University of Toronto Department of Economics ECO 2061H Economic Theory - Macroeconomics (MA) Winter 2012

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Answers to Assignment 2

1. The household's problem is,

$$\max_{c_t,\ell_t,I_t} E_0 \sum_{t=0}^{\infty} e^{-\rho t} \left[\ln \left(c_t \right) + \frac{(1-\ell_t)^{\theta}}{\theta} \right]$$

s.t.

$$c_t + I_t = w_t \ell_t (1 - \tau_t) + R_t K_t + T_t$$
$$K_{t+1} = (1 - \delta) K_t + I_t$$

where T_t are lump-sum taxes, and $R_t = r_t + \delta$. Set-up the Lagrangian,

$$\pounds = E_0 \left\{ \sum_{t=0}^{\infty} e^{-\rho t} \left[\ln \left(c_t \right) + \frac{(1-\ell_t)^{\theta}}{\theta} \right] + \sum_{t=0}^{\infty} \lambda_t \left[w_t \ell_t (1-\tau_t) + R_t K_t + T_t + (1-\delta) K_t - c_t - K_{t+1} \right] \right\}$$

(a) The first order conditions are,

$$E_t \left\{ e^{-\rho t} \frac{1}{c_t} - \lambda_t \right\} = 0$$
$$E_t \left\{ -e^{-\rho t} \left(1 - \ell_t \right)^{\theta - 1} + \lambda_t w_t (1 - \tau_t) \right\} = 0$$
$$E_t \left\{ -\lambda_t + \lambda_{t+1} [R_{t+1} + 1 - \delta] \right\} = 0$$

(b) Combining the first and the third, the second and the third and the first and the second you get respectively,

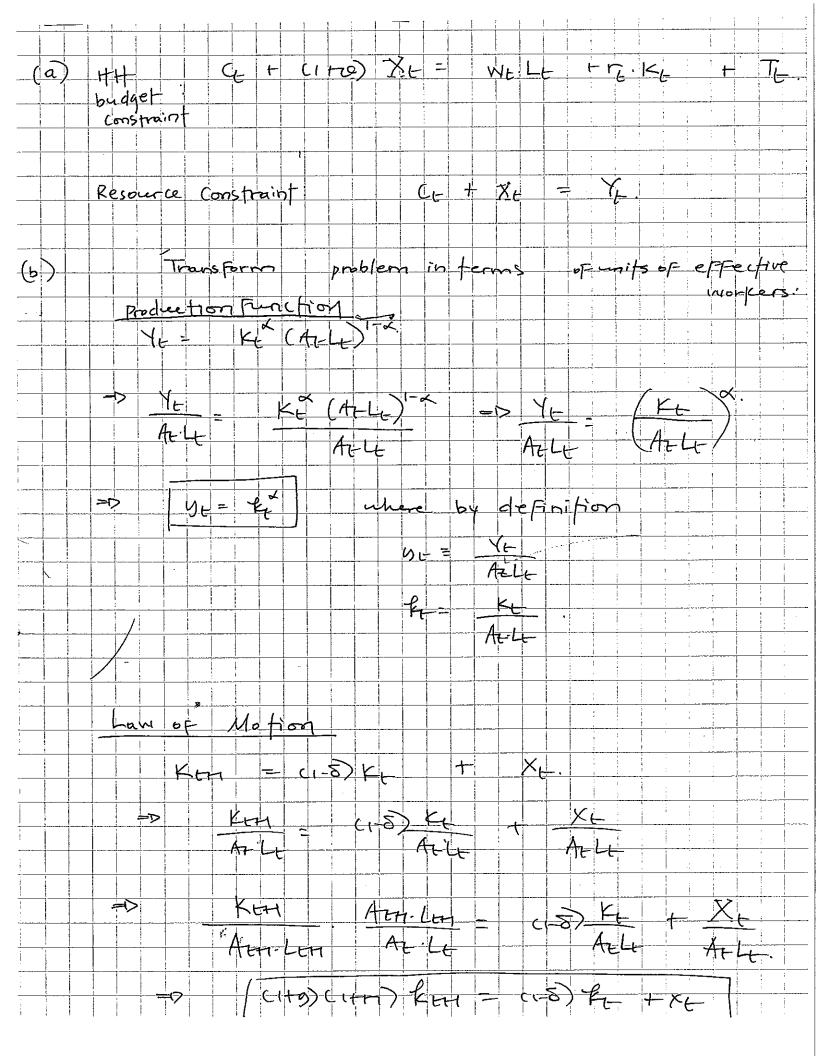
$$\frac{1}{c_t} = e^{-\rho} E_t \left\{ \frac{1}{c_{t+1}} (1+r_{t+1}) \right\}$$
$$e^{-\rho} E_t \left\{ (1+r_{t+1}) \left(\frac{1-\ell_{t+1}}{1-\ell_t} \right)^{\theta-1} \frac{w_t}{w_{t+1}} \frac{(1-\tau_t)}{(1-\tau_{t+1})} \right\} = 1$$
$$(1-\ell_t)^{\theta-1} = \frac{w_t (1-\tau_t)}{c_t}$$

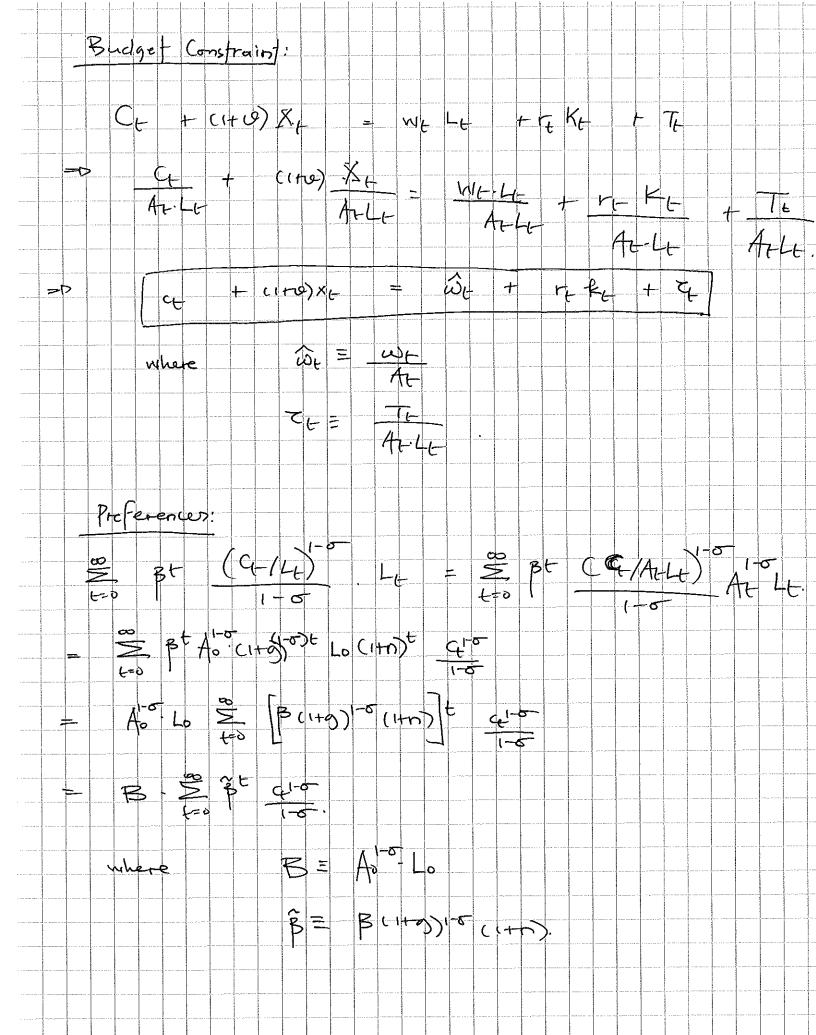
where I have used that $R_t = r_t + \delta$. The first of these equations reflects the trade-off between consumption today and consumption tomorrow. The second reflects the trade-off between leisure (labor supply) today and leisure (labor supply) tomorrow. The third reflects the trade-off between leisure and consumption today. See class notes for the intuition.

(c) We can rewrite the Euler equation as,

$$\frac{1}{c_t} = e^{-\rho} \left\{ E_t \left(\frac{1}{c_{t+1}} \right) E_t (1 + r_{t+1}) + Cov \left(\frac{1}{c_{t+1}}, 1 + r_{t+1} \right) \right\}$$

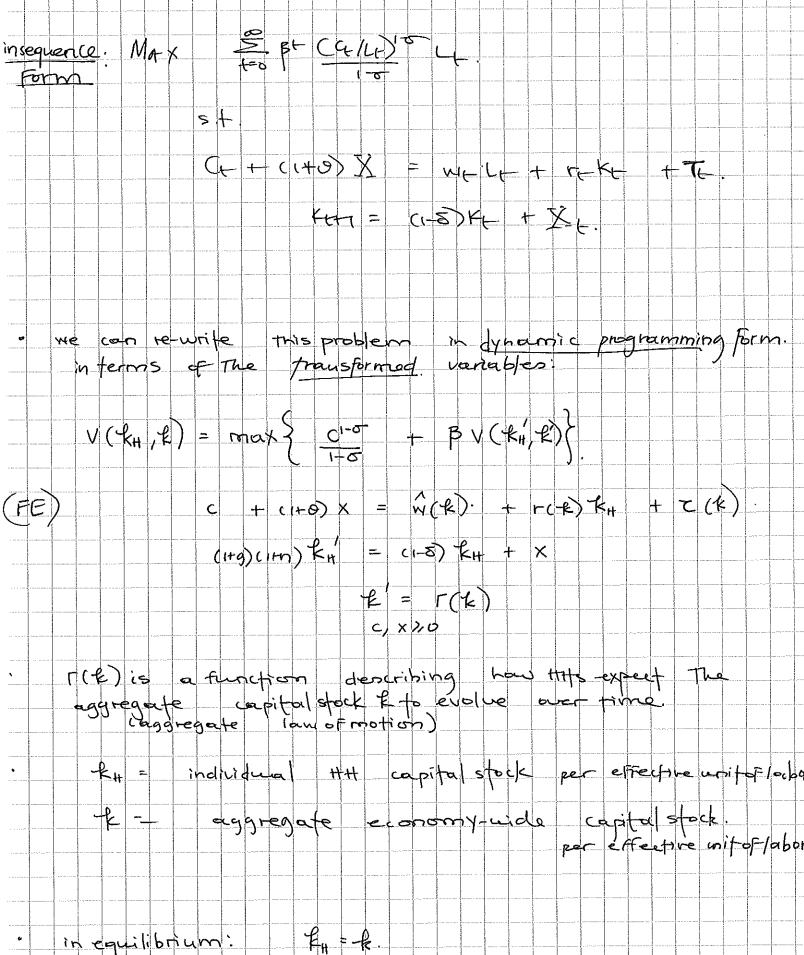
Denote by c_{t+1}^+ the level of consumption for which $Cov(c_{t+1}^+, 1 + r_{t+1}) > 0$ and thus $Cov(\frac{1}{c_{t+1}^+}, 1 + r_{t+1}) < 0$. Denote by c_{t+1}^- the level of consumption for which $Cov(c_{t+1}^-, 1 + r_{t+1}) < 0$ and thus $Cov(\frac{1}{c_{t+1}^-}, 1 + r_{t+1}) > 0$. From the right hand side of the above equation this implies that $\frac{1}{c_t^+} < \frac{1}{c_t^-}$, and consequently $c_t^- < c_t^+$. Intuitively: when the covariance between consumption and the interest rate is positive (you get a good return in good times) then the marginal utility of consumption and the interest rate is negative. Consequently the expected marginal benefit from giving up more consumption today (and thus saving) is lower; consequently you will save less and consume more today.





Firm Problem (static each period) $MAX \quad \begin{cases} K_{t}^{2}(A_{t}L_{t})^{1-\alpha} - w_{t} \cdot L_{t} - r_{t}K_{t} \end{cases}$ FONC: $K_{E}: \propto K_{F}^{\infty 1} (A_{E}L_{E})^{1-\alpha} = \Gamma_{E}.$ $L_{F}: (1-\alpha) K_{E}^{\infty} A_{E}^{1-\alpha} L_{E}^{-\alpha} = W_{E}.$ Re-write these conditions interms of transformed variables: $T_{L} = \propto \begin{pmatrix} E_{L} \\ A_{L} \\ -\infty \end{pmatrix} \qquad = \qquad T_{L} = \propto \cdot E_{L} \\ T_{L} = \propto \cdot E_{L} \\ T_{L} = \sim \cdot E_{L} \\ T_{L}$ uhere as defined above: ME ME. At We can thus write factor prices as functions of the economics acguegate capital. $r(\mathbf{k}) = \propto \mathbf{k}^{\mathbf{x}-1}$ $\hat{\omega}(\mathbf{E}) = (\mathbf{F}_{\mathbf{Z}})\mathbf{E}^{\mathbf{X}}$





in equilibrium:

Ref. of RCE:

A RCE is a list of functions V(KIT, E) g"(KIT, E), g"(

• Given $\hat{\omega}(k), r(k), z(k)$ and the orggregate law of motion r(k) the value tunction $V(k_{H}, k)$ solven the ATFS (FE): where $g^{c}(k_{H}, k), g^{v}(k_{H}, k), g^{k'}(k_{H}, k)$ are the optimal decision rules.

given W(R), r(R) the decision Function FE(R) solver the Finis problem.

· markets clear

 $g^{\epsilon}(k,k) + g^{\star}(k,k) = k^{\star}.$ $k = f^{k}(k)$

the government's budget constraint is satisfied

 $(g, g^{\times}(\mathcal{L}, \mathcal{E})) = z(\mathcal{L})$

indir. and acy. laws of motion are consistent:

 $g^{E'}(-k, k) = \Gamma(-k)$

