i.e. that each of the numbers is equi probable) setting the size of the test at 0.05.

6a. The poverty rate in a city is thought to be 0.15. After some poverty reduction programs were introduce a random sample of 400 inhabitants was taken and 50 of them were deemed to be in poverty. Test the hypothesis that the policies were a failure by performing a one sided lower tailed test of population proportions setting the size of the test at .05.
6b. What is the power of the test when the true poverty rate after the programs were introduced was .1?

7. In a statistics course the 9 girls in the class obtained an average mark of 72 with a sample variance of 81, the 16 boys in the class obtained an average mark of 70 with a sample variance of 81. Are the girls smarter than the guys? (set the size of the test at .025)

8. Samples independently drawn from populations A and B yielded the following:
$$\bar{x}_A = 24.6 \quad \bar{x}_B = 28.2 \quad \sum(x_{iA} - \bar{x}_A)^2 = 20.3 \quad \sum(x_{iB} - \bar{x}_B)^2 = 30.1 \quad n_A = 10 \quad n_B = 15.$$
Stating any necessary assumptions test (at $\alpha = .05$) H_0 $\mu_A = \mu_B$ against H_1 $\mu_A \neq \mu_B$.

9. In a sample of one hundred week long observations the number of crashes observed at the intersection of Mississauga Road and Dundas was as follows:

Number of crashes observed during a week.	Number of weeks observed
0	25
1	53
2	14
3	6
4	2

a) Test the hypothesis that the weekly number of crashes is distributed as a Poisson distribution with a mean of 1.8 setting the size of the test at .05.

b) Test the hypothesis that the distribution is Poisson setting the size of the test at .05.

10. A 1 meter long high pressure water hose is an important component in a dishwashing machine. In 200 hose breakdowns service engineers have recorded 45 fractures in the first 10 centimeters and 32 fractures in the last 10 centimeters the remaining 123 fractures were in the middle 80 centimeters of the hose. Test the hypothesis that the fractures are uniformly distributed throughout the hose setting the size of the test at .05.