Exercise 3

1. poisson dist^{n} : $P(X=x) = \frac{\lambda^x}{x!} e^{-\lambda}$

$$E(X) = Var(X) = \lambda .$$

$$P(X=x) = \frac{2^x}{x!} e^{-2}$$

$$E(X) = Var(X) = 2 .$$

$$P(X=0) = \frac{2^0}{0!} e^{-2} = e^{-2}$$

$$P(X=1 \mid X \geq 1) = \frac{P(X=1 \ \& \ X \geq 1)}{P(X \geq 1)} = \frac{P(X=1)}{1-P(X=0)} = \frac{\frac{2^1}{1!} e^{-2}}{1-e^{-2}} = \frac{2e^{-2}}{1-e^{-2}}$$

2. $X \sim N(65, 100)$

① $P(X \leq 50) = P\left(\frac{X-65}{\sqrt{100}} \leq \frac{50-65}{\sqrt{100}}\right) = P\left(\frac{X-65}{10} \leq -1.5\right)$

$\frac{X-65}{10} \sim N(0,1)$

so check the standard normal dist^{n} table and get the solution

② $P(75 \leq X \leq 85) = P\left(\frac{75-65}{\sqrt{100}} \leq \frac{X-65}{\sqrt{100}} \leq \frac{85-65}{\sqrt{100}}\right)$

$$= P\left(1 \leq \frac{X-65}{\sqrt{100}} \leq 2\right) = 0.9772 - 0.8413$$

③ $P(X \geq x) = 10\% = P\left(\frac{X-65}{\sqrt{100}} \geq \frac{x-65}{\sqrt{100}}\right) = 10\%$

$\therefore \frac{x-65}{\sqrt{100}} = 1.28 \Rightarrow x = 77.8$

④ $P(X \geq 90 \mid X \geq 65) = \frac{P(X \geq 90 \ \& \ X \geq 65)}{P(X \geq 65)} = \frac{P(X \geq 90)}{0.5}$

$= \frac{P\left(\frac{X-65}{\sqrt{100}} \geq \frac{90-65}{\sqrt{100}}\right)}{0.5} = 2P\left(\frac{X-65}{10} \geq 2.5\right)$

$\downarrow$

check table

3 a). $E(X) = \sum x p(x) = 2000000 * 0.5 + 3000000 * 0.3 + 5000000 * 0.2$

$= 2900000$

$Var(X) = E(X^2) - [E(X)]^2$

$= (2000000)^2 * 0.5 + (3000000)^2 * 0.3 + (5000000)^2 * 0.2 - (2900000)^2$

$= 9.7 * 10^{12} - (2900000)^2 = 1.29 * 10^{12}$

b). $y = 0.2X - 56000$

$E(y) = E(0.2X - 56000) = 0.2E(X) - 56000 = 524000$

$Var(y) = Var(0.2X - 56000) = (0.2)^2 Var(X) = 5.16 * 10^{10}$

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a). $Z \sim N(0,1)$

$P(Z>1) = 1 - P(Z\leq 1) = 1-0.8413$

$P(0.5<Z<2) = P(Z<2) - P(Z\leq 0.5) = 0.9772 - 0.6915$

b). $X \sim N(10,16)$

$P(12<X<18) = P\left(\frac{12-10}{\sqrt{16}} < \frac{X-10}{\sqrt{16}} < \frac{18-10}{\sqrt{16}}\right)$

$= P(0.5 < Z < 2) = 0.9772 - 0.6915$.

$P(X<b) = 0.3 \Rightarrow P\left(\frac{X-10}{\sqrt{16}} < \frac{b-10}{\sqrt{16}}\right) = 0.3$

$\Rightarrow P\left(Z < \frac{b-10}{4}\right) = 0.3$

$\Rightarrow \frac{b-10}{4} = -0.52 \Rightarrow b = 7.92$

5①. $X \sim B(5, 0.005)$

$P(X=0) = C_5^0 (0.005)^0 * (1-0.005)^{5-0} = 0.9752$

② $P = (1-0.005)^4 * 0.005 = 0.0049$.

6. $X \sim N(500, 3600)$

a). $P(X \geq 530) = P\left(\frac{X-500}{\sqrt{3600}} \geq \frac{530-500}{\sqrt{3600}}\right)$

$= P(Z \geq 0.5)$

$= 1 - 0.6915 = 0.3085$

b). $P(X \geq x) = 10\%$

$\Rightarrow P\left(\frac{X-500}{\sqrt{3600}} \geq \frac{x-500}{\sqrt{3600}}\right) = 10\%$

$\Rightarrow P\left(Z \geq \frac{x-500}{60}\right) = 10\%$

$\Rightarrow \frac{x-500}{60} = 1.28 \Rightarrow x = 576.8$

c). $P(\bar{X} > 520)$

$X \sim N(500, 3600)$ $E( )$

$\bar{X} = \frac{\Sigma X}{36}$

$\therefore E(\bar{X}) = E\left(\frac{\Sigma X}{36}\right) = \frac{1}{36} * \Sigma E(X) = E(X) = 500$

$Var(\bar{X}) = Var\left(\frac{1}{36}\Sigma X\right) = \frac{1}{(36)^2} \Sigma Var(X) = \frac{1}{36} Var(X) = \frac{1}{36} * 3600 = 100$

$\therefore \bar{X} \sim N(500, 100)$

$\therefore P(\bar{X} > 520) = P\left(Z > \frac{520-500}{\sqrt{100}}\right) = P(Z > 2)$

7. a) $\frac{25}{2*100} = 0.125$

b) P(leak in the 1st 10 c m | leak in the 1st 30 cm)

$$= \frac{P(\text{leak in the 1st 10cm \& 1st 30 cm})}{P(\text{leak in the 1st 30 cm})}$$

$$= \frac{P(\text{leak in the 1st 10 cm})}{P(\text{leak in the 1st 30cm})} = \frac{\frac{10}{2*100}}{\frac{30}{2*100}} = \frac{1}{3}.$$

c). P (leak in the mid 10 cm | not leak in the 1st & last 10 cm)

$$= \frac{P(\text{leak in the mid 10 cm})}{P(\text{not leak in the 1st \& last cm})}$$

$$= \frac{P(\text{leak in the mid 10 cm})}{1 - P(\text{leak in the 1st 10cm}) - P(\text{leak in the last cm})}$$

$$= \frac{\frac{10}{200}}{1 - \frac{10}{200} - \frac{10}{200}} = \frac{1}{18}$$

d) $200 * 0.2475 = 49.5$

8. $X \sim B(12, 0.1)$

$$P(X=9) = C_{12}^{9}(0.1)^9 * (1-0.1)^{12-9}$$

9 $P(X=x) = \frac{\lambda^x}{x!}e^{-\lambda} = \frac{0.8^x}{x!}e^{-0.8}$

a). $P(X=1) = \frac{0.8^1}{1!}e^{-0.8} = 0.8e^{-0.8}$

b) $P(X \geqslant 1) = 1-P(X=0) = 1-\frac{0.8^0}{0!}e^{-0.8} = 1-e^{-0.8}$

c) $P(X \geqslant 2 \mid X \geqslant 1) = \frac{P(X \geqslant 2 \;\&\; X \geqslant 1)}{P(X \geqslant 1)} = \frac{P(X \geqslant 2)}{P(X \geqslant 1)} = \frac{1-\cancel{P(X=0)}-P(X=1)}{1-P(X=0)}$

$$= \frac{1-e^{-0.8}-0.8e^{-0.8}}{1-e^{-0.8}}$$

d). $P(X_1=0) * P(X_2=2) + P(X_1=1) * P(X_2=1) + P(X_1=2) * P(X_2=0)$

$$= e^{-0.8} * \frac{0.8^2}{2!}e^{-0.8} + 0.8e^{-0.8} * 0.8 * e^{-0.8} + \frac{0.8^2}{2!}e^{-0.8} * e^{-0.8}$$

10. There's obviously a typo in the question, so I assume it takes on the value "1" (instead of 0) with probability 0.2.

suppose the prob. of taking on value 2 is p, then prob. of taking on value 3 is $1-0.2-p=0.8-p$.

$$E(X)=1*0.2+2*p+3*(0.8-p)=1.8$$

$$\Rightarrow 0.2+2p+2.4-3p=1.8$$

$$\Rightarrow p=0.8$$

$\therefore$

X	1	2	3
P(x)	0.2	0.8	0

$$Var(X)=E(X^2)-[E(X)]^2=1^2*0.2+2^2*0.8+3^2*0-(1.8)^2$$

$$=0.2+3.2-3.24=0.16$$