

Eco220. Exercise set 2

1. The probability that either Jack and/or Jill complete the statistics course is four times the probability that neither of them will complete it, the chance that Jack completes the course is twice the chance that they both complete it and the chance that Jill completes the course is three times the chance that they both complete it.

Find:

- a) $P(\text{Jack completes the course})$
- b) $P(\text{Jill completes the course})$
- c) $P(\text{Jack or Jill but not both complete the course})$
- d) $P(\text{Jack completes the course but Jill does not})$
- e) $P(\text{Jill completes the course but Jack does not})$
- f) $P(\text{Jill completes the course} \mid \text{Jack completes the course})$
- g) $P(\text{Jack completes the course} \mid \text{Jill completes the course})$
- h) $P(\text{Jill completes the course} \mid \text{Jack does not complete the course})$
- i) $P(\text{Jack completes the course} \mid \text{Jill does not complete the course})$
- j) Are the events “Jill completes the course” and “Jack completes the course” independent
- k) Are the events “Jack completes the course” and “Jill completes the course” mutually exclusive

2) Suppose in question 1 the probability that either Jack and/or Jill complete the statistics course is equal to the probability that neither of them will complete it but everything else remains the same. Do parts a) to i) what answers remain unchanged and why?

3) Prove the following

- a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- b) If $P(B) > 0$, $P(A|B) \geq 1 + P(A)/P(B)$
- c) If $P(A) > 0$ and $P(B) > 0$, $P(A|B)/P(B|A) = (1 - P(A^c))/(1 - P(B^c))$

4). A professor said he would pick one student randomly with replacement from a class of 20 to answer each of 4 questions and Wendy got picked three of the four times. What is the chance of this happening? What is the chance that a student would not to be picked at all? Given that a student gets picked, what is the chance of her being picked three of four times?

5). The chance that Jimmy passes his English Exam is .5 and the chance that he passes his French exam is .4, the chance that he passes them both is .3. Given that he fails his French exam what is the chance that he will fail his English Exam. Are passing or failing the two exams independent or not? Explain why.

6). Bag one contains two white chips and one red chip and bag two contains two red chips and one white chip. A chip is drawn from bag one, if it is red it is placed in bag two, if it is white it is returned to bag one. Then a chip is drawn from bag two, what is the probability of the chip being drawn from bag two being red.

7). A fair coin is tossed three times and the events A and B are defined as follows:

A: {At least one head is observed}
B: {The number of heads observed is odd}

- a. Identify the sample points in the events A , B , $A \cup B$, A^c , and $A \cap B$.
- b. Find $P(A)$, $P(B)$, $P(A \cup B)$, $P(A^c)$, and $P(A \cap B)$ by summing the probabilities of the appropriate sample points.
- c. Find $P(A \cup B)$ using the additive rule. Compare your answer to the one you obtained in part b.
- d. Are the events of A and B mutually exclusive? Why?

8) Let A and B be two events defined on S . If the probability that at least one of the events occurs is 0.3 and the probability that A occurs but B does not is 0.1 what is the probability of B ? If the probability of A and B occurring is 0.1 what is the probability that B occurs but A does not?

9. A card is drawn from a standard deck of cards.

- a) What is the chance that it is an ace.
- b) What is the chance that it is a diamond
- c) What is the chance that it is a picture
- d) What is the chance that it is at least a 4.
- e) What is the chance that it is at most a 10.
- f) What is the chance that it is at least a 4 given that it is not a picture.
- g) What is the chance that it is an ace given that it is a diamond.
- h) What is the chance that it is a diamond given that it is an ace.
- i) What is the chance that it is a queen given that it is a picture.
- j) Comparing a) and b) with g) and h) what does this tell you about values of cards and suits of cards?

10) It was said that a certain president of the United States "Could not walk and chew gum at the same time". If it were true for that president would "walking" and "chewing gum" be independent events or would they be mutually exclusive? Why?