Ex2:

Ex2-A

1. $P(A\cup B) = 4(1-P(A\cup B)) \Rightarrow P(A\cup B) = 0.8$

$P(A) = 2P(A\cap B)$

$P(B) = 3P(A\cap B)$

$P(A\cup B) = P(A)+P(B)-P(A\cap B) = 2P(A\cap B)+3P(A\cap B)-P(A\cap B)$

$\Rightarrow P(A\cap B) = 0.2$

$P(A) = 0.4$

$P(B) = 0.6$.

a). $P(A) = 0.4$

b). $P(B) = 0.6$

c). $P(A\cup B) - P(A\cap B) = 0.8 - 0.2 = 0.6$

d). $P(A) - P(A\cap B) = 0.4 - 0.2 = 0.2$

e). $P(B) - P(A\cap B) = 0.6 - 0.2 = 0.4$

f). $P(B|A) = \dfrac{P(A\cap B)}{P(A)} = 0.5$

g). $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{1}{3}$

h) $P(B|A^c) = \dfrac{P(A^c\cap B)}{P(A^c)} = \dfrac{P(B)-P(A\cap B)}{1-P(A)} = \dfrac{2}{3}$

i). $P(A|B^c) = \dfrac{P(A\cap B^c)}{P(B^c)} = \dfrac{P(A)-P(A\cap B)}{1-P(B)} = 0.5$

j) $P(A\cap B) = 0.2$

$P(A)P(B) = 0.4 \times 0.6 = 0.24 \neq P(A\cap B) \Rightarrow$ Not indep.

k). $P(A\cap B) = 0.2 \neq 0 \Rightarrow$ not mutually exclusive.

Ex2-(2).

2). $P(A\cup B) = 1 - P(A\cup B) \Rightarrow P(A\cup B) = 0.5$

$P(A\cap B) = 0.25$

$P(A) = 0.25$

$P(B) = 0.375$

a) Same calculation as i).

$\wr$ $\qquad\qquad$ $\wr$

k). $\qquad\qquad$ ''

You will find f). g). have same solution as i), because $P(A) = 2P(A\cap B)$ $P(B) = 3P(A\cap B)$ unchanged.

3) a). $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

Proof: $A = (A\cap B) \cup (A\cap B^c)$

$(A\cap B) \cap (A\cap B^c) = \phi$

By A2 $\quad P(A) = P(A\cap B) + P(A\cap B^c)$

$P(B) = P(B\cap A) + P(B\cap A^c)$

$A\cup B = (A\cap B) \cup (A\cap B^c) \cup (B\cap A^c)$

mutually exclusive

$\therefore P(A\cup B) = P(A\cap B) + P(A\cap B^c) + P(B\cap A^c)$

$= P(A\cap B) + [P(A) - P(A\cap B)] + [P(B) - P(A\cap B)]$

$= P(A) + P(B) - P(A\cap B)$

b). $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A) + P(B) - P(A\cup B)^{\geq 0}}{P(B)_{>0}} \leq \dfrac{P(A)+P(B)}{P(B)} = 1 + \dfrac{P(A)}{P(B)}$

Ex 2 - ③

c). $\dfrac{P(A|B)}{P(B|A)} = \dfrac{\dfrac{P(A\cap B)}{P(B)}}{\dfrac{P(A\cap B)}{P(A)}} = \dfrac{P(A)}{P(B)} = \dfrac{1-P(A^c)}{1-P(B^c)}$

4). ① $C_4^3 \left(\frac{1}{20}\right)^3 * \frac{19}{20} = \frac{76}{20^4}$

② $\left(\frac{19}{20}\right)^4$

③ P(picked 3 of 4 times | gets picked)

$= \dfrac{P(\text{picked 3 of 4 times} \cap \text{gets picked})}{P(\text{gets picked})}$

$= \dfrac{76}{20^4} \Big/ \left(\dfrac{19}{20}\right)^4 = \dfrac{76}{19^4}$

5). $P(A) = 0.5$

$P(B) = 0.4$

$P(A\cap B) = 0.3$

$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.6$

$P(A^c|B^c) = \dfrac{P(A^c\cap B^c)}{P(B^c)} = \dfrac{1-P(A\cup B)}{1-P(B)} = \dfrac{2}{3}$

$P(A\cap B) = 0.3 \neq P(A)P(B) = 0.2 \Longleftrightarrow$ Not indep

6). P(Red from bag 1 $\cap$ red from bag 2) + P(white from bag 1 $\cap$ Red from 2)

= P(Red from 1) * P(red from 2 | red from 1) + P(red from 2 | white from 1) * P(white from 1)

$= \frac{1}{3} * \frac{3}{4} + \frac{2}{3} * \frac{2}{3} = \frac{25}{36}$

Ex2 - 4

7) a). $A = \{HHH \quad HHT \quad HTH \quad THH \quad HTT \quad THT \quad TTH\}$

$B = \{HHH, HTT, THT, TTH\}$

$A \cup B = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

$A^c = \{TTT\}$

$A \cap B = \{HHH, HTT, THT, TTH\}$

b). $P(A) = \frac{7}{8}$

$P(B) = \frac{4}{8}$

$P(A \cup B) = \frac{7}{8}$

$P(A^c) = \frac{1}{8}$

$P(A \cap B) = \frac{1}{2}$

c). $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{1}{2} - \frac{1}{2} = \frac{7}{8}$

d). Not mutually exclusive as $A \cap B \neq \phi$.

8). $P(A \cup B) = 0.3$

$P(A \cap B^c) = P(A) - P(A \cap B) = 0.1$

$\underset{0.3}{\underset{\parallel}{P(A \cup B)}} = \underbrace{P(A) + P(B) - P(A \cap B)}_{0.1} \Rightarrow P(B) = 0.2$

$P(A \cap B) = 0.1 \Rightarrow P(B) - P(A \cap B) = P(B \cap A^c) = 0.2 - 0.1 = 0.1$

9) a). $\frac{4}{52}$ Ex 2-(5).

b). $\frac{13}{52}$

c). $\frac{12}{52}$

d). $1 - \frac{12}{52} = \frac{10}{13}$

e). $\frac{40}{52}$

f). $\frac{28}{40}$

g). $\frac{4}{52}$

h). $\frac{13}{52}$

i) $\frac{4}{12}$

10). A & B can not happen at the same time.
~~It's~~ they are mutually exclusive by def^{n}.